Lasers and Optoelectronics

ECE 4300

Fall 2016

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About ECE 4300: Lasers and Optoelectronics

Instructors:
Prof. Debdeep Jena (djena@cornell.edu), ECE and MSE, Cornell University
Prof. Clif Pollock, ECE, Cornell University.

Course Website:
https://djena.engineering.cornell.edu/2016_ece4300.htm
Homework assignments and postings will appear on this website. Please bookmark it.

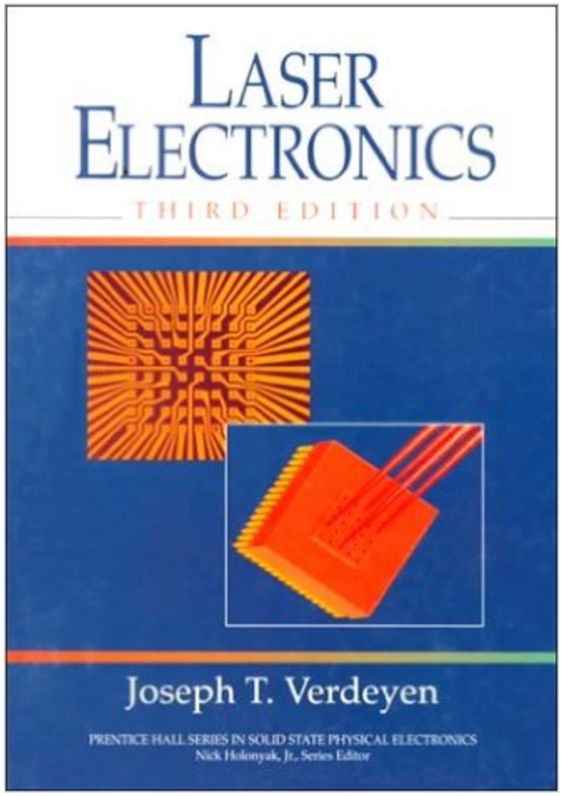
Class Hours:
MWF 10:10 - 11:00 am.
Location: Bard Hall 140.
Office hours: TBD.

Prerequisites:
ECE 3030 or permission of instructor.

About ECE 4300: Lasers and Optoelectronics

Course contents:
Introduction to the operation, physics, and application of lasers. The course covers diffraction-limited optics, Gaussian beams, optical resonators, the interaction of radiation with matter, stimulated emission, rate equations, and laser design. Examples of coherent radiation to nonlinear optics, communication, and leading-edge research are frequently used. Course concludes with a lab where students design and then build a laser.

Textbook:
The following text is required for the course:
Laser Electronics by Joseph T. Verdeyen, 3rd Edition.



Important Note: Several figures and text that appear in these slides are from the assigned textbook. The slides are not meant to replace the text: please read the book!

About ECE 4300: Lasers and Optoelectronics

Outcomes:

- Be able to analytically design and physically construct a functional laser with simple optics.
- Understand the general operating principles of laser systems, and be knowledgeable of specific systems (e.g. tunable, ultrafast, high power, fiber and semiconductor lasers).
- Understand how to design and the physics behind continuous wave operation, mode locking, Q-switching, and harmonic generation.
- Be able to design a laser optic system using mirrors, lenses and gain media based on Gaussian beam analysis.

About ECE 4300: Lasers and Optoelectronics

Homeworks:

- Homework assignments are an integral part of learning in this course. Approximately one problem set will be assigned every two weeks.
- You are allowed to work with other students in the class on your homeworks. The name(s) of the student(s) you worked with must be included in your homework. But what you turn in must be in your *own* writing, and have your *own* plots and figures. Turning in plots/figures/text that are exact replicas of others *is considered cheating* (see below).
- Assignments must be turned in before class on the due date. The time the assignment is turned in should be written. There will be a 10% penalty each day of delay, and assignments will not be accepted beyond 3 days after the due date. There will be no exceptions to this rule.
- Present your solutions *neatly*. Do not turn in rough unreadable worksheets - learn to **take pride in your presentation**. Show the relevant steps, so that partial points can be awarded. $\boxed{\text{BOX}}$ your final answers where applicable. Draw figures wherever necessary. Please print out the question sheet(s) and staple to the top of your homework. Write your name, email address, and date/time the assignment is turned in on the cover.
- Grading of the ECE 4300 assignments will be done by a course grader, with support from the instructors.

About ECE 4300: Lasers and Optoelectronics

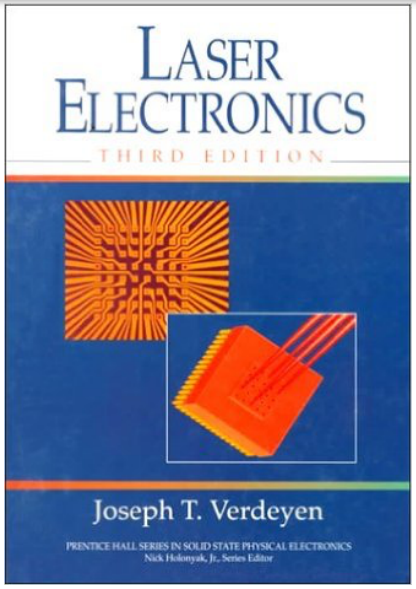
Cheating Policy:
Collaboration in homework assignments is allowed, but you must adhere to the requirements described in the homeworks section above. Collaboration in exams is considered cheating. Please read Cornell's policy on cheating here: `http://cuinfo.cornell.edu/aic.cfm`. Now there is no escaping the fact that lasers are *just plain cool*. So let's not spoil that by cheating! No matter how familiar we are with lasers, or how deeply we understand them, they remain an endless source of wonder and amazement - that such a thing actually exists. So let's approach the course in that spirit & enjoy discovering the secrets of this beautiful device!

Exams and Grades:
Other than the assignments, there will be two written prelim exams, and a written final exam. Here is the approximate breakup of scores that will go towards your final grade:

35% Assignments
15% Prelim 1 [Friday September 30th, 2016]
20% Prelim 2 [Monday, October 31st, 2016]
30% Final [TBD]

Demonstrations and Laboratories:
A few demonstrations will be performed in the course. In one of the assignments students will design and build a laser.

Course Outline



What is a Laser?

- *Source of coherent light*
- *Coherence in space: real space & wavelength space*
- *Coherence in time: real time and frequency*
- *Laser wavelength can be made tunable*
- *Both light and matter are extremely far from equilibrium*
- *Can generate ultrashort time pulses*

Laser Physics in a Nutshell

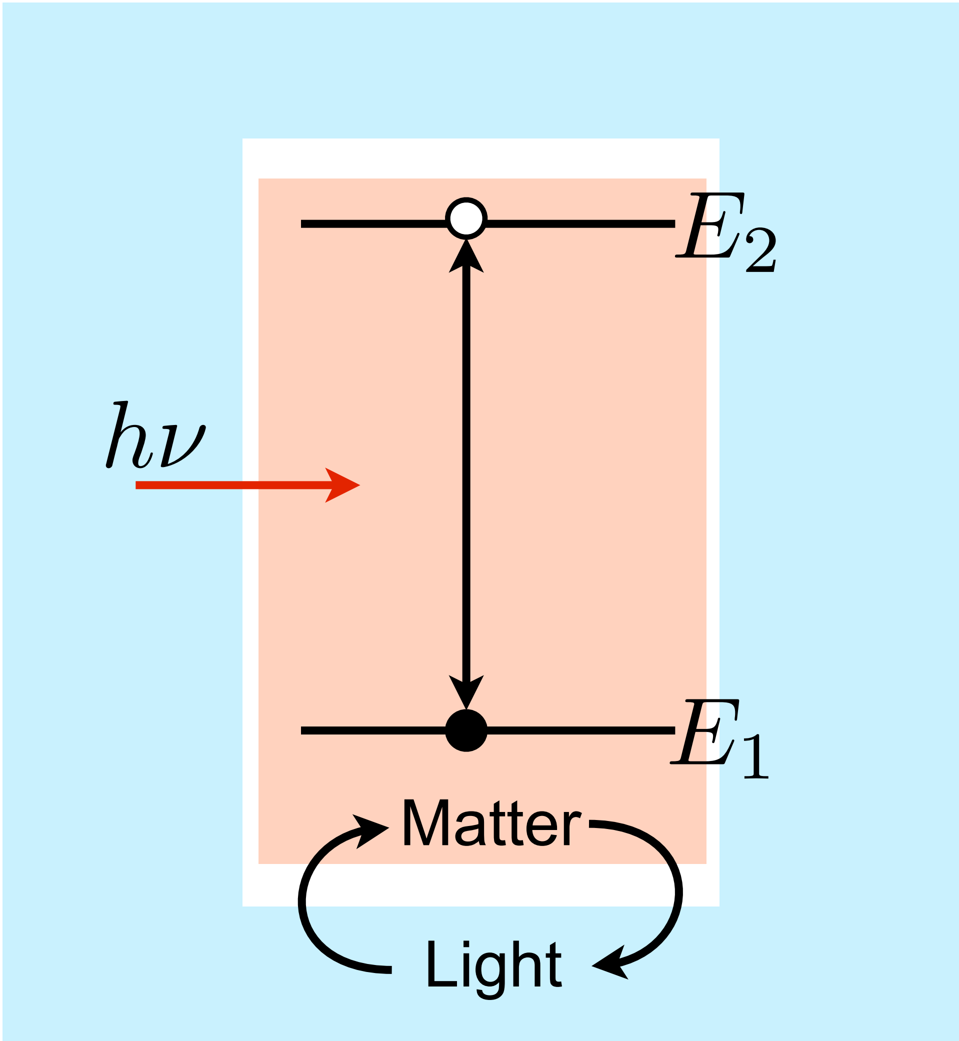
Quantum Mechanics needed to get started on Lasers:

$$E = \hbar\omega = h\nu$$

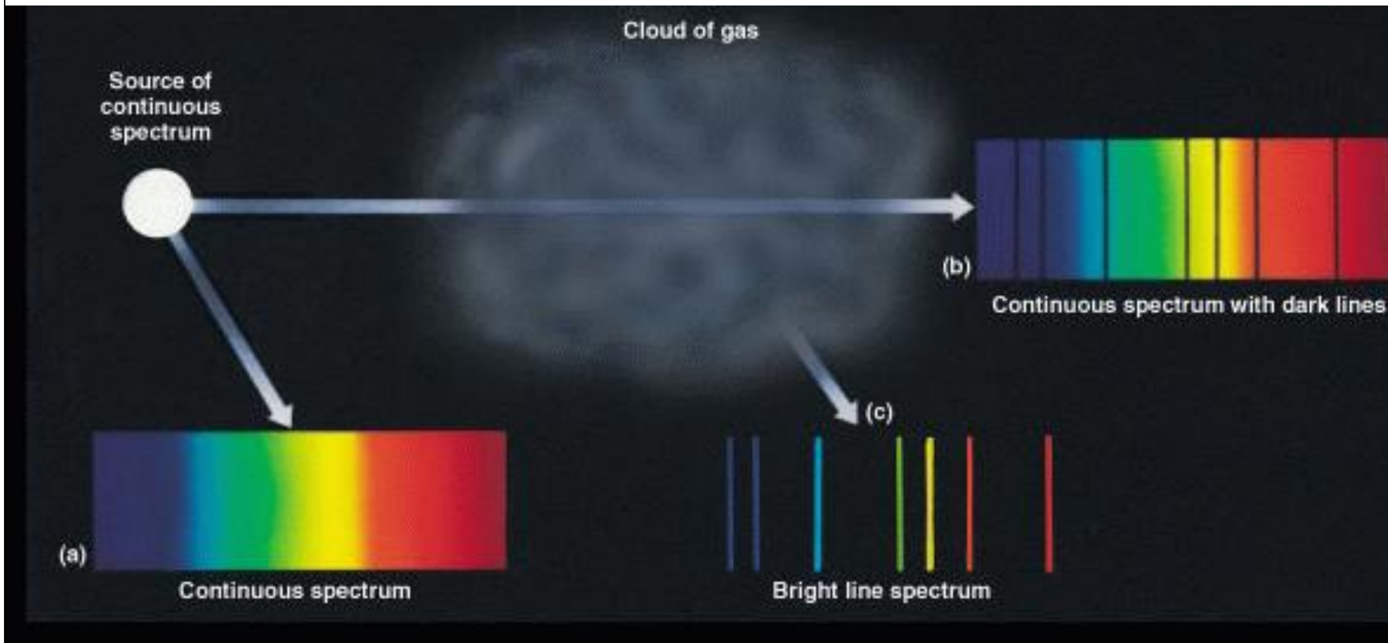
$$\nu = \frac{c}{\lambda}$$

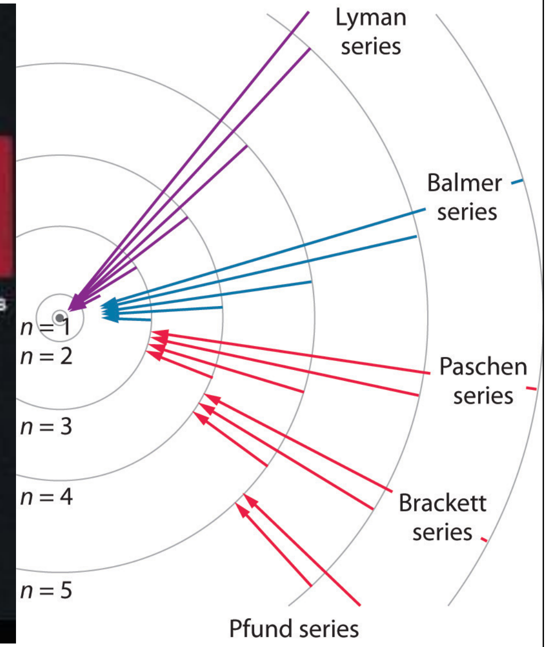
$$E = \frac{1240}{\lambda(\text{nm})}\ \text{eV}$$

$$E_2 - E_1 = h\nu$$



"Light-Matter" Interaction

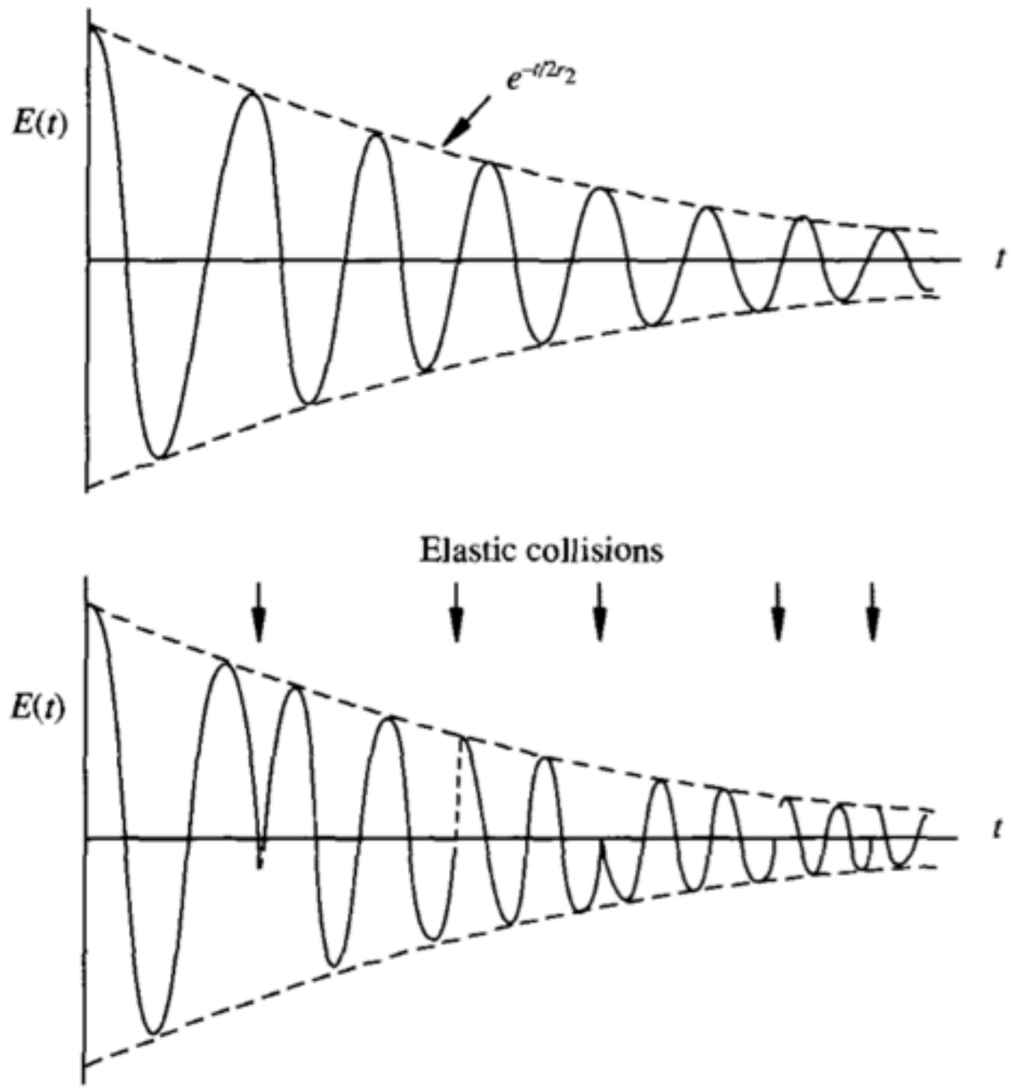




Wikipedia

- *Absorption and emission spectra of atoms was the first hint that electron energies are* ***quantized****.*
- *Lead to the development of quantum mechanics, and finds a wide range of applications today.*

Decoherence in spontaneous emission



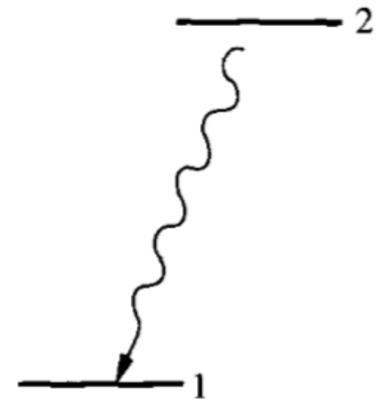
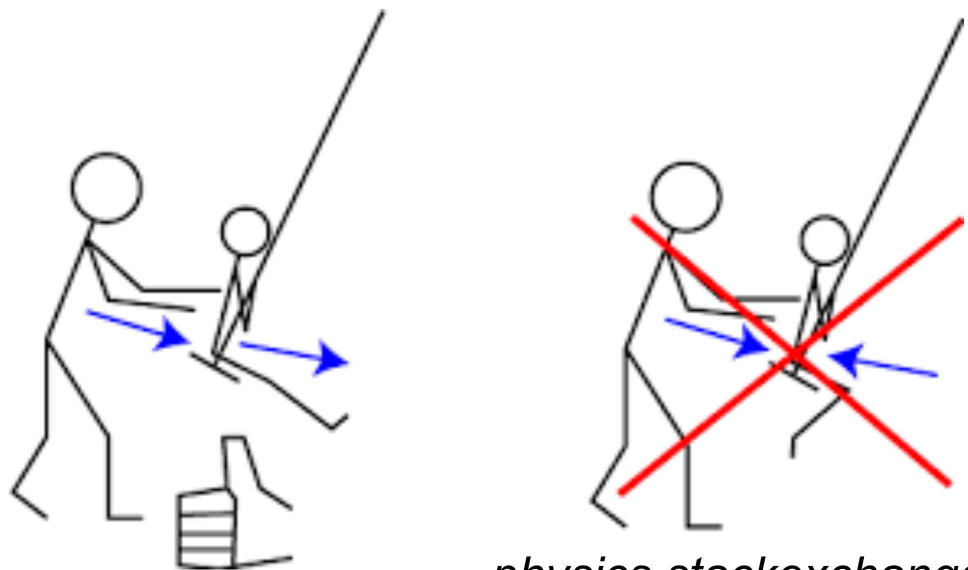


FIGURE 7.8. Classical picture of the effect of (b) elastic collisions on the emission radiation shown in (a).

Tool against decoherence: GAIN

$$\text{Resonator} + \text{Gain} \implies \text{Oscillator}$$



physics.stackexchange

How can one amplify photons?

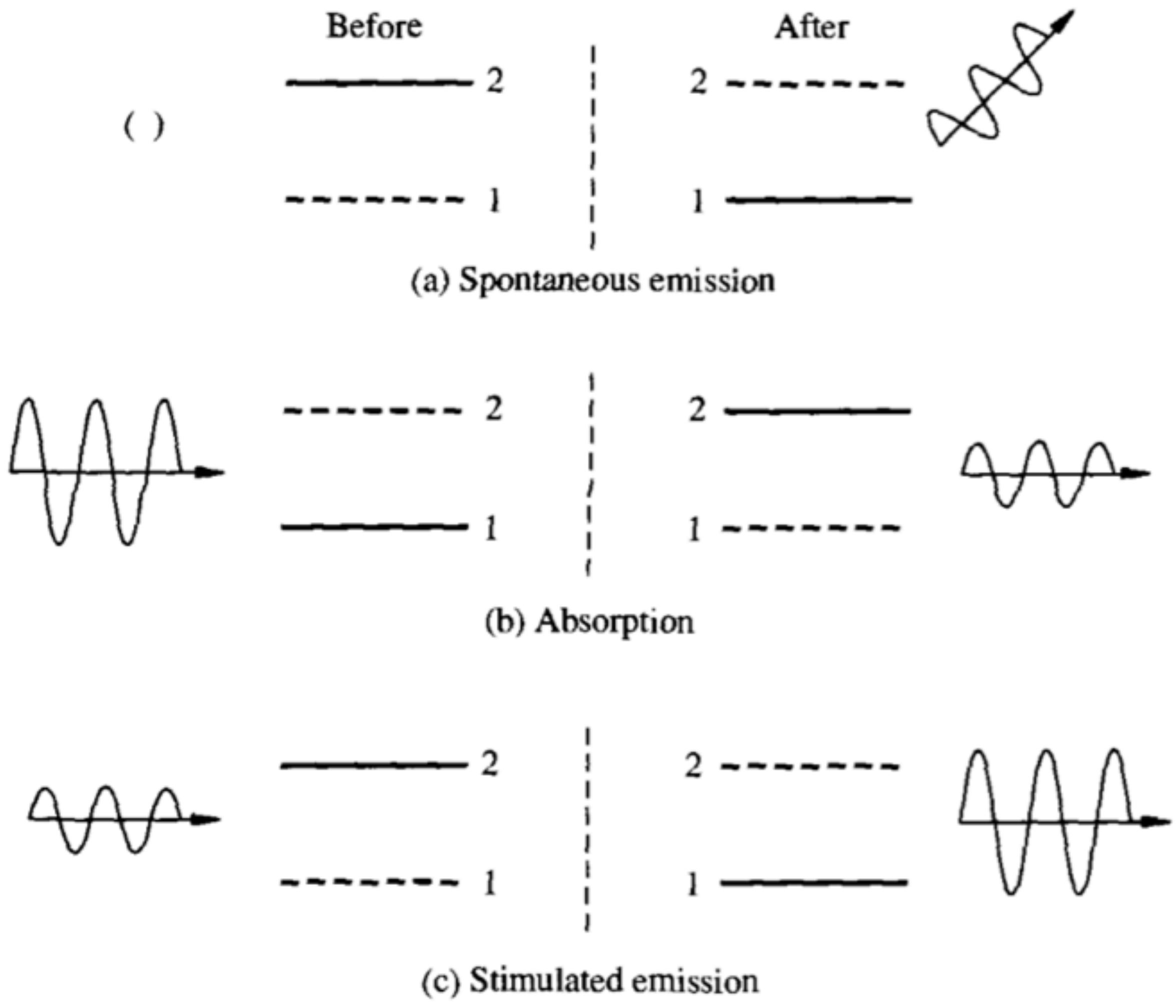


FIGURE 7.5. Effect of radiation on an atom.

Stimulated Emission = Light Amplification

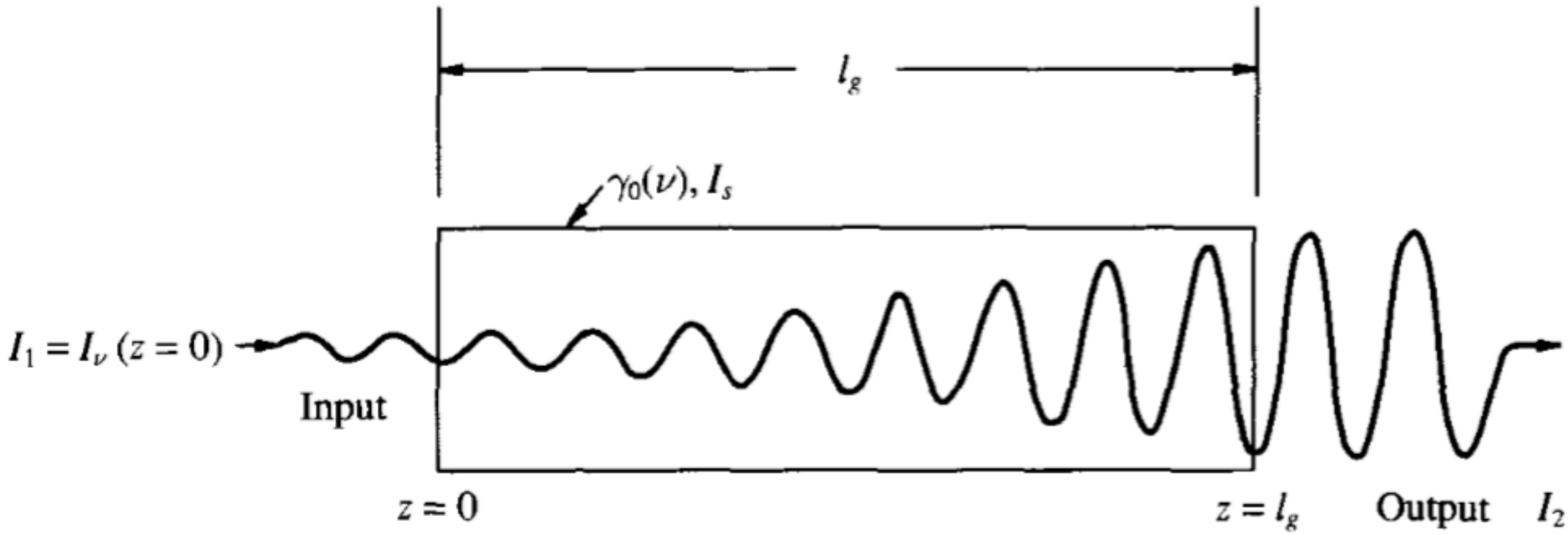


FIGURE 8.7. An optical amplifier.

How can one *amplify* photons?

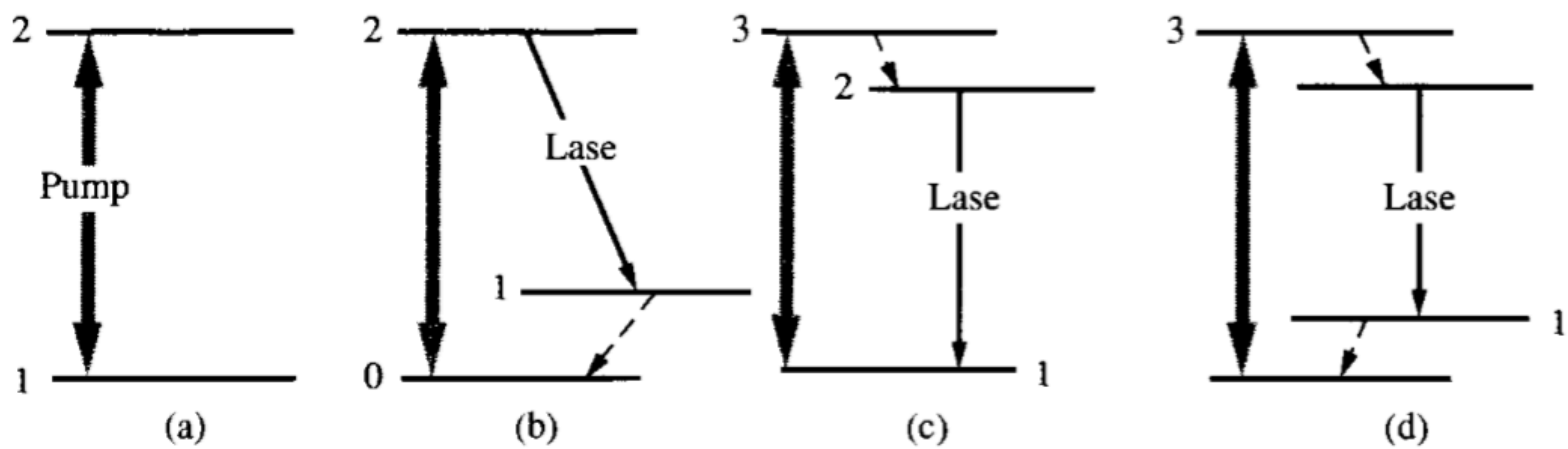


FIGURE 9.1. Possible arrangement of the energy levels of a laser: (a) represents a two level system, (b) and (c) are three level lasers, and (d) is a four-level laser. All double-headed arrows represent the pumping route, the dashed single-headed arrows represent relaxation by any cause, and the solid arrows between 2 and 1 represent stimulated emission by the laser radiation.

(c)

FIGURE 8.3. Evolution of laser oscillation from spontaneous emission: (a) initial; (b) intermediate; and (c) final.

The basic structure of a Laser

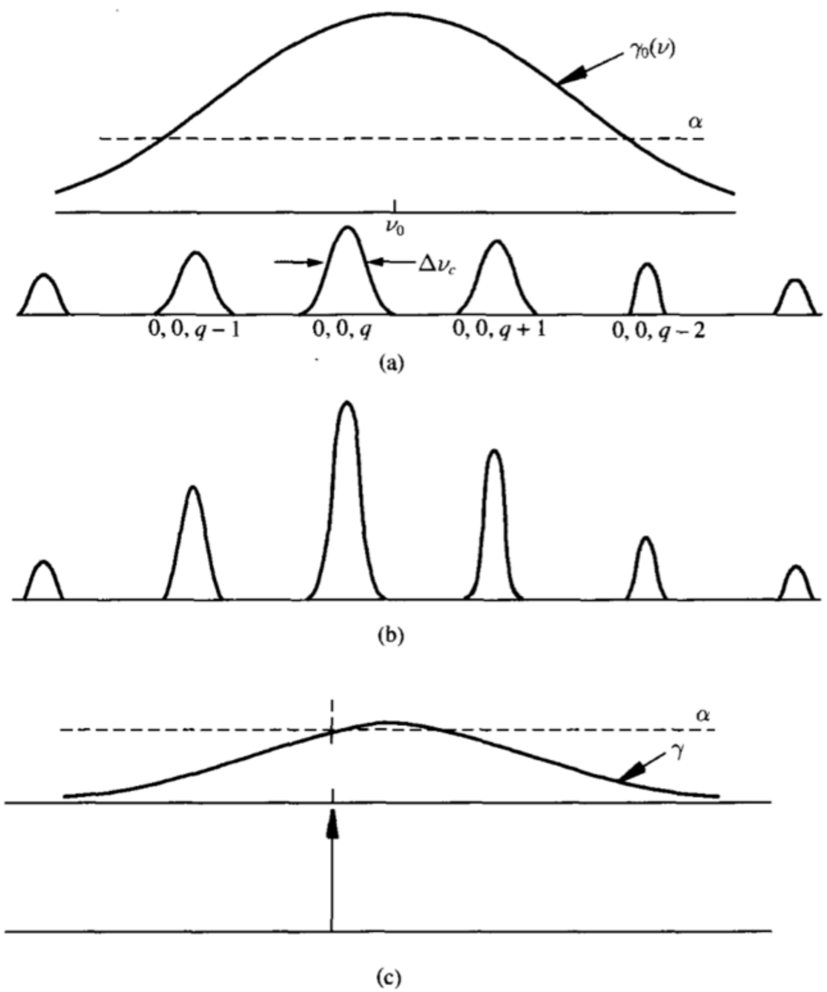
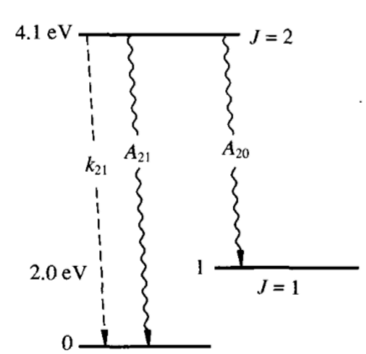
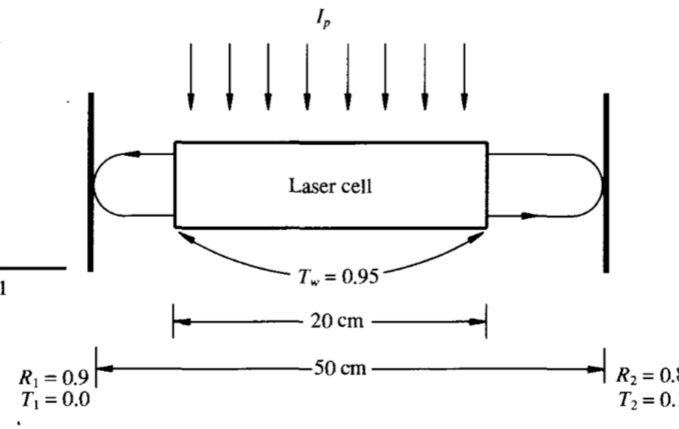


FIGURE 8.3. Evolution of laser oscillation from spontaneous emission: (a) initial; (b) intermediate; and (c) final.





Laser Physics in a Nutshell

G_0 is the small signal power gain per pass

S = the fraction of the power surviving each pass

$1 - S = L$, the fraction of the power lost per pass

$$\boxed{\text{the net round trip gain} \geq 1}$$

$$G^2S^2 > 1 \qquad \text{or} \qquad G > 1/S$$

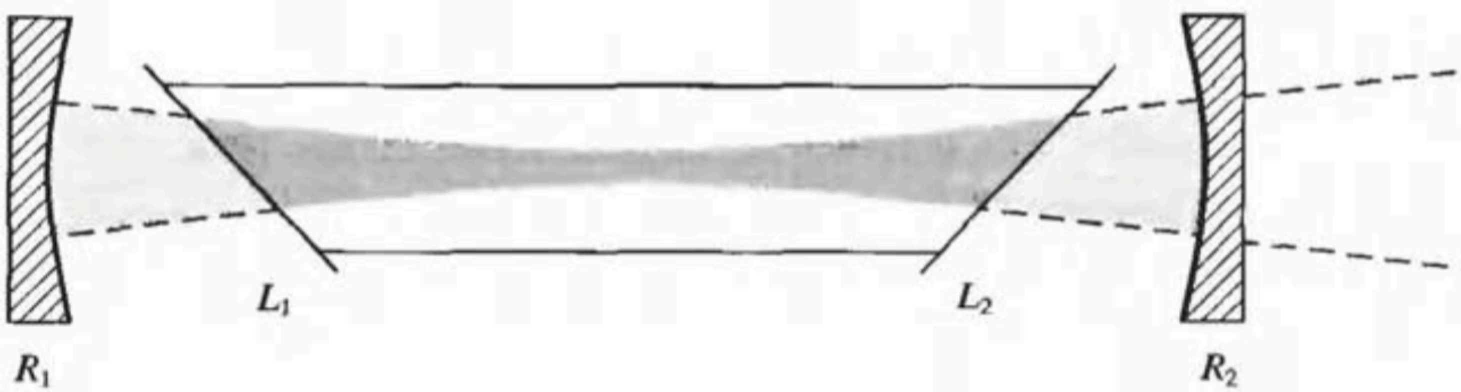
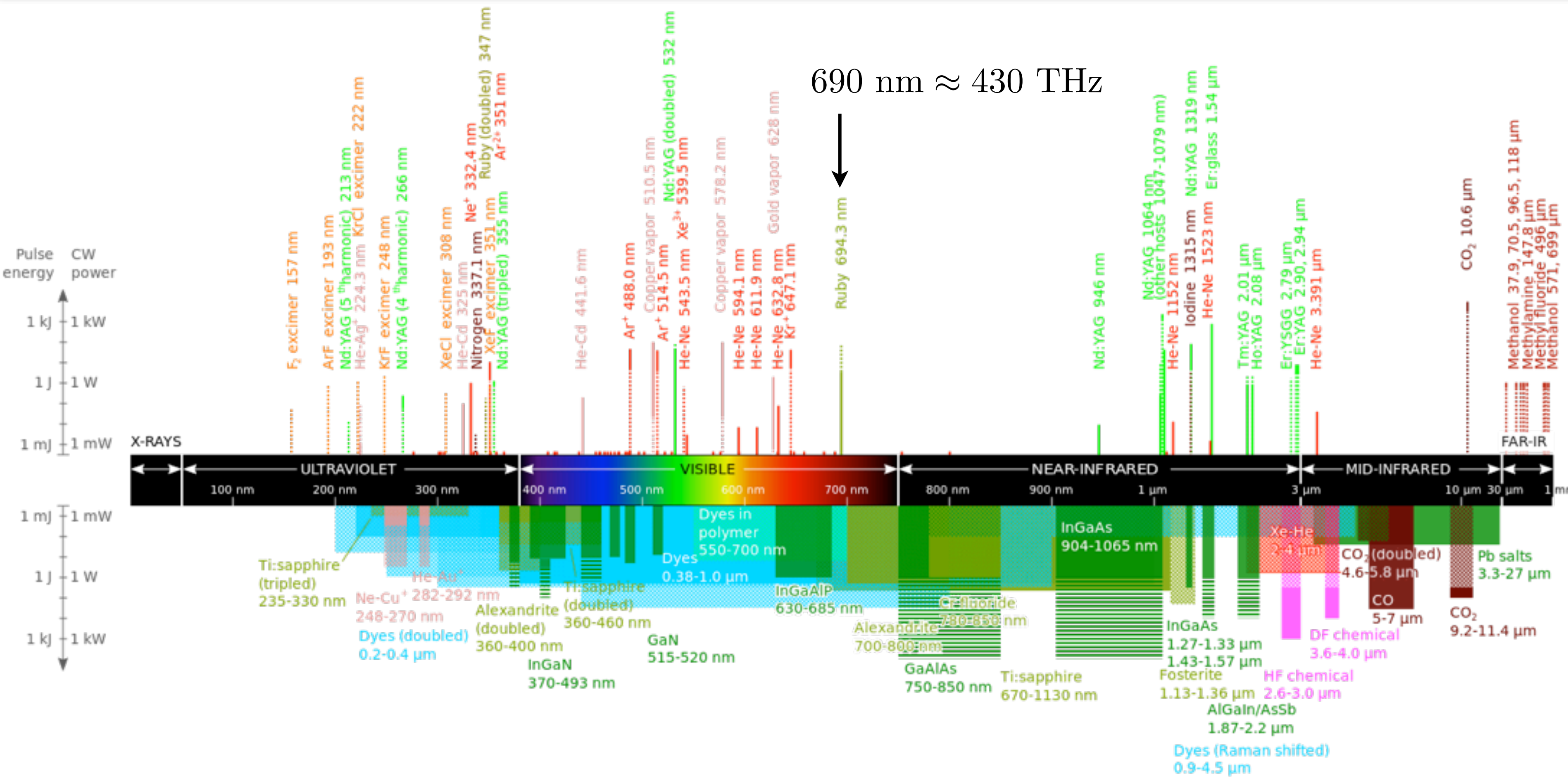


FIGURE 0.1. Schematic of a simple laser.

$$G(1 - L_2)R_2(1 - L_2)G(1 - L_1)R_1(1 - L_1) \geq 1$$

Types of Lasers



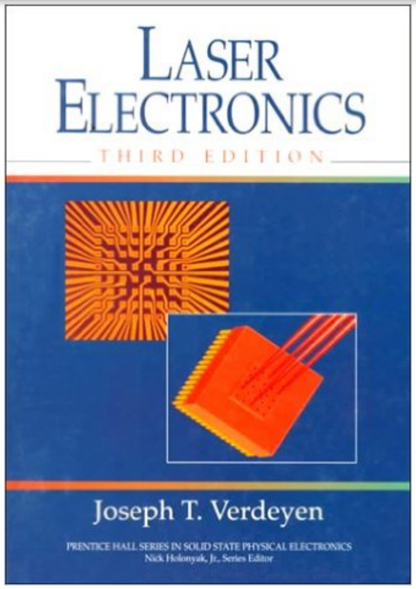
From Wikipedia

Course Outline

0 Preliminary Comments

1 Review of Electromagnetic Theory

2 Ray Tracing in an Optical System



Light Emerges from Maxwell's Equations

Maxwell

$$
\begin{aligned}
\nabla \cdot \mathbf{D} &= \rho, && \text{Gauss's law}\\
\nabla \cdot \mathbf{B} &= 0, && \text{Gauss's law}\\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, && \text{Faraday's law}\\
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, && \text{Ampere's law.}
\end{aligned}
$$

$$\nabla \cdot \mathbf{J} = -\partial\rho/\partial t, \quad \text{Continuity Equation}$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

Lorentz force

$$
\begin{aligned}
(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\mathbf{E} &= 0\\
(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\mathbf{B} &= 0
\end{aligned}
$$

Wave equations: Predict the existence of Light!

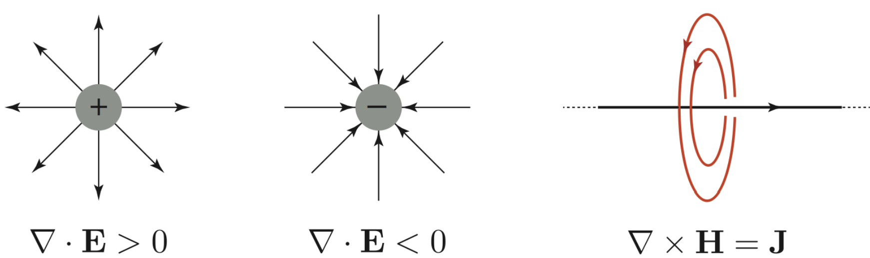


FIGURE 20.1: Electrostatic Fields.

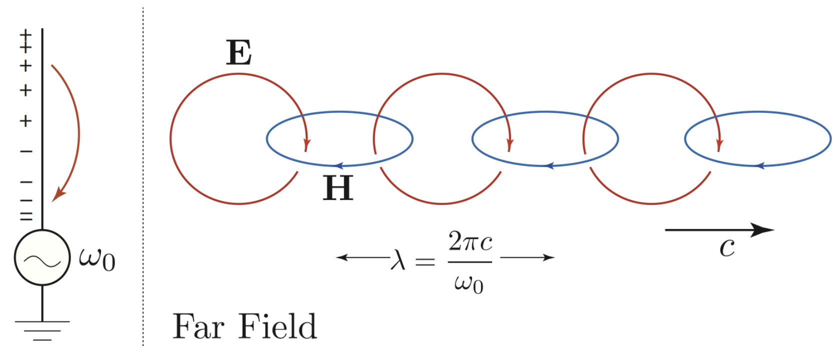


FIGURE 20.2: Antenna producing an electromagnetic wave.

Electrostatics

Electrodynamics

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}$$

A: Magnetic vector potential

$$H(\omega) = \Omega \cdot [\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 H^2] = \Omega \cdot [\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 \omega^2 A^2]$$

Classical energy density in an electromagnetic wave

Light Emerges from Maxwell's Equations

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho, \\ \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \end{aligned}$$

Monochromatic lightwave

$$\boxed{\begin{aligned} \mathbf{k} \cdot \mathbf{E} &= 0, \\ \mathbf{k} \cdot \mathbf{B} &= 0, \\ \mathbf{k} \times \mathbf{E} &= \omega \mathbf{B}, \\ \mathbf{k} \times \mathbf{B} &= -\frac{\omega}{c^2}\mathbf{E}. \end{aligned}}$$

20.4 Maxwell's equations in (k, ω) space

Consider an electromagnetic wave of a fixed frequency ω. Since $\mathbf{E}, \mathbf{B} \propto e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, we make two observations. Time derivatives of Faraday and Ampere's laws give $\frac{\partial}{\partial t} e^{-i\omega t} = -i\omega e^{-i\omega t}$, which means we can replace $\frac{\partial}{\partial t} \to -i\omega$, $\frac{\partial^2}{\partial t^2} \to (-i\omega)^2$, and so on. Similarly, the vector operators div and curl act on the $e^{i\mathbf{k}\cdot\mathbf{r}}$ part only, giving $\nabla \cdot (e^{i\mathbf{k}\cdot\mathbf{r}} \hat{\eta}) = i\mathbf{k} \cdot (e^{i\mathbf{k}\cdot\mathbf{r}} \hat{\eta})$ and $\nabla \times (e^{i\mathbf{k}\cdot\mathbf{r}} \hat{\eta}) = i\mathbf{k} \times (e^{i\mathbf{k}\cdot\mathbf{r}} \hat{\eta})$. These relations may be verified by straightforward substitution. Thus, we can replace $\nabla \to i\mathbf{k}$. With these observations, Maxwell equations in free

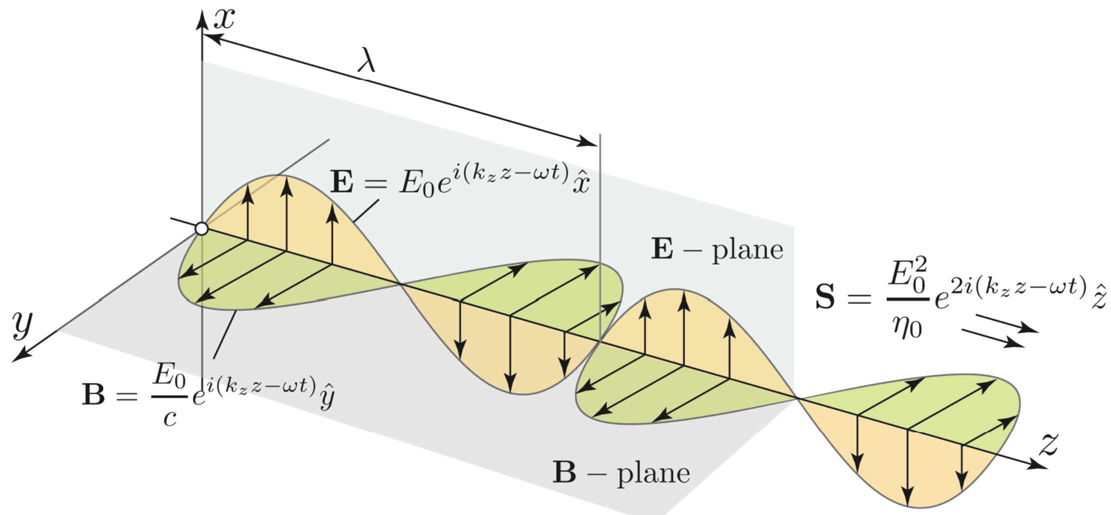


FIGURE 20.3: Electromagnetic wave.

$$\boxed{\mathbf{S} = \langle \mathbf{S}(\mathbf{r}, t) \rangle = \frac{1}{2} \mathrm{Re}[\mathbf{E} \times \mathbf{H}^\star] = \frac{E_0^2}{2\eta} \hat{z} = \frac{\eta}{2} H_0^2 \hat{z},}$$

Poynting Vector

Review of Maxwell's equations for Light

$$\boxed{\begin{aligned}\nabla \times \mathbf{h} &= \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{e}}{\partial t} + \frac{\partial \mathbf{p}}{\partial t} \\ \nabla \times \mathbf{e} &= -\mu_0 \frac{\partial \mathbf{h}}{\partial t}\end{aligned}}$$

Time-dependent Maxwell Equations

$$\mathbf{e}(\mathbf{r}, t) = Re\left[\mathbf{E}(\mathbf{r})e^{j\omega t}\right] \qquad \mathbf{h}(\mathbf{r}, t) = Re\left[\mathbf{H}(\mathbf{r})e^{j\omega t}\right]$$

$$\mathbf{j}(\mathbf{r}, t) = Re\left[\mathbf{J}(\mathbf{r})e^{j\omega t}\right] \qquad \mathbf{p}(\mathbf{r}, t) = Re\left[\mathbf{P}(\mathbf{r})e^{j\omega t}\right]$$

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\epsilon_0\mathbf{E} + j\omega\mathbf{P} = \mathbf{J} + j\omega\mathbf{D}$$

$$\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H}$$

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$$

$$\boxed{\mathbf{P} = \epsilon_0\chi\mathbf{E}}$$

The Polarization Vector

$$\begin{aligned}\mathbf{D} &= \epsilon_0\mathbf{E} + \mathbf{P} \\ &= \epsilon_0(1+\chi)\mathbf{E} \\ &= \epsilon_0\epsilon_\tau\mathbf{E} = \epsilon_0 n^2\mathbf{E}\end{aligned}$$

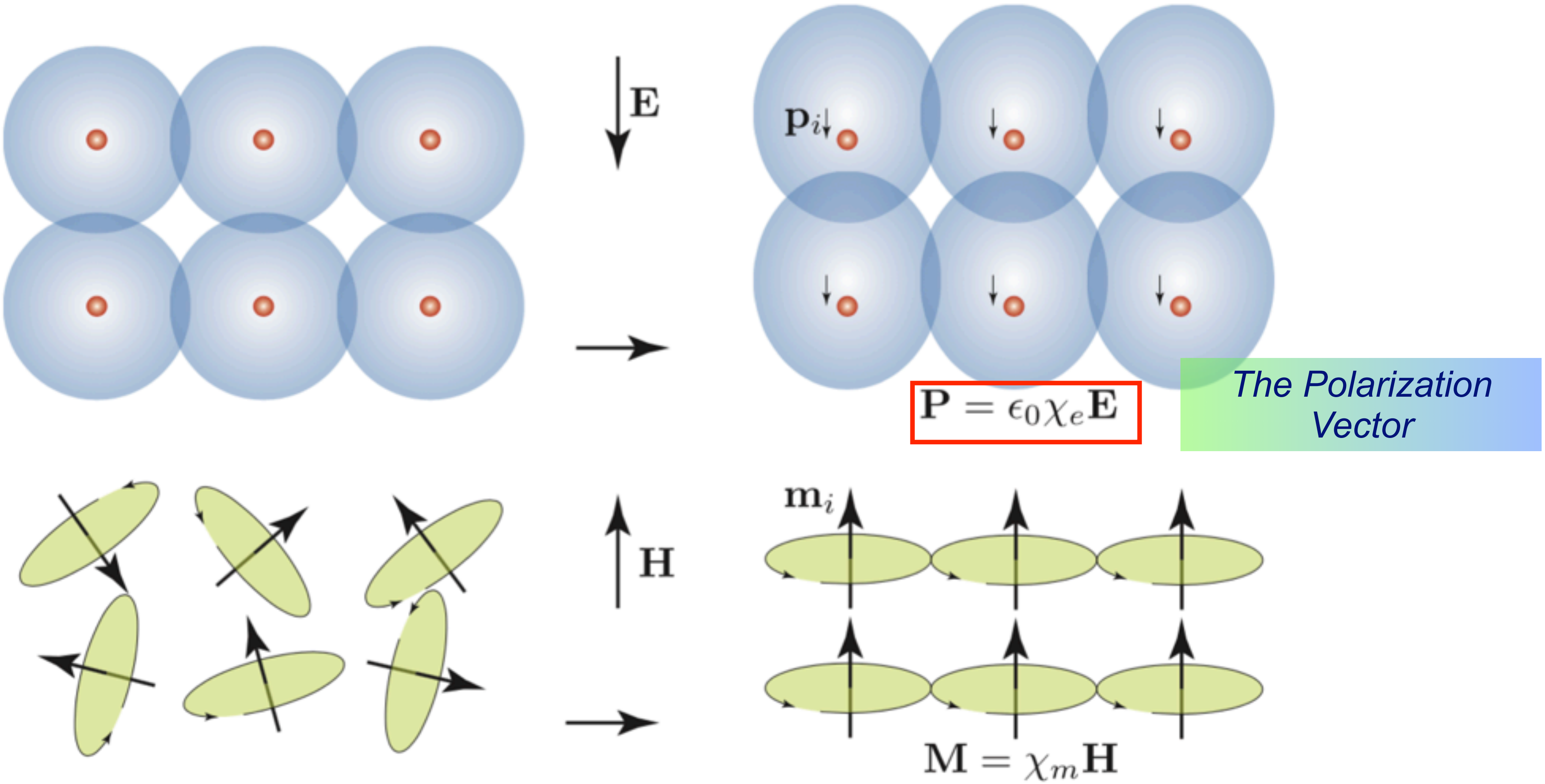
$$F(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}\, dt$$

$$\nabla \times \nabla \times \mathbf{e} = \mu_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{h}) = -\mu_0\epsilon_0 \frac{\partial^2 \mathbf{e}}{\partial t^2}$$

$$\boxed{\nabla^2\mathbf{e} - \frac{1}{c^2}\frac{\partial^2\mathbf{e}}{\partial t^2} = 0}$$

$$c^2 = 1/\mu_0\epsilon_0$$

Review of Maxwell's equations for Light



Review of Maxwell's equations for Light

$$\nabla^2 \mathbf{e} - \frac{1}{c^2}\frac{\partial^2 \mathbf{e}}{\partial t^2} = 0$$

$$\nabla^2 \mathbf{h} - \frac{1}{c^2}\frac{\partial^2 \mathbf{h}}{\partial t^2} = 0$$

Light propagation in free space

any function of the form $f(t - \mathbf{a}_n \cdot \mathbf{r}/c)$ is a solution

$$\mathbf{e}(\mathbf{r}, t) = Re\left\{[\mathbf{E}(\omega, \mathbf{k}_0)]\exp\left[j\omega\left(t - \frac{\mathbf{a}_n \cdot \mathbf{r}}{c}\right)\right]\right\}$$

$$\mathbf{e}(\mathbf{r}, t) = Re\left\{\left[\mathbf{E}(\omega, \mathbf{k}_0)\right]\exp(j\omega t)(-j\mathbf{k}_0 \cdot \mathbf{r})\right\}$$

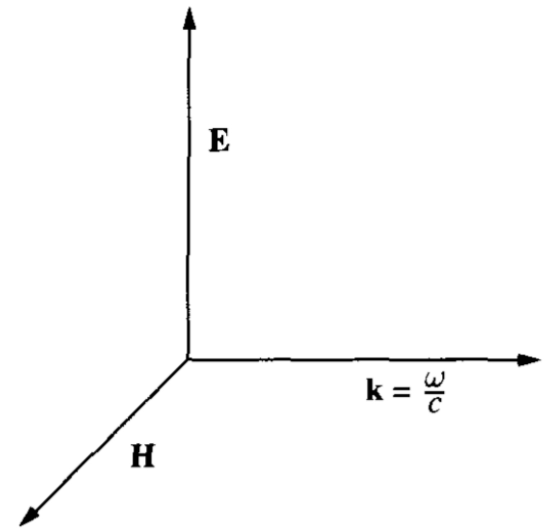
$$|\mathbf{k}_0| = \frac{\omega}{c} = \frac{2\pi}{\lambda_0}$$

$$\begin{Bmatrix} \mathbf{e} \\ \mathbf{h} \end{Bmatrix} = \begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix} \exp(j\omega t)\exp(-j\mathbf{k}_0 \cdot \mathbf{r})$$

$$\mathbf{k}_0 \cdot \mathbf{r} = (k_x\mathbf{a}_x + k_y\mathbf{a}_y + k_z\mathbf{a}_z)\cdot(x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z)$$
$$= k_x x + k_y y + k_z z$$

$$\nabla \rightarrow = \mathbf{jk}$$
$$\nabla\cdot \rightarrow -\mathbf{jk}\cdot$$
$$\nabla\times \rightarrow -\mathbf{jk}\times$$

$$\mathbf{k}_0 \times \mathbf{E} = +\omega\mu_0\mathbf{H}$$
$$\mathbf{k}_0 \times \mathbf{H} = -\omega\epsilon_0\mathbf{E}$$



$$\eta_0 = \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} = \text{wave impedance of free space}$$

$$\frac{|\mathbf{E}|}{|\mathbf{H}|} = \frac{\omega\mu_0}{|\mathbf{k}_0|} = \frac{|\mathbf{k}_0|}{\omega\epsilon_0} = \eta_0 = \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \cong 377\ \Omega$$

Review of Maxwell's equations for Light

$$\mathbf{S} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^* = \frac{1}{2}\mathbf{E} \times \frac{(\mathbf{k}_0 \times \mathbf{E})^*}{\omega\mu_0} = \frac{1}{2}\mathbf{E} \cdot \mathbf{E}^* \frac{\mathbf{k}_0}{\omega\mu_0}$$

Poynting Vector, Power delivered by light

$$\nabla \times \mathbf{h} = \epsilon_0 \frac{\partial \mathbf{e}}{\partial t} + \frac{\partial \mathbf{p}_l}{\partial t} + \frac{\partial \mathbf{p}_a}{\partial t} = \epsilon_0 n^2 \frac{\partial \mathbf{e}}{\partial t} + \frac{\partial \mathbf{p}_a}{\partial t}$$

$$\nabla \times \mathbf{e} = -\mu_0 \frac{\partial \mathbf{h}}{\partial t}$$

$$\nabla^2 \mathbf{e} - \left(\frac{n}{c}\right)^2 \frac{\partial^2 \mathbf{e}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{p}_a}{\partial t^2}$$

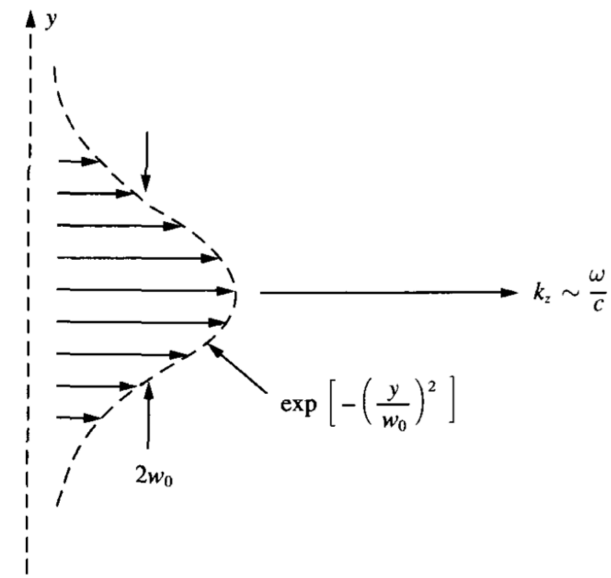
Wave equation for light propagation in material media

Gaussian Beams of Light

$$E(y) = E_0 \exp\left[-\left(\frac{y}{w_0}\right)^2\right]$$

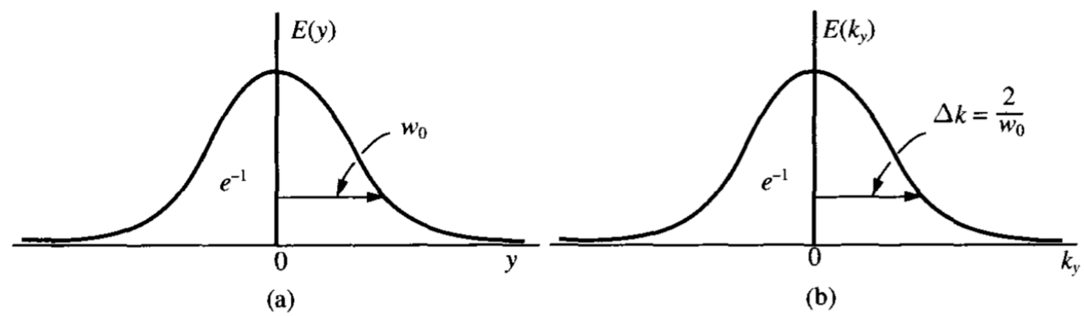
$$\boxed{(\Delta y)^2 = \frac{\int_{-\infty}^{+\infty}(y-0)^2 E^2(y)dy}{\int_{-\infty}^{+\infty} E^2(y)dy}}$$

"Uncertainty" has a PRECISE meaning!



$$E(k_y) = \pi^{1/2} w_0 E_0 \exp\left[-\left(\frac{k_y w_0}{2}\right)^2\right]$$

$$\boxed{(\Delta k_y)^2 = \frac{\int_{-\infty}^{+\infty}(k_y-0)^2 E^2(k_y)dk_y}{\int_{-\infty}^{+\infty} E^2(k_y)dk_y}}$$



$$\Delta y \cdot \Delta k_y = 1/2.$$

Uncertainty Relations for all Waves

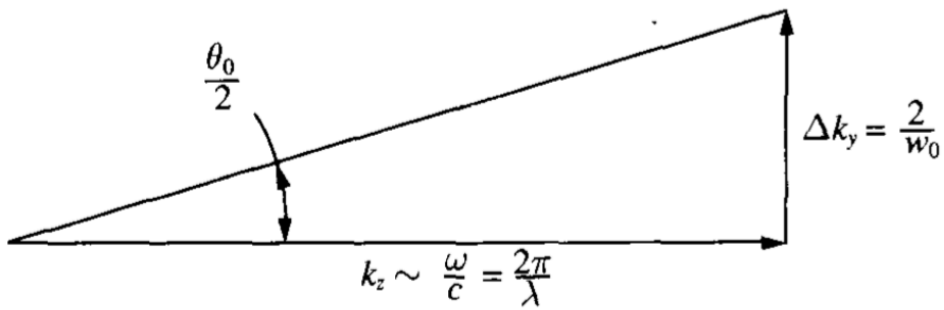
Item	Physical	Conjugate variable	Relation
ω	Angular frequency	t (time)	$\Delta\omega\Delta t \geq \frac{1}{2}$
k_x	Propagation along x	x	$\Delta k_x \Delta x \geq \frac{1}{2}$
k_y	Propagation along y	y	$\Delta k_y \Delta y \geq \frac{1}{2}$
k_z	Propagation along z	z	$\Delta k_z \Delta z \geq \frac{1}{2}$
E	$\hbar\omega =$ energy	t	$\Delta E \Delta t \geq h/4\pi$
p_x	Momentum along x	x	$\Delta p_x \Delta x \geq h/4\pi$
p_y	Momentum along y	y	$\Delta p_y \Delta y \geq h/4\pi$
p_z	Momentum along z	z	$\Delta p_z \Delta z \geq h/4\pi$

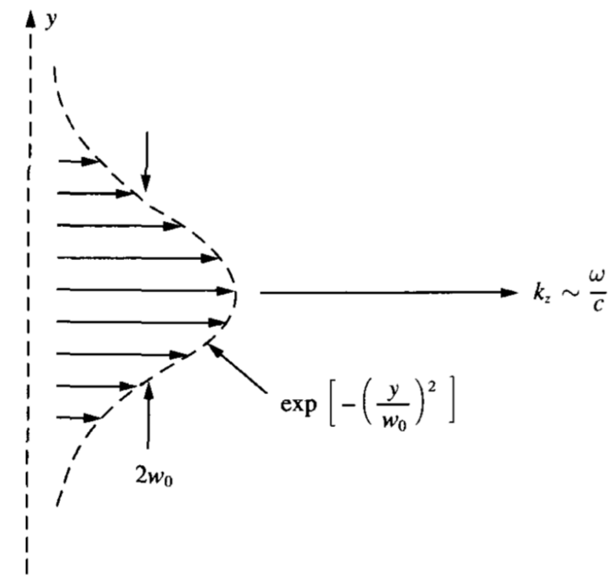
Gaussian Beam Spreading

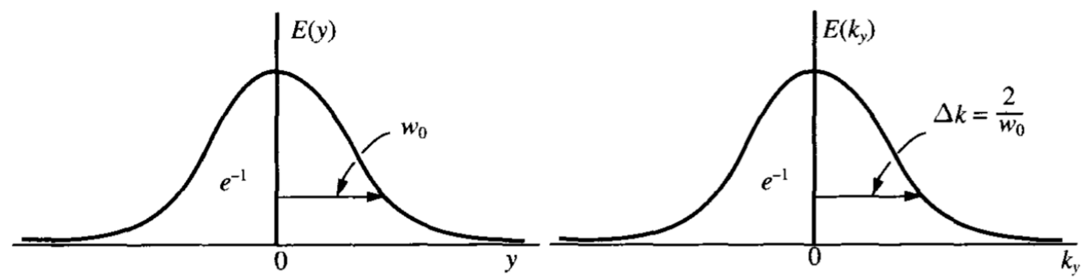
$$\frac{\theta_0}{2} = \frac{\Delta k_y}{k_z} = \frac{\lambda}{\pi w_0}$$

$$\boxed{\theta_0 = \frac{2\lambda}{\pi w_0}}$$

Beam spreading angle
~ wavelength/Diameter







(a) (b)

It is instructive to consider some numbers here. Let $\lambda = 694.3$ nm and $2w_0 = 0.1$ cm; then θ_0 is 8.8×10^{-4} rad. To achieve the same beam spread at 10-cm wavelength would require an antenna aperture $2w_0$ of 144 m. Such a small divergence of an optical beam justifies the simple ray-tracing approach of Chapter 2.

Wave Propagation in Anisotropic Media

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_0 \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_2 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{bmatrix}$$

$$\mathbf{k} \times \mathbf{h} = -\omega \mathbf{D}$$

$$\mathbf{k} \cdot (\mathbf{k} \times \mathbf{H}) \equiv 0 = -\omega \mathbf{k} \cdot \mathbf{D}$$

$$k^2 = \omega^2 \mu_0 \epsilon_0 \epsilon_1$$

$$\frac{k_0^2}{k^2} = \frac{1}{\epsilon_1} = \frac{1}{n_1^2}$$

$$\mathbf{k} = k(\cos\theta\, \mathbf{a}_z + \sin\theta\, \mathbf{a}_y)$$

$$\mathbf{D} = D(-\cos\theta\, \mathbf{a}_y + \sin\theta\, \mathbf{a}_z)$$

$$E_y = \frac{D}{\epsilon_0 \epsilon_1}[-\cos\theta]$$

$$E_z = \frac{D}{\epsilon_0 \epsilon_2}[\sin\theta]$$

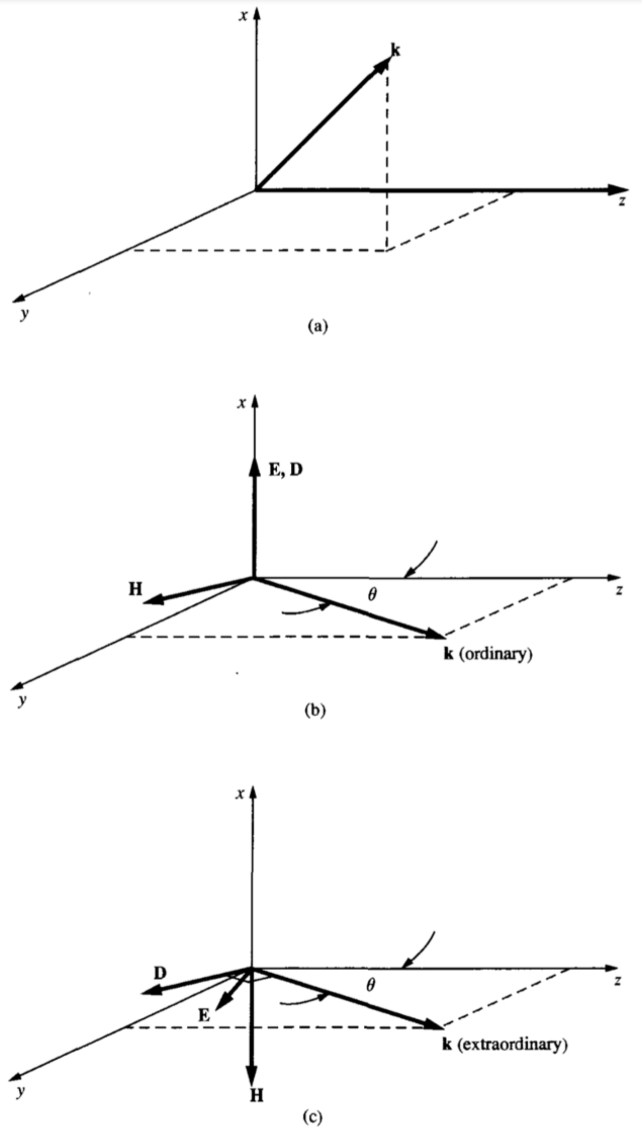


FIGURE 1.5. Orientation of **k**, **E**, and **D** for a uniaxial crystal. (a) The general problem. (b) The ordinary wave. (c) The extraordinary wave.

$$k^2 = \omega^2 \mu_0 \frac{\mathbf{D} \cdot \mathbf{D}}{\mathbf{E} \cdot \mathbf{D}}$$

$$\left(\frac{k_0}{k}\right)^2 = \frac{1}{n_{\text{eff}}^2} = \epsilon_0 \frac{\mathbf{E} \cdot \mathbf{D}}{\mathbf{D} \cdot \mathbf{D}}$$

$$\boxed{\frac{1}{n_{\text{eff}}^2} = \frac{\cos^2\theta}{n_1^2} + \frac{\sin^2\theta}{n_2^2}}$$

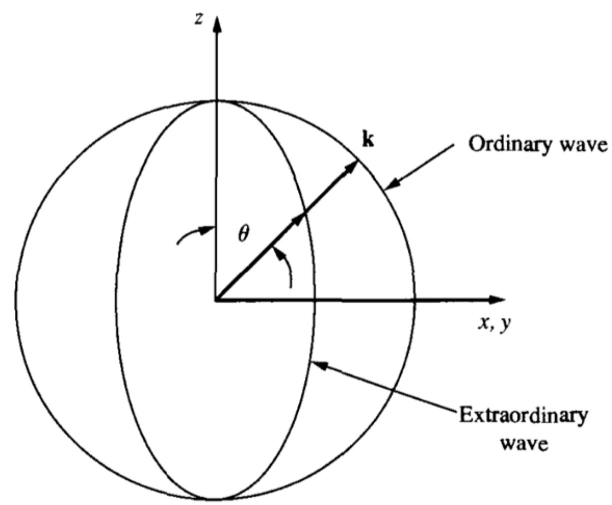


FIGURE 1.6. The index ellipsoid for a uniaxial crystal.

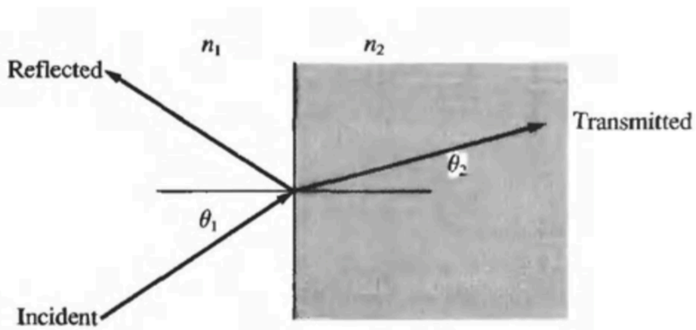
Boundary Conditions in Optics

$$\mathbf{a}_n \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$$

$$\mathbf{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_{s2}$$

$$\mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_{s2}$$

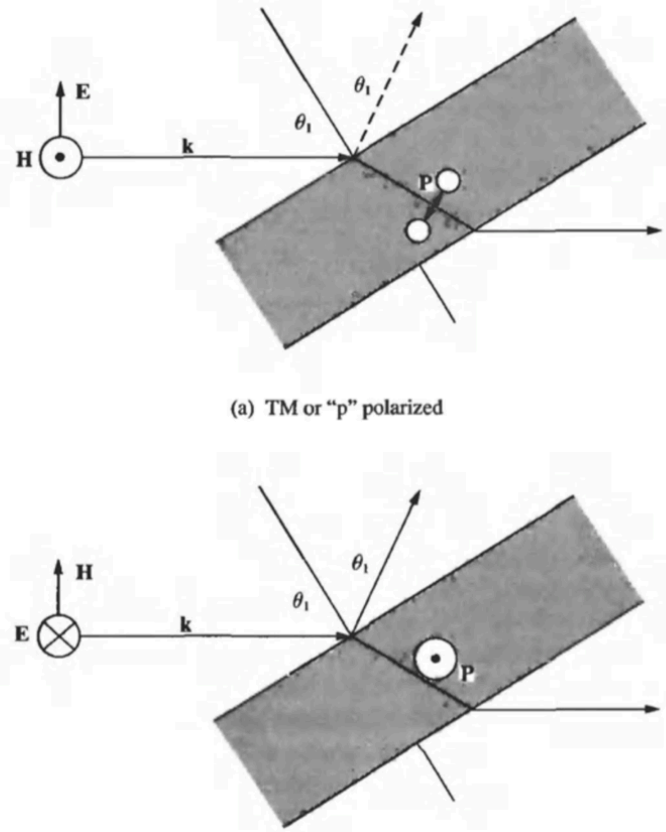
$$\mathbf{a}_n \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$$



$$\phi = (\omega/c) n_1 \sin\theta_1$$

$$(\omega/c)\, n_1 \sin\theta_1 = (\omega/c)\, n_2 \sin\theta_2$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$



(a) TM or "p" polarized

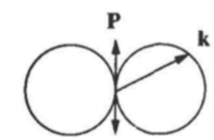
(b) TE or "s" polarized

$$\theta_1 + \theta_2 = \frac{\pi}{2}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(Snell's law)}$$

$$n_1 \sin\theta_1 = n_2 \sin(\pi/2 - \theta_1) = n_2 \cos\theta_1$$

$$\boxed{\tan\theta_1 = \frac{n_2}{n_1} \qquad \text{(Brewster's angle)}}$$



(c) Dipole radiation

FIGURE 1.8. Brewster's angle windows.

Coherent Electromagnetic Radiation

Longitudinal Phase Coherence

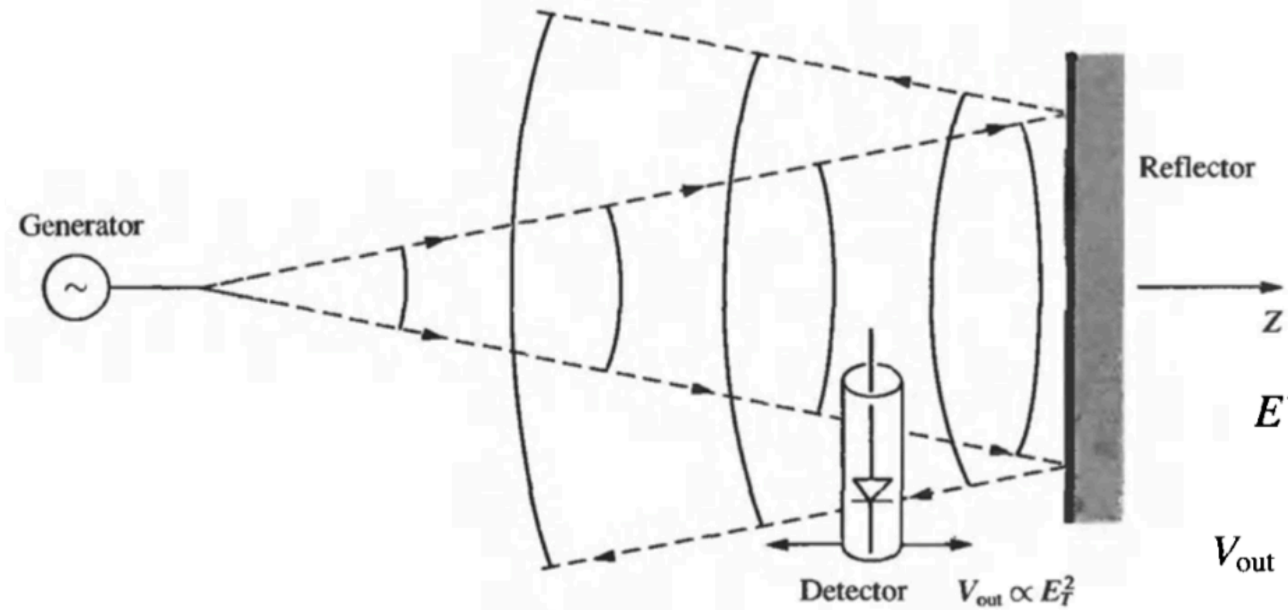


FIGURE 1.9. Simple interference experiment.

$$E^+ = E_0 \exp(-jkz)$$

$$E^- = -E_0 \exp(+jkz)$$

$$V_{out} \propto E_T E_T^* = 4E_0^2 \sin^2 kz$$

$$E^+ = E_0 \exp\left\{-j\left[kz + \Delta\phi(t)\right]\right\}$$

$$V_{out} \propto E_T^2 = 4E_0^2 \sin^2\left[kz + \frac{\Delta\phi(t)}{2}\right]$$

$$\Delta t = \frac{2z}{c} = \frac{2 \times 3}{3 \times 10^8} = 20\text{ns}$$

$$\Delta\phi = \left.\frac{d\phi}{dt}\right|_{max} \Delta t = 10^{-4} \times 2\pi \times 10^{+9} \times 20 \times 10^{-9} = 0.004\pi \Rightarrow 0.72^\circ$$

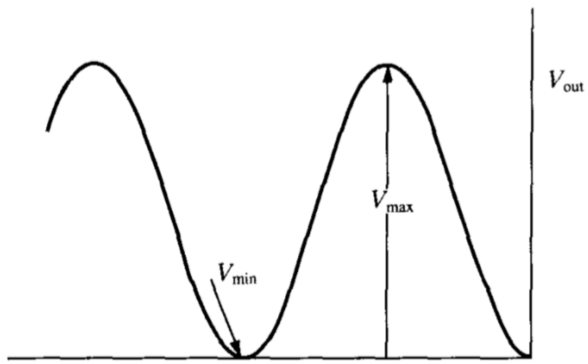


FIGURE 1.10. Measurements of the VSWR. (NOTE: Most detectors produce an output [i.e., voltage] proportional to the power sampled by the antenna. Consequently, the quantity V_{max}/V_{min} would correspond to the power standing-wave ratio.)

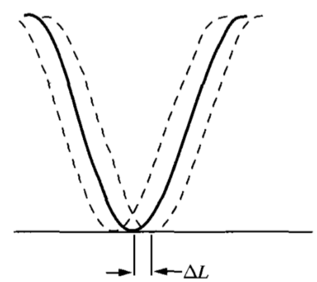


FIGURE 1.11. "Jittering" of the minimum position owing to the random jumps in phase of the later portion of the wave.

Coherent Electromagnetic Radiation

Transverse Phase Coherence

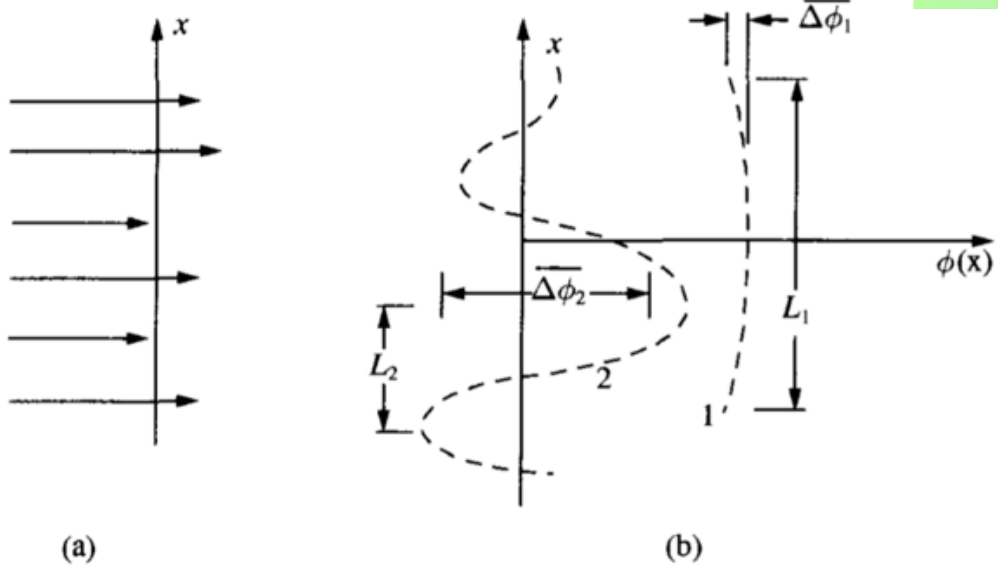


FIGURE 1.17. Two beams of the same size but with radically different variations of phase in the transverse direction.

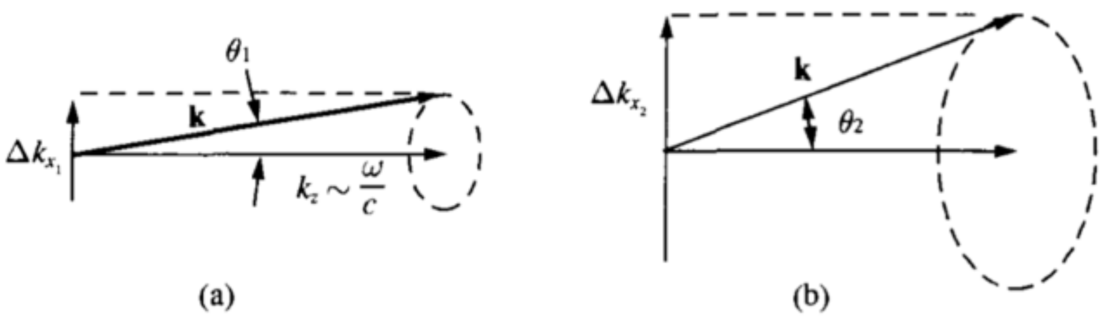
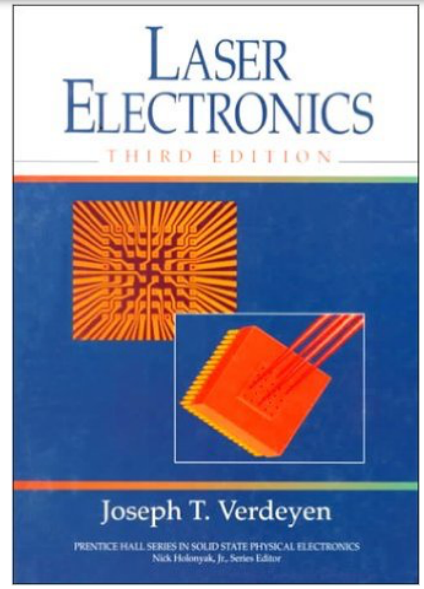


FIGURE 1.18. Beam spreads for the two beams of Fig. 1.17.

$$\theta \sim \frac{\Delta k_x}{k_z} \qquad \theta_1 \sim \frac{\overline{\Delta\phi_1}}{L_1}\frac{\lambda}{2\pi} \qquad \theta_2 = \frac{\overline{\Delta\phi_2}}{L_2}\frac{\lambda}{2\pi}$$

Course Outline



Ray Tracing for Coherent Beams: Optical Cavity Design

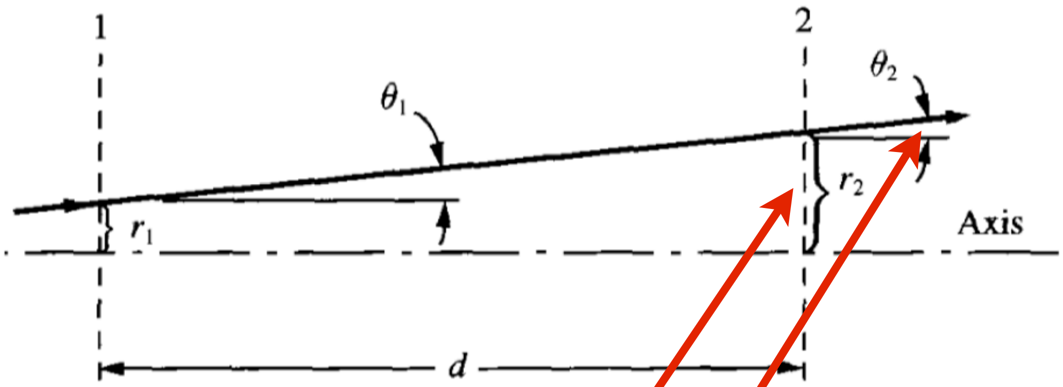


FIGURE 2.1. Ray in a homogeneous dielectric of length d.

height

slope

$$\begin{bmatrix} r_{\text{out}} \\ r'_{\text{out}} \end{bmatrix} = \boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix}} \begin{bmatrix} r_{\text{in}} \\ r'_{\text{in}} \end{bmatrix}$$

The "ABCD" Matrix

Ray Tracing for Coherent Beams: Optical Cavity Design

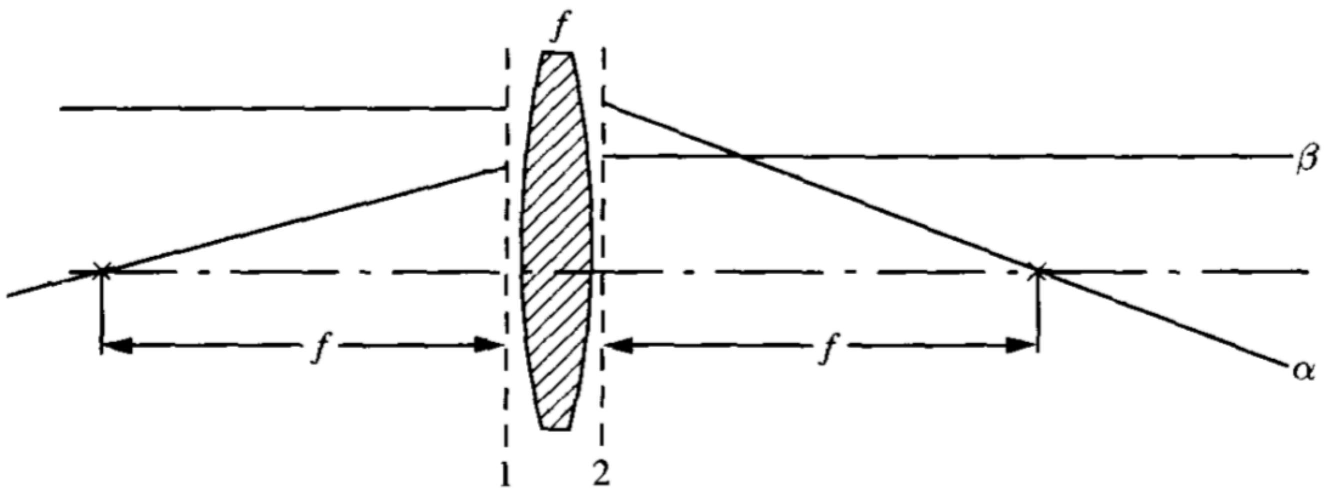


FIGURE 2.4. Paper experiment with a "thin" lens.

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$$

Ray Tracing for Coherent Beams: Optical Cavity Design

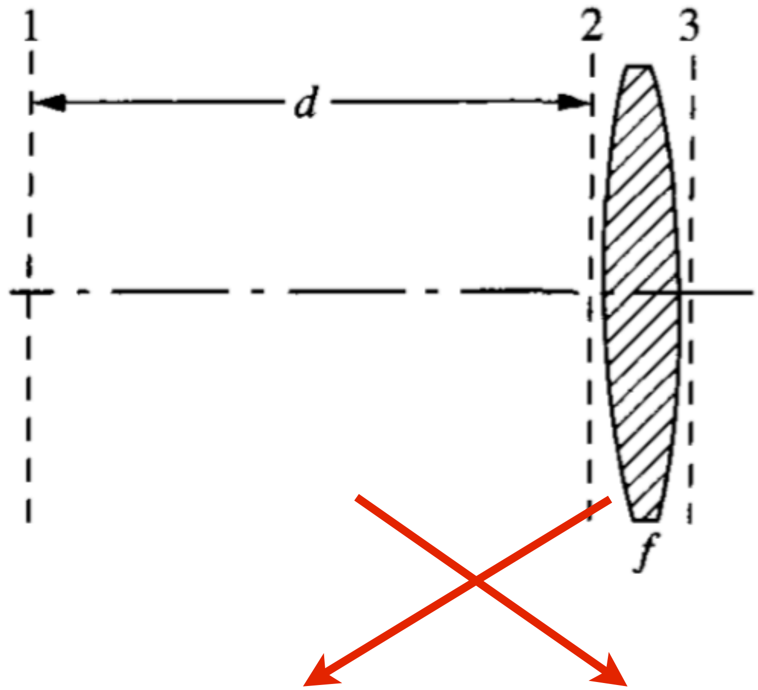


FIGURE 2.5. Combination of a lens plus free space.

3 <= 2 2 <= 1

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ -\dfrac{1}{f} & 1 - \dfrac{d}{f} \end{bmatrix}$$

Ray Tracing: Optical Cavity Design

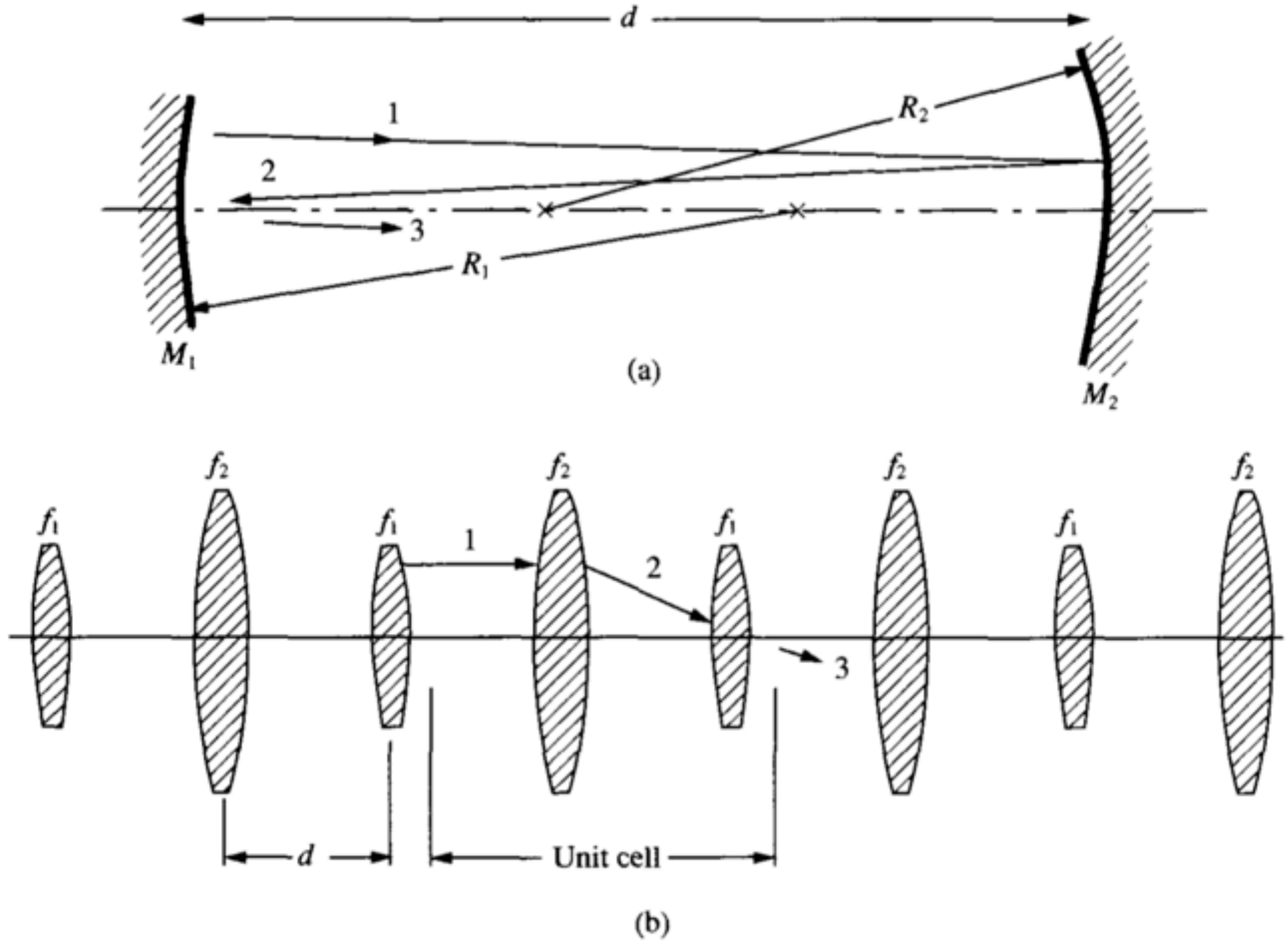


FIGURE 2.7. (a) Optical cavity showing a ray bouncing back and forth between the mirrors; (b) lens-waveguide equivalent to the mirror system shown in (a).

Stability Criteria for Laser Optical Cavities

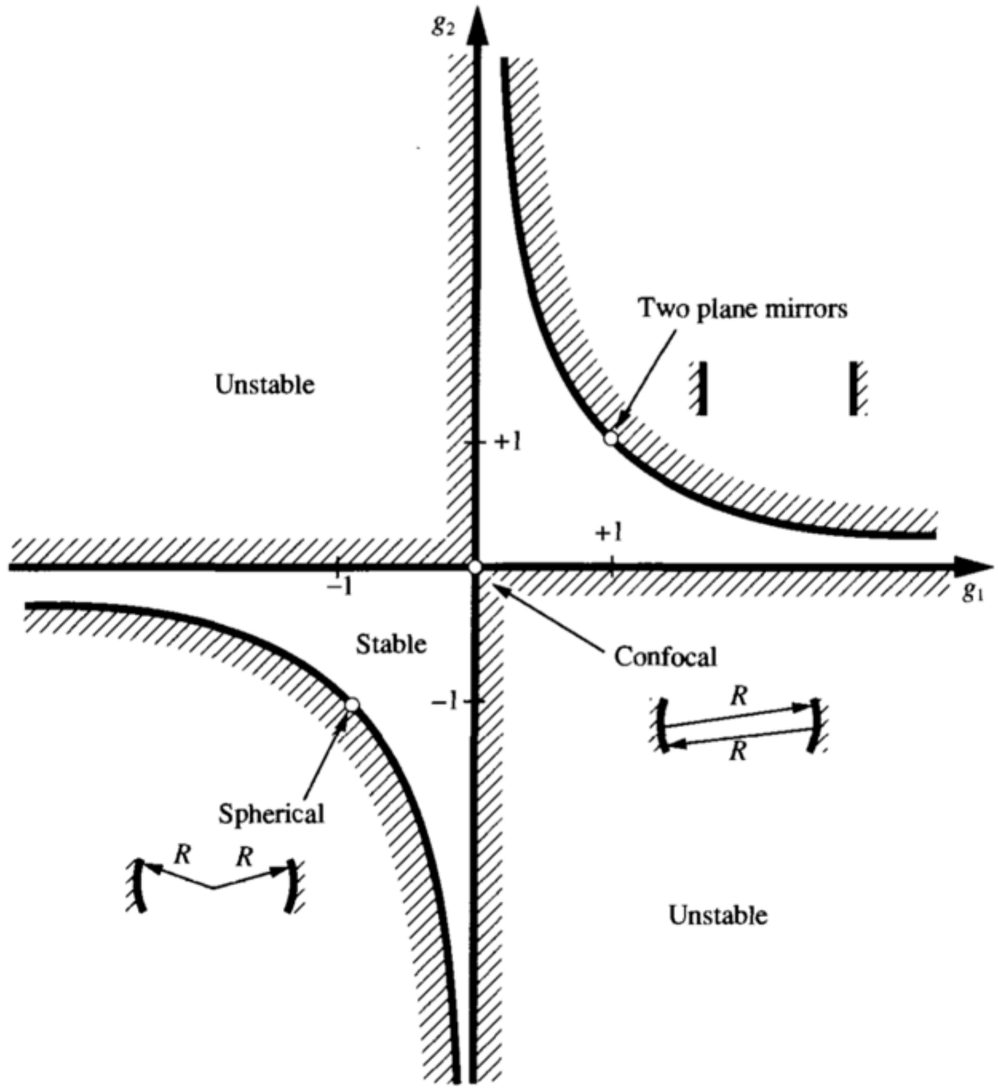
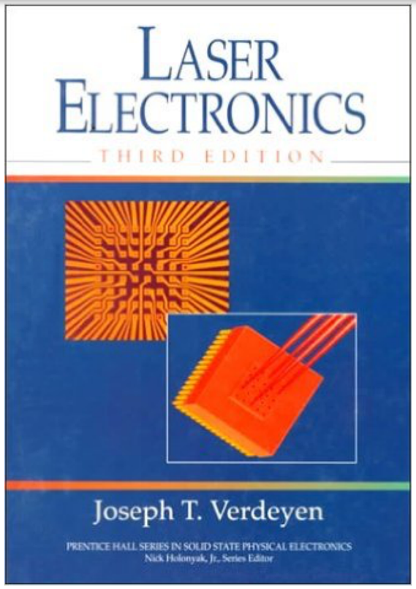


FIGURE 2.9. Stability diagram for the cavity of Fig. 2.7.

Course Outline



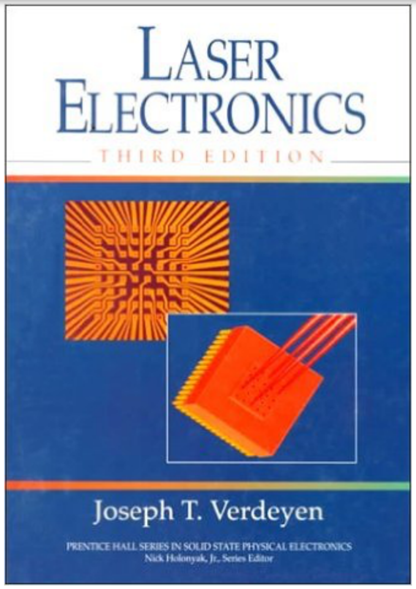
Course Outline

3 Gaussian Beams

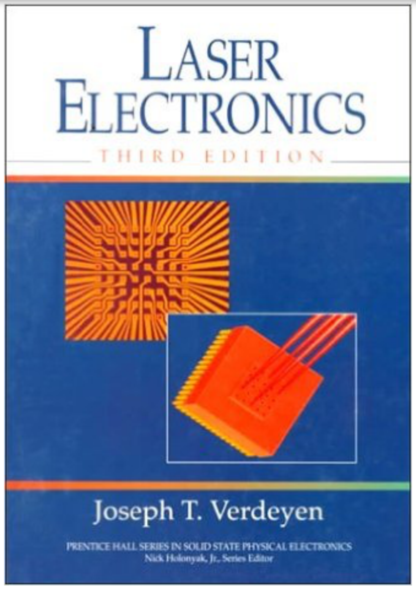
✓

5 Optical Cavities

✓



Course Outline



Higher order modes: Hermite-Gaussian Laser Beams

$$\frac{E(x,y,z)}{E_0} = \left\{\frac{w_0}{w(z)}\exp\left[-\frac{r^2}{w^2(z)}\right]\right\} \quad \text{amplitude factor}$$

$$\times \exp\left\{-j\left[kz - \tan^{-1}\left(\frac{z}{z_0}\right)\right]\right\} \quad \text{longitudinal phase}$$

$$\times \exp\left[-j\frac{kr^2}{2R(z)}\right] \quad \text{radial phase}$$

where

$$w^2(z) = w_0^2\left[1 + \left(\frac{\lambda_0 z}{\pi n w_0^2}\right)^2\right] = w_0^2\left[1 + \left(\frac{z}{z_0}\right)^2\right]$$

$$R(z) = z\left[1 + \left(\frac{\pi n w_0^2}{\lambda_0 z}\right)^2\right] = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$$

$$z_0 = \frac{\pi n w_0^2}{\lambda_0}$$

$$\frac{E(x,y,z)}{E_{m,p}} = H_m\left[\frac{2^{1/2}x}{w(z)}\right] H_p\left[\frac{2^{1/2}y}{w(z)}\right]$$

$$\times \frac{w_0}{w(z)}\exp\left[-\frac{x^2+y^2}{w^2(z)}\right]$$

$$\times \exp\left\{-j\left[kz - (1+m+p)\tan^{-1}\left(\frac{z}{z_0}\right)\right]\right\}$$

$$\times \exp\left[-j\frac{kr^2}{2R(z)}\right]$$

$$H_m(u) = (-1)^m e^{u^2}\frac{d^m e^{-u^2}}{du^m}$$

Higher order modes: Hermite-Gaussian Laser Beams

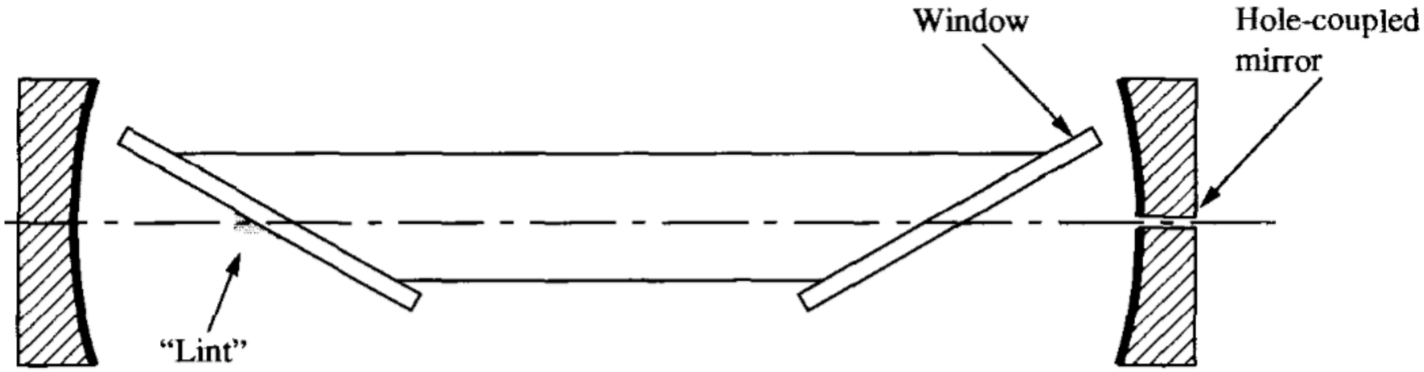


FIGURE 3.4. Simple laser.

$$\frac{E(x,y,z)}{E_{m,p}} = H_m\left[\frac{2^{1/2}x}{w(z)}\right] H_p\left[\frac{2^{1/2}y}{w(z)}\right] \times \frac{w_0}{w(z)}\exp\left[-\frac{x^2+y^2}{w^2(z)}\right] \times \exp\left\{-j\left[kz-(1+m+p)\tan^{-1}\left(\frac{z}{z_0}\right)\right]\right\} \times \exp\left[-j\frac{kr^2}{2R(z)}\right]$$

$$H_m(u) = (-1)^m e^{u^2}\frac{d^m e^{-u^2}}{du^m}$$

$$H_0(u) = 1 \Rightarrow 1$$

$$H_1(u) = 2(u) \Rightarrow u$$

$$H_2(u) = (2u^2-1)2 \Rightarrow 2u^2-1$$

Higher order modes: Hermite-Gaussian Laser Beams

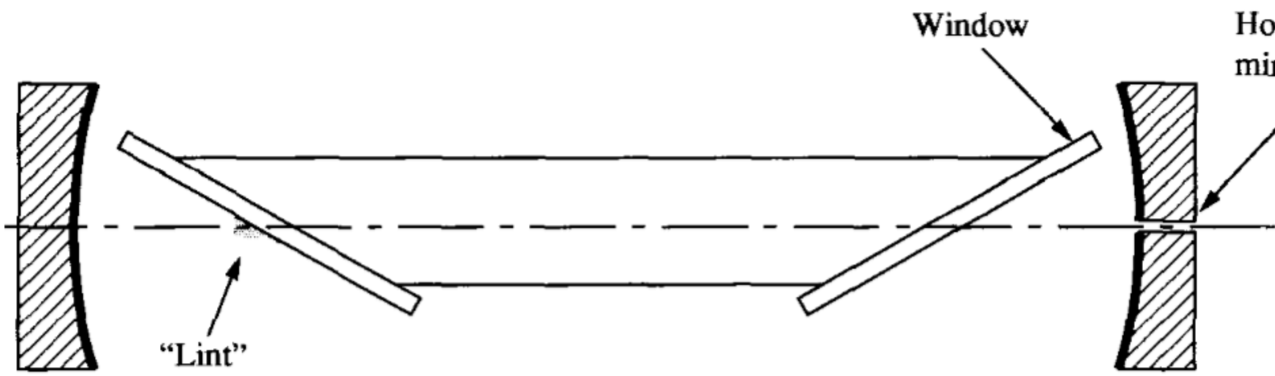


FIGURE 3.4. Simple laser.

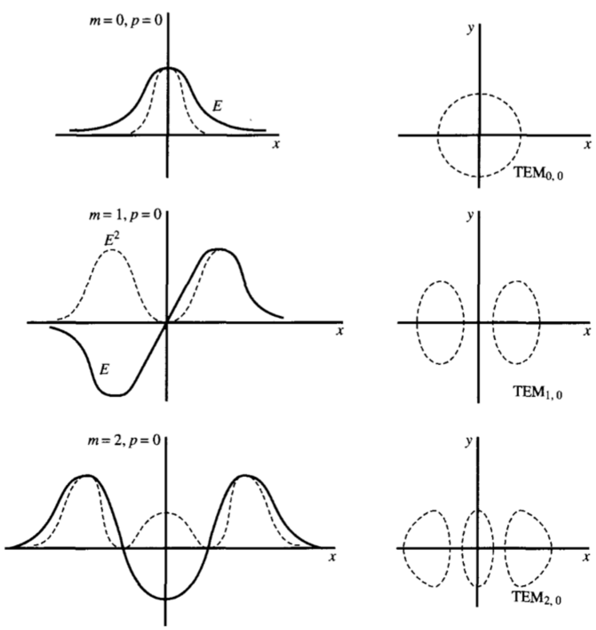


FIGURE 3.5. The field E, intensity E^2, and "dot" pattern of various modes.

The answer is that *all* Hermite-Gaussian beams have *exactly* the same divergence (or far-field angle), given by

$$\theta = \frac{2\lambda_0}{\pi n w_0} \tag{3.7.1}$$

where w_0 is the minimum spot size for the $\mathrm{TEM}_{0,0}$ mode.

Mode Volume

$$E_0^2 V = \int_0^d \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(x, y, z) E^*(x, y, z)\, dx\, dy\, dz$$

$$E_0^2 V_{m,p} = E_0^2 \int_0^d \frac{w_0^2}{w^2(z)} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H_m^2\left(\frac{2^{1/2}x}{w}\right) e^{-2x^2/w^2} \times H_p^2\left(\frac{2^{1/2}y}{w}\right) e^{-2y^2/w^2}\, dy\, dx\, dz$$

This equation can be rearranged and put in a more conventional form by the substitution

$$u = \frac{2^{1/2}x}{w} \quad \text{or} \quad \frac{2^{1/2}y}{w}$$

$$V_{m,p} = \int_0^d \frac{w_0^2}{2}\, dz \left[\int_{-\infty}^{+\infty} H_m^2(u) e^{-u^2}\, du\right] \left[\int_{-\infty}^{+\infty} H_p^2(u) e^{-u^2}\, du\right] \tag{5.4.3}$$

Now

$$\int_{-\infty}^{+\infty} e^{-u^2} H_m^2(u)\, du = 2^m m! \pi^{1/2}$$

Hence, the mode volume is given by

$$\boxed{V_{m,p} = \frac{\pi w_0^2}{2} d(m!p!2^{m+p})} \qquad (Note : 0! = 1) \tag{5.4.4}$$

The first factor in (5.4.4) has the satisfying interpretation: $\sim$ area$(\pi w_0^2/2)$ $\times$ length(d), whereas the last is the modification of this basic volume for the higher-order modes.

Mode Volume: Example

If we take the example considered previously and restrict our attention to the $TEM_{0,0}$ mode, the mode volume is quite small:

$$w_0 = 0.94 \text{ mm} \qquad R_2 = 20 \text{ m}$$
$$d = 1 \text{ m} \qquad R_1 = \infty$$

Therefore

$$V_{0,0} = 1.38 \text{ cm}^3$$

With this number we can quickly estimate the number of atoms that can possibly interact with the mode and thus contribute to the laser output power. For instance, suppose that we had a pressure of 0.1 torr for neon for this example, with each atom being excited (on the average) of 10 times per second (by the gas discharge) and thus producing a photon at 632.8 nm. What is the maximum power that we could expect from this laser?

$$\text{Energy per photon} = h\nu = \frac{hc}{\lambda_0} = 3.14 \times 10^{-19} \text{ J} = 1.96 \text{ eV}$$

$$\times$$

$$\text{Number of neon atoms} = 0.1(3.54 \times 10^{16}) V_{0,0} \qquad = 4.88 \times 10^{15}$$

$$\times$$

$$\left\{\begin{array}{l} \text{Average excitation per atom} \\ \| \\ \text{Average emission per atom} \end{array}\right\} \qquad = 10 \text{ sec}^{-1}$$

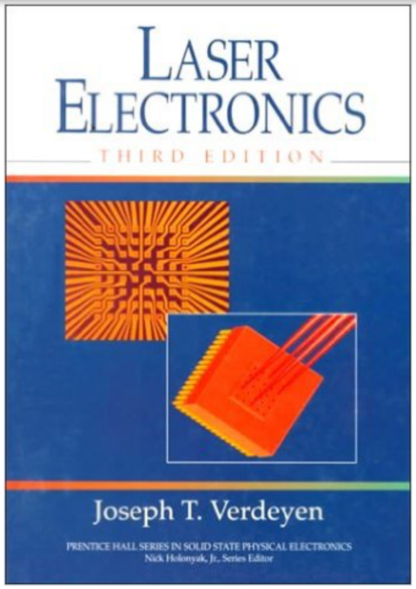
$$\|$$

$$\text{Power} \qquad = 15.3 \text{ mW}$$

This is typical for a laser. There are only a couple ways to increase this power. We could increase the length d to make the mode volume large. But there is a practical limit: A 10-m-long laser would be most unwieldy. If we could excite the atoms at a faster rate, the power would be higher. But as we will see later, the 10(second)$^{-1}$ rate is already optimistic. Thus we are left with changing the mode volume.

We could go to the higher-order modes, and for some applications this is a viable method of extracting more power. But as was pointed out in Sec. 3.6, we are still limited by the divergence of the fundamental mode. Unstable resonators have a much larger mode volume and can be used with high gain lasers. These are covered in Chapter 12.

Course Outline



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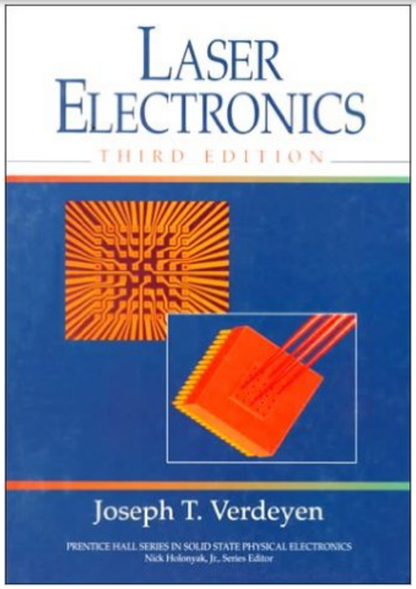

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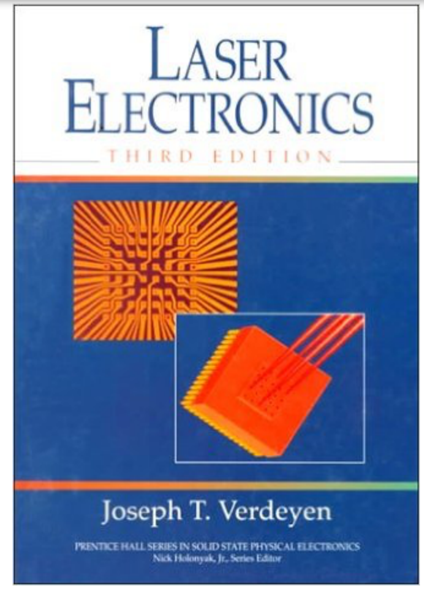


Resonant Cavity with Gain

$$T(\theta) = \frac{I_{\text{trans}}}{I_{\text{inc}}} = \left\{ \frac{(1-R_1)(1-R_2)}{(1-\sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2}\sin^2\theta} \right\}$$

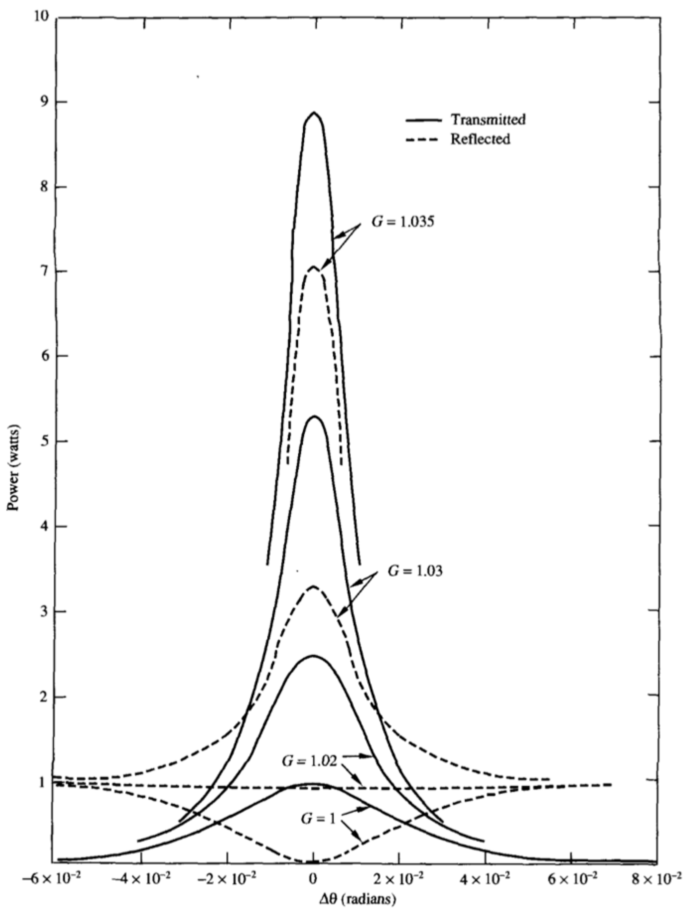
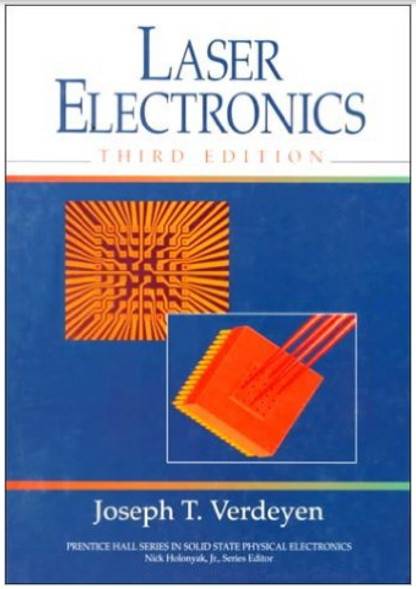


FIGURE 6.8. Response of an active cavity. The parameter G is the single pass power amplification factor.

$$T = \frac{G_0(1-R_1)\cdot(1-R_2)}{(1-G_0\sqrt{R_1 R_2})^2 + 4G_0\sqrt{R_1 R_2}\sin^2\theta}$$

$$\text{FWHM} = 2\Delta\theta_{1/2} = \frac{1-G_0\sqrt{R_1 R_2}}{G_0^{1/2}\sqrt[4]{R_1 R_2}}$$

Course Outline



Blackbody Radiation

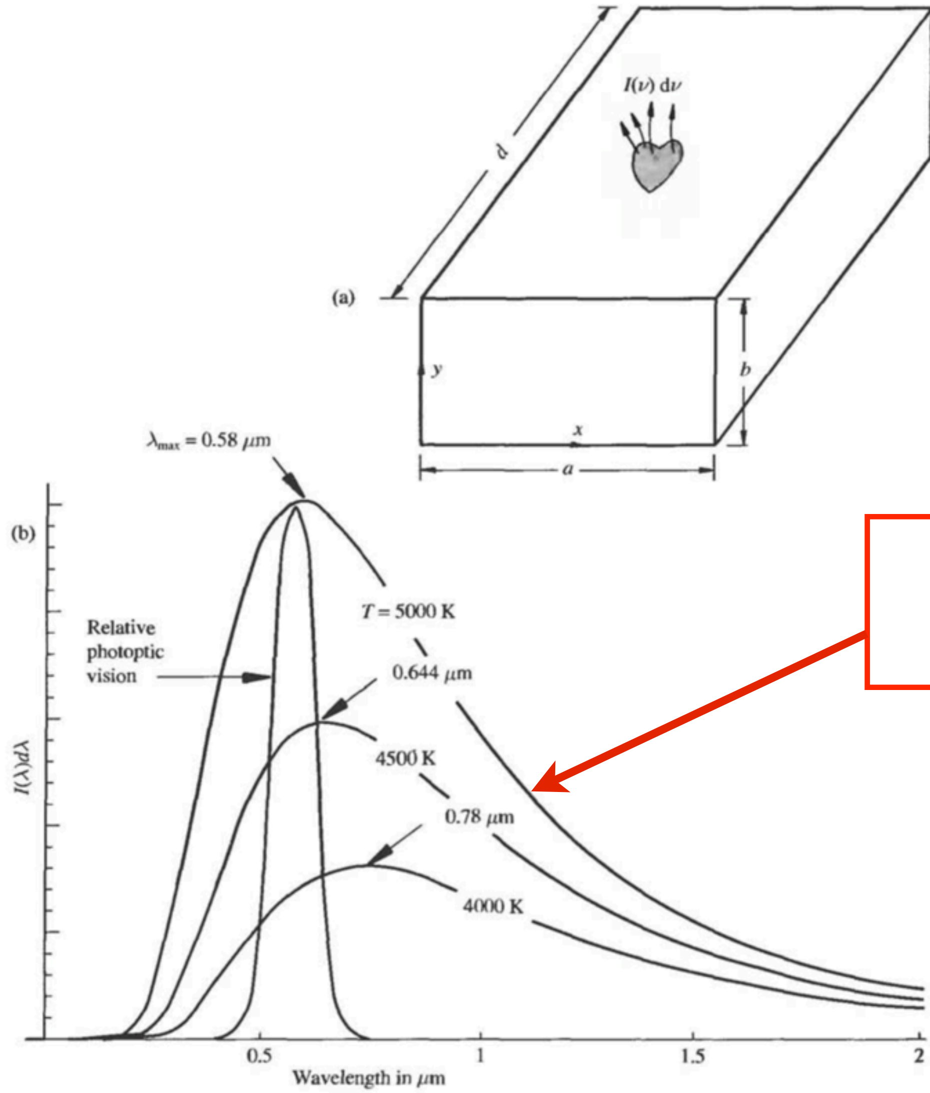
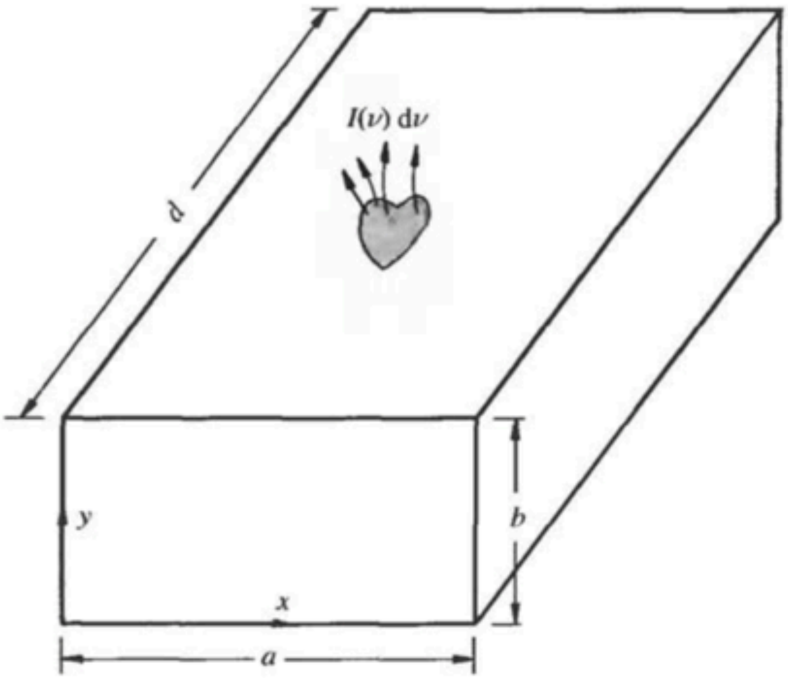


FIGURE 7.2. (a) The cavity used to measure the blackbody spectrum; (b) $I(\lambda)d\lambda$ from experiment. (NOTE: $\lambda_{max}T = 2.898 \times 10^7$ Å-°K.) The bell-shaped curve in (b) illustrates the response of the eye.

$$\boxed{\rho(\nu)\,d\nu = \frac{8\pi h\nu^3\,d\nu}{c^3}\,\frac{1}{e^{h\nu/kT}-1}}$$

Blackbody Radiation



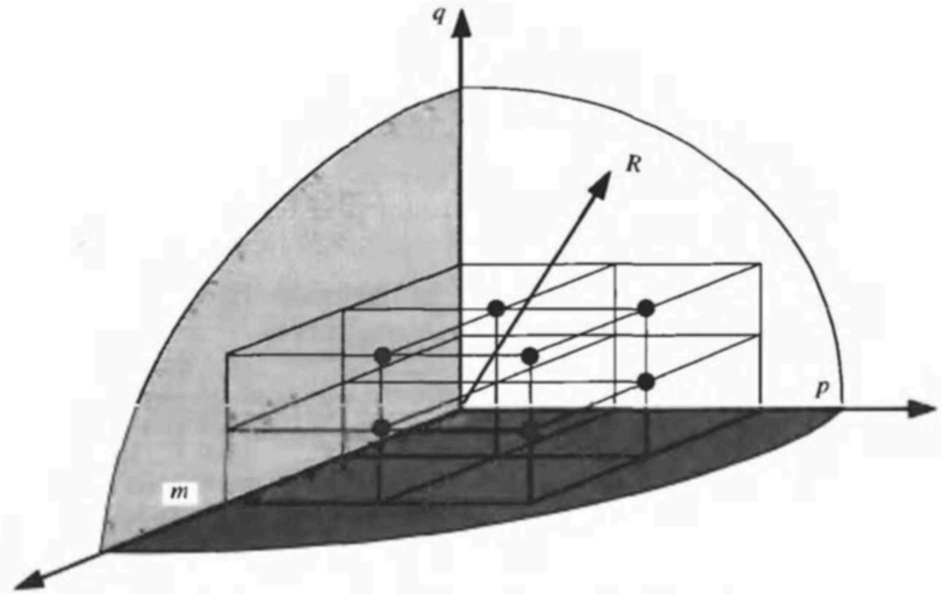


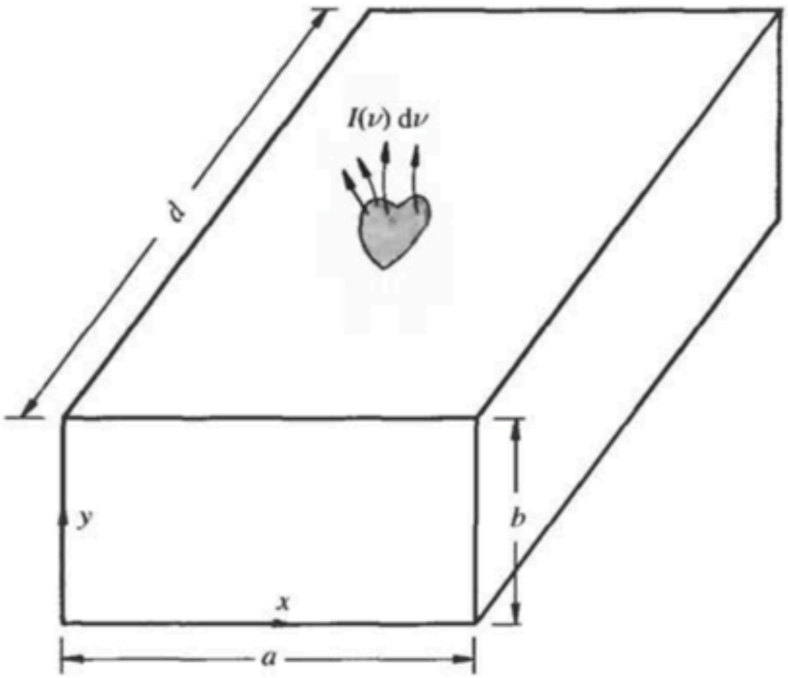
FIGURE 7.3. The mode diagram for cubic cavity.

Mode Density

$$p(\nu)d\nu = \frac{1}{V}\frac{dN}{d\nu}d\nu = \frac{8\pi n^2 \bar{n}_g}{c^3}\nu^2 d\nu \cong \frac{8\pi n^3}{c^3}\nu^2 d\nu$$

$$\bar{n}_g = n + \nu\frac{dn}{d\nu} \doteq n - \lambda\frac{dn}{d\lambda}$$

Blackbody Radiation: Planck's Derivation



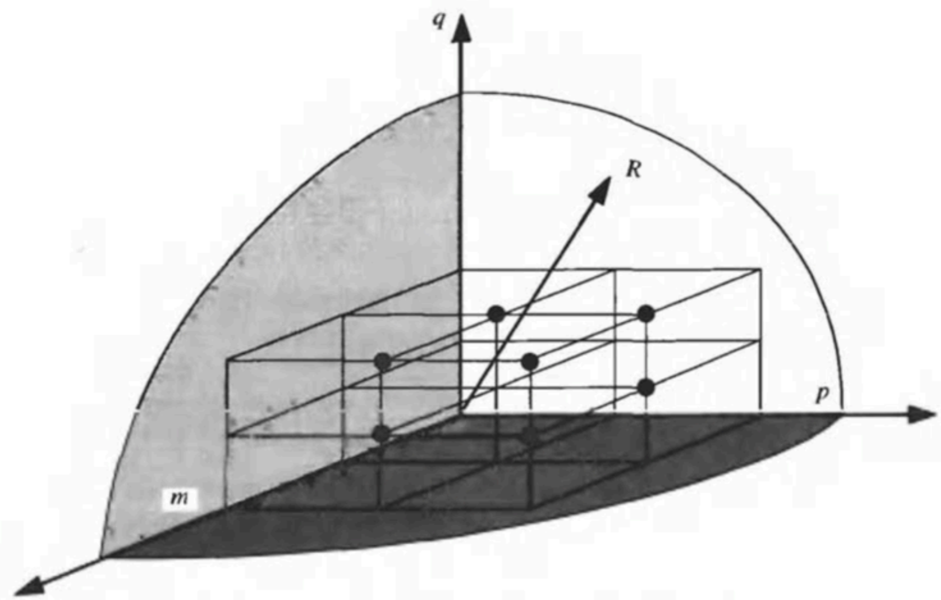
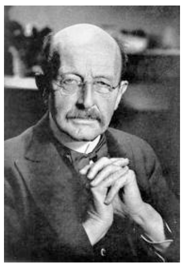


FIGURE 7.3. The mode diagram for cubic cavity.



$$\epsilon_n = nh\nu \qquad \text{(the quantum hypothesis)}$$

The Blackbody Spectrum

$$\boxed{\rho(\nu) = \left(\frac{8\pi n^2 n_g \nu^2}{c^3}\right) \cdot (h\nu) \cdot \frac{1}{e^{h\nu/kT} - 1} = \frac{8\pi n^2 n_g \nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}}$$

Blackbody Radiation: Einstein's Derivation

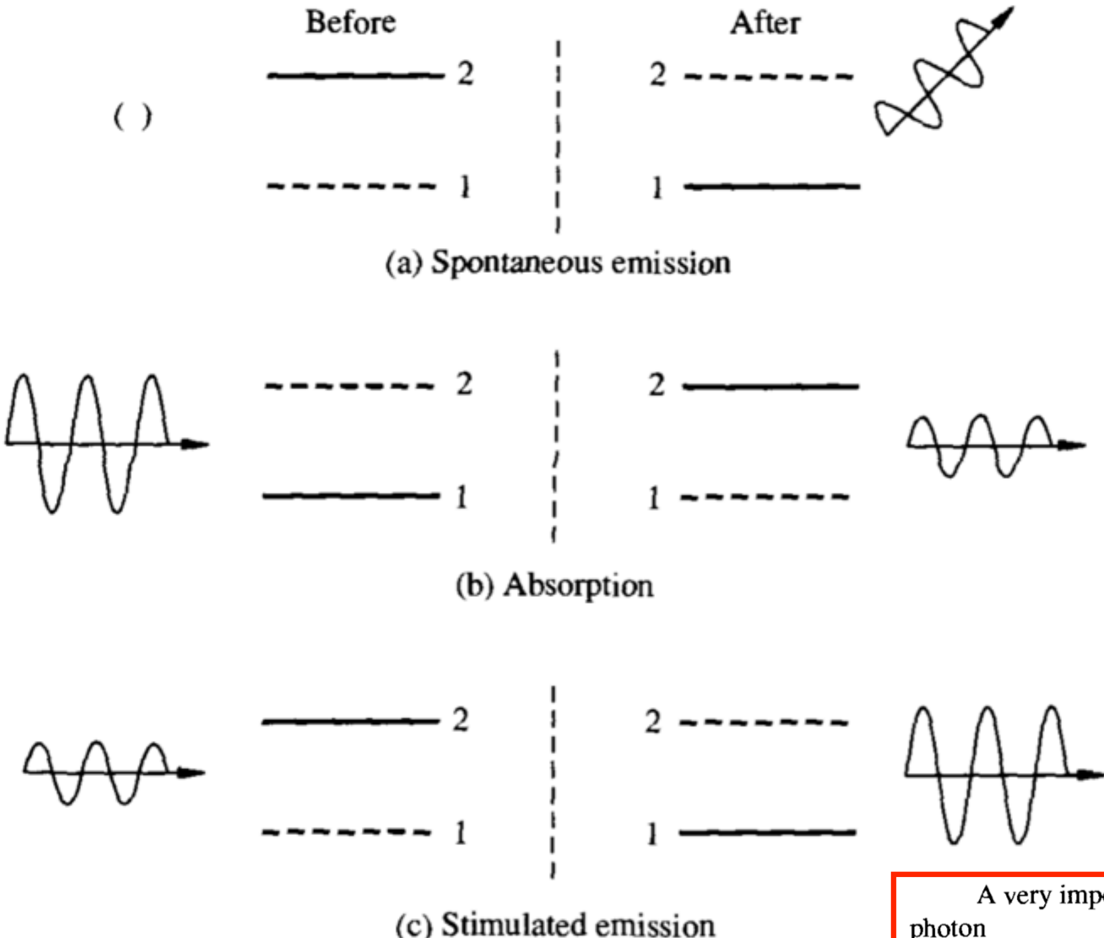


FIGURE 7.5. Effect of radiation on an atom.

A very important point to remember from Fig. 7.5 is that stimulated emission adds a photon

1. *At the same frequency* of the stimulating wave
2. *In the same polarization* of the stimulating wave
3. *In the same direction* of travel of the stimulating wave
4. *In the same phase* of the stimulating wave

Blackbody Radiation: Einstein's Derivation

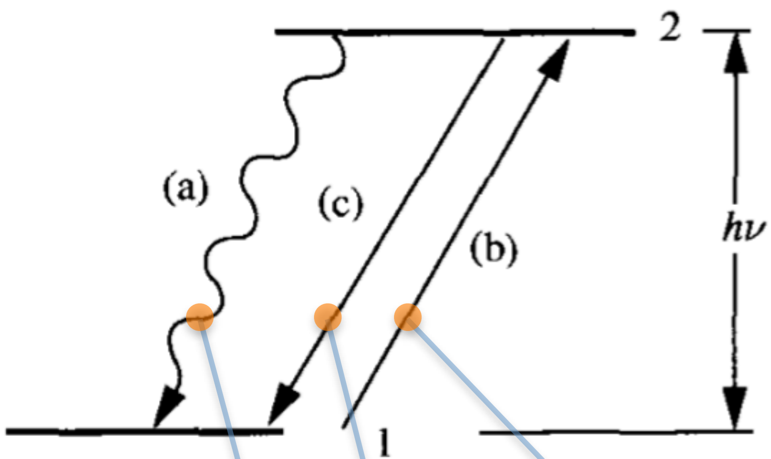


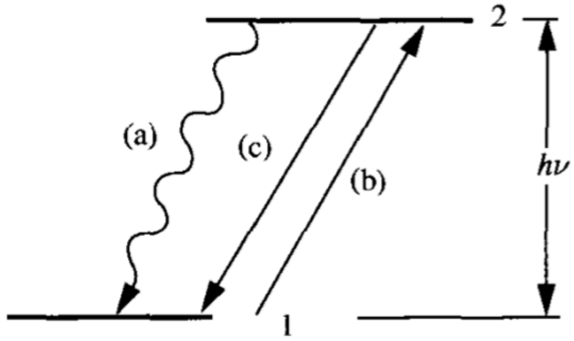
FIGURE 7.4. Radiative processes in a two-level system.

$$\left.\frac{dN_2}{dt}\right|_{\substack{\text{spontaneous}\\\text{emission}}} = -A_{21}N_2$$

$$\left.\frac{dN_2}{dt}\right|_{\text{absorption}} = +B_{12}N_1\rho(\nu) = -\left.\frac{dN_1}{dt}\right|_{\text{absorption}}$$

$$\left.\frac{dN_2}{dt}\right|_{\substack{\text{stimulated}\\\text{emission}}} = -B_{21}N_2\rho(\nu) = -\left.\frac{dN_1}{dt}\right|_{\substack{\text{stimulated}\\\text{emission}}}$$

Blackbody Radiation: Einstein's Derivation



$$\left.\frac{dN_2}{dt}\right|_{\substack{\text{spontaneous}\\\text{emission}}} = -A_{21}N_2$$

$$\left.\frac{dN_2}{dt}\right|_{\text{absorption}} = +B_{12}N_1\rho(\nu)$$

$$\left.\frac{dN_2}{dt}\right|_{\substack{\text{stimulated}\\\text{emission}}} = -B_{21}N_2\rho(\nu)$$

$$\boxed{\left.\frac{dN_2}{dt}\right|_{\text{radiative}} = -A_{21}N_2 + B_{12}N_1\rho(\nu) - B_{21}N_2\rho(\nu)} = -\left.\frac{dN_1}{dt}\right|_{\text{radiative}} \qquad (7.3.4)$$

At *equilibrium*, the time rate of change must be zero.

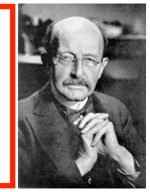
$$\frac{N_2}{N_1} = \frac{B_{12}\rho(\nu)}{A_{21} + B_{21}\rho(\nu)} \qquad (7.3.5)$$

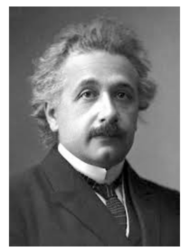
Einstein invoked classic Boltzmann statistics to provide another equation for the ratio of the two populations in states 2 and 1 and set that value equal to (7.3.5):

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-h\nu/kT} = \frac{B_{12}\rho(\nu)}{A_{21} + B_{21}\rho(\nu)} \qquad (7.3.6)$$

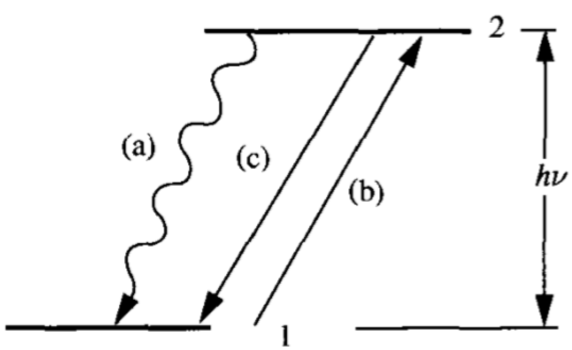
Blackbody Radiation: Einstein's Derivation

$$\boxed{\rho(\nu) = \left(\frac{8\pi n^2 n_g \nu^2}{c^3}\right) \cdot (h\nu) \cdot \frac{1}{e^{h\nu/kT} - 1} = \frac{8\pi n^2 n_g \nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}}$$





$$\boxed{\rho(\nu) = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\frac{B_{12} g_1}{B_{21} g_2} e^{h\nu/kT} - 1}}$$



$$\frac{B_{12} g_1}{B_{21} g_2} = 1 \quad \text{or} \quad \boxed{g_2 B_{21} = g_1 B_{12}}$$

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi n^2 n_g h \nu^3}{c^3}}$$

Line Shape

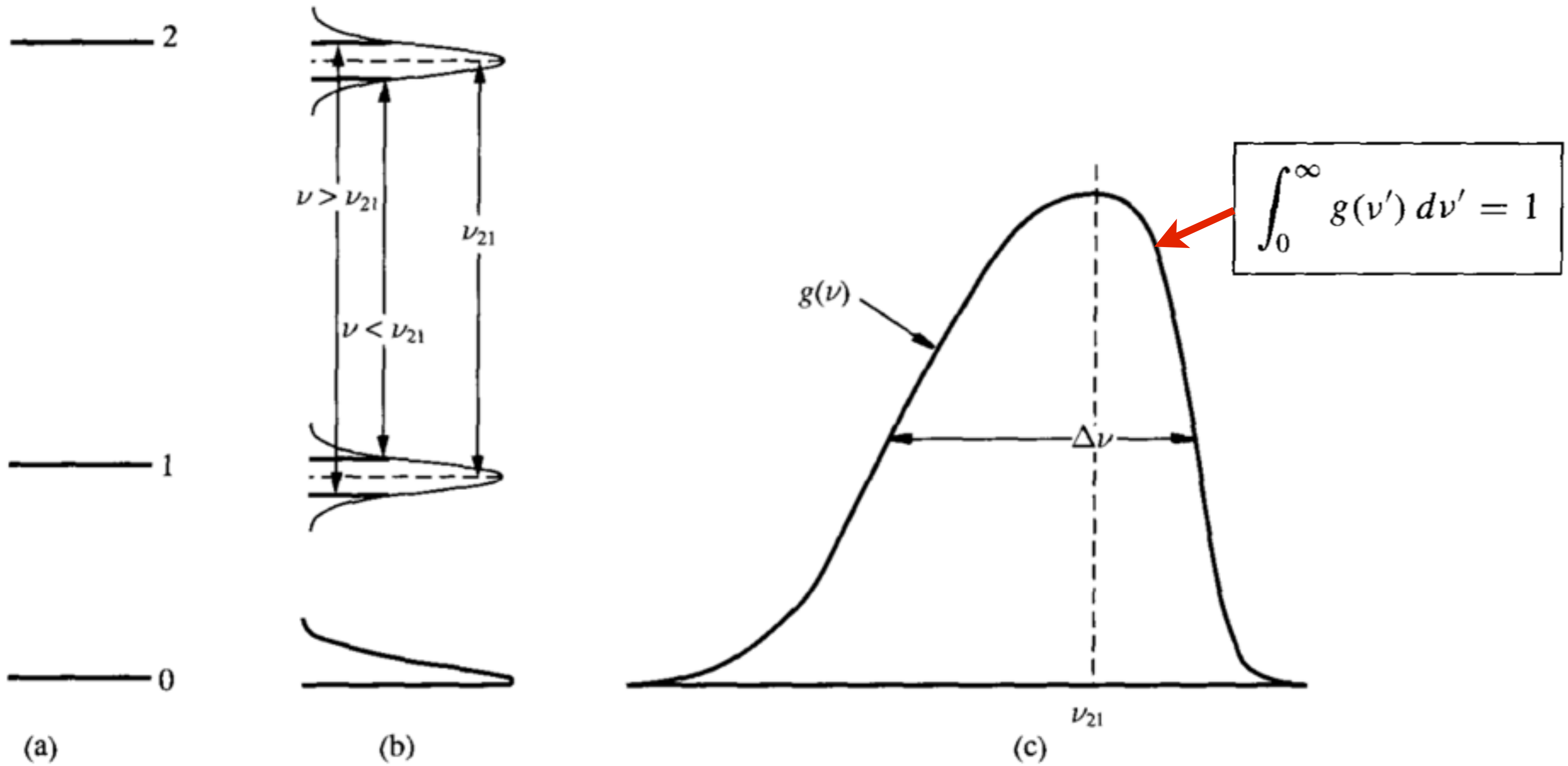


FIGURE 7.6. The evolution of the energy level diagram in (a) emitting a zero width line to broadened levels in (b) yielding the spectral line shape shown in (c).

Optical transitions of an atom are "dressed" by the electromagnetic surrounding. This dressing changes the optical transition of the atom and gives it a ***Line Shape****.*

Line Shape

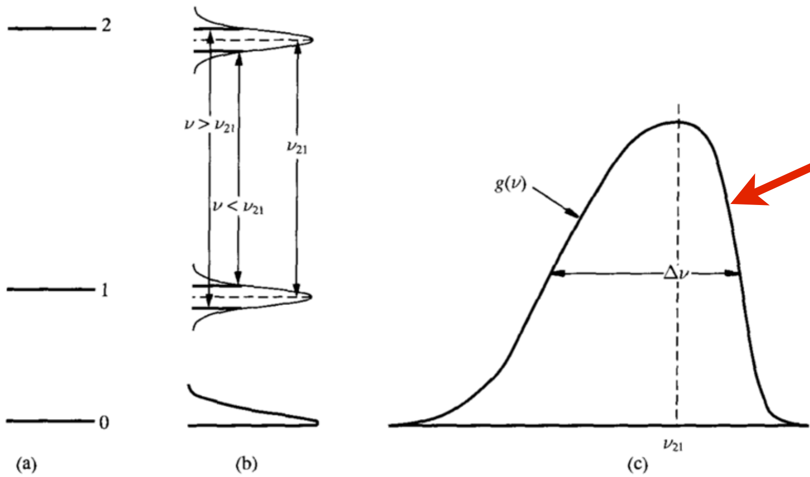


FIGURE 7.6. The evolution of the energy level diagram in (a) emitting a zero width line to broadened levels in (b) yielding the spectral line shape shown in (c).

$$\boxed{\int_0^{\infty} g(\nu')\, d\nu' = 1}$$

$$\left.\frac{dN_2}{dt}\right|_{\text{radiative}} = -A_{21}N_2 + B_{12}N_1\rho(\nu) - B_{21}N_2\rho(\nu)$$

$$\left.\frac{dN_2}{dt}\right|_{\text{radiative}} = -A_{21}N_2[\int_0^{\infty} g(\nu')\, d\nu' = 1] + B_{12}N_1 \int_0^{\infty} \rho(\nu')g(\nu')\, d\nu' - B_{21}N_2 \int_0^{\infty} \rho(\nu')g(\nu')\, d\nu'$$

$$\rho(\nu') \approx \rho_\nu \delta(\nu' - \nu) \qquad \text{(i.e., only one frequency)}$$

$$\boxed{\left.\frac{dN_2}{dt}\right|_{\text{radiative}} = -A_{21}N_2 - B_{21}N_2\rho_\nu g(\nu) + B_{12}N_1\rho_\nu g(\nu)}$$

$$\rho_\nu \left(\frac{\text{joules}}{\text{volume}}\right) = \frac{I_\nu(\text{watts/area} = \text{joules} \cdot \text{time}^{-1} \cdot L^{-2})}{\text{photon velocity} = L/T}$$

$$\rho_\nu = \frac{I_\nu}{\text{group velocity} = c/n_g} \qquad \text{(elementary EM theory)}$$

Line Shape: Effect on Radiative Transition Rate

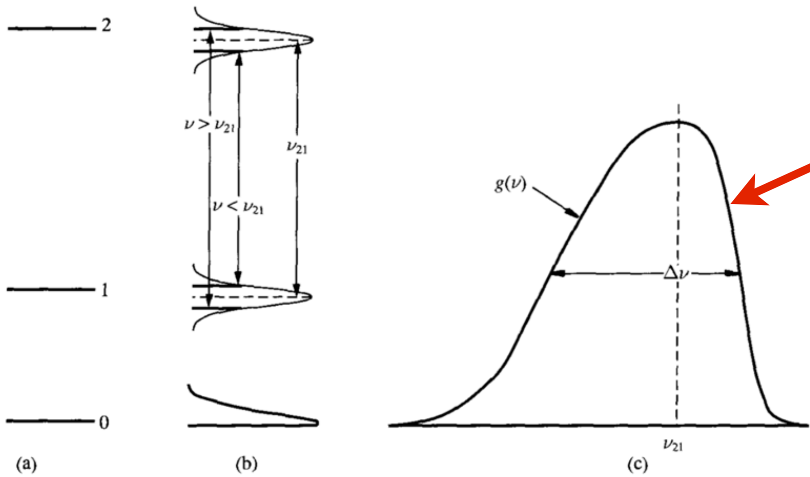


FIGURE 7.6. The evolution of the energy level diagram in (a) emitting a zero width line to broadened levels in (b) yielding the spectral line shape shown in (c).

$$\boxed{\int_0^\infty g(\nu')\,d\nu' = 1}$$

$$\rho(\nu') \approx \rho_\nu \delta(\nu' - \nu) \qquad \text{(i.e., only one frequency)}$$

$$\left.\frac{dN_2}{dt}\right|_{\text{radiative}} = -A_{21}N_2 - B_{21}N_2\rho_\nu g(\nu) + B_{12}N_1\rho_\nu g(\nu)$$

$$\boxed{\left.\frac{dN_2}{dt}\right|_{\text{radiative}} = -A_{21}N_2 - \frac{\sigma(\nu)I_\nu}{h\nu}\left[N_2 - \frac{g_2}{g_1}N_1\right]}$$

$$\boxed{\text{stimulated emission cross section} = \sigma(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)}$$

Amplification *by an Atomic System*

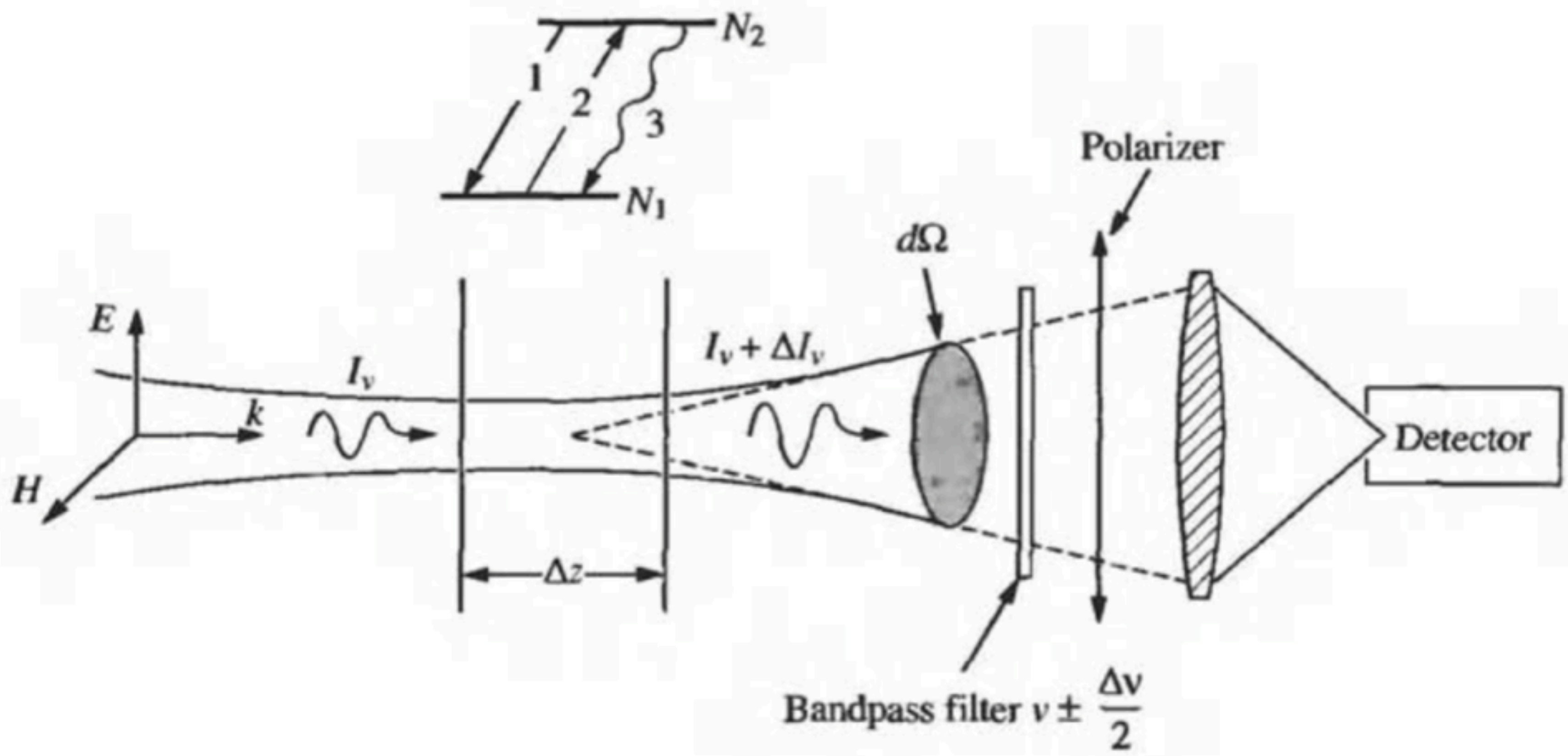


FIGURE 7.7. Measurement of the gain of an atomic system.

Amplification *by an Atomic System*

Thus the output consists of the input intensity *plus* that added by the following processes:

1. *Stimulated emission*: the amount of radiation stimulated by the incoming wave. Since this is stimulated emission, the frequency, phase, and direction of the added signal are the same as the incoming wave (this is indicated by process 1 on the energy-level diagram)

minus:

2. *Absorption*: the amount of radiation absorbed by the atoms in state 1

plus:

3. *Spontaneous emission*: the amount of radiation emitted spontaneously by the atoms in state 2 in the direction of the input wave and in the same frequency within bandwidth $\Delta\nu$ of our detector (this is indicated by the wavy line going from state 2 to state 1).

$$\boxed{\left.\frac{dN_2}{dt}\right|_{\text{radiative}} = -A_{21}N_2 - \frac{\sigma(\nu)I_\nu}{h\nu}\left[N_2 - \frac{g_2}{g_1}N_1\right]}$$

$$\begin{array}{cccccccccccc}
 & 1 & & 2 & & 3 & & 4 & & 5 & & 6 \\
\Delta I_\nu = & +h\nu & \times & B_{21}\dfrac{I_\nu}{(c/n_g)} & \times & g(\nu) & \times & 1 & \times & 1 & \times & N_2\Delta z \\
 & -h\nu & \times & B_{12}\dfrac{I_\nu}{(c/n_g)} & \times & g(\nu) & \times & 1 & \times & 1 & \times & N_1\Delta z \\
 & +h\nu & \times & A_{21}\Delta\nu & \times & g(\nu) & \times & \dfrac{1}{2} & \times & \dfrac{d\Omega}{4\pi} & \times & N_2\Delta z
\end{array}$$

Further manipulation yields

$$\frac{\Delta I_\nu}{\Delta z} \rightarrow \frac{dI_\nu}{dz} = \left[\frac{h\nu}{(c/n_g)}(B_{21}N_2 - B_{12}N_1)g(\nu)\right]I_\nu + \frac{1}{2}\left[h\nu A_{21}N_2 g(\nu)\Delta\nu\frac{d\Omega}{4\pi}\right]$$

Amplification *by an Atomic System*

$$\boxed{\frac{dI_\nu}{dz} = \left\{\left[A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)\right]\left[N_2 - \frac{g_2}{g_1}N_1\right]I_\nu\right\} \triangleq \gamma(\nu)I_\nu}$$

$$\boxed{N_2 > \frac{g_2}{g_1}N_1 \qquad \text{for gain}}$$

$$\boxed{\gamma(\nu) \triangleq \frac{1}{I_\nu}\left(\frac{dI_\nu}{dz}\right)} \qquad \gamma(\nu) \triangleq \frac{[dI_\nu/dz] = \text{net power emitted per unit of volume}}{I_\nu = \text{power per unit of area traversing that volume}}$$

$$\gamma(\nu) = \frac{1}{I_\nu}\cdot h\nu\left\{\frac{dN_2}{dt}\bigg|_{(\text{stim}-\text{abs})} = \frac{\sigma(\nu)I_\nu}{h\nu}\left[N_2 - (g_2/g_1)N_1\right]\right\}$$

$$= \sigma(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right]$$

Broadening of Spectral Lines

All transitions have a finite width if, for no other reason, to ensure compliance with the uncertainty principle. There are two generic classifications of processes that contribute to the width of a spectral line. They are

1. Homogeneous broadening: a mechanism that applies to all atoms.
2. Inhomogeneous broadening: one that is caused by some identifiable difference between groups of atoms.

$$P_{2,1}(E) = \frac{(\Delta E_{2,1})}{2\pi\left[(E - E_{2,1})^2 + (\Delta E_{2,1}/2)^2\right]}$$

for band 1: $E = x + E_1$ and $a = \Delta E_1/2$

for band 2: $E = x + E_1 + h\nu$ and $b = \Delta E_2/2$

and define $\delta = h\nu - (E_2 - E_1)$

Then

$$g(h\nu) = \int_{-\infty}^{+\infty} \left\{ \frac{\Delta E_1}{2\pi} \frac{1}{[x^2 + a^2]} \right\} \left\{ \frac{\Delta E_2}{2\pi} \frac{1}{[(x+\delta)^2 + b^2]} \right\} dx$$

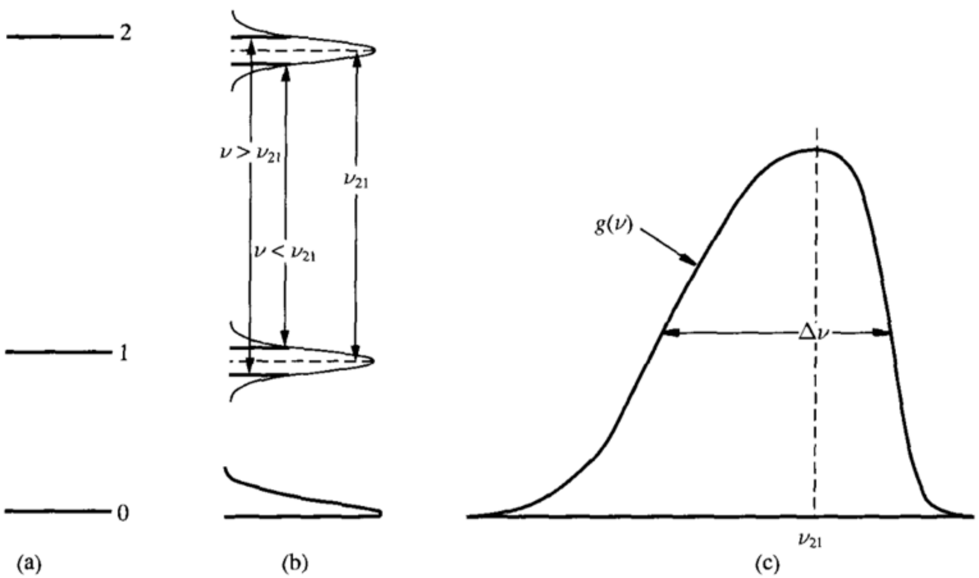


FIGURE 7.6. The evolution of the energy level diagram in (a) emitting a zero width line to broadened levels in (b) yielding the spectral line shape shown in (c).

$$g(\nu) = \frac{\Delta\nu}{2\pi\left[(\nu_0 - \nu)^2 + (\Delta\nu/2)^2\right]}$$

$$\Delta\nu = \frac{1}{2\pi}\left\{\frac{1}{\tau_2} + \frac{1}{\tau_1}\right\}$$

Homogeneous Broadening of Spectral Lines

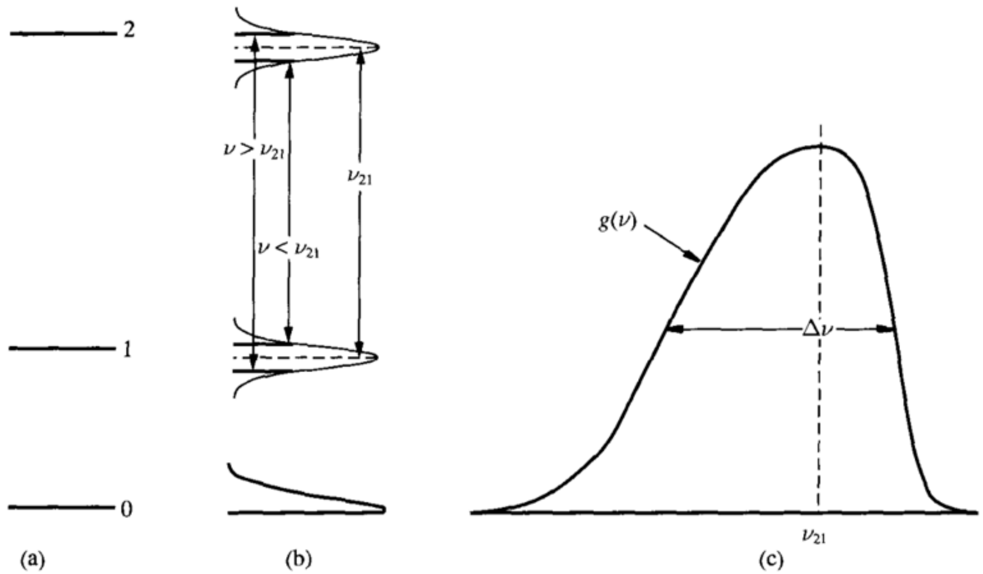


FIGURE 7.6. The evolution of the energy level diagram in (a) emitting a zero width line to broadened levels in (b) yielding the spectral line shape shown in (c).

$$P_{2,1}(E) = \frac{(\Delta E_{2,1})}{2\pi\left[(E - E_{2,1})^2 + (\Delta E_{2,1}/2)^2\right]}$$

$$(\Delta E_{1,2}) \cdot (\tau_{1,2}) = 1$$

$$g(\nu) = \frac{\Delta\nu}{2\pi\left[(\nu_0 - \nu)^2 + (\Delta\nu/2)^2\right]}$$

$$\Delta\nu = \frac{1}{2\pi}\left\{\frac{1}{\tau_2} + \frac{1}{\tau_1}\right\}$$

$$\frac{dN_2}{dt} = \left[\sum_{j<2} A_{2j}\right] N_2 - k_2 N_2 \triangleq -\frac{N_2}{\tau_2}$$

$$\frac{1}{\tau_{\mathrm{rad}}} = \sum_{j<2} A_{2j}$$

$$\phi_{21} = \frac{\text{rate of decay from states 2 to 1}}{\text{sum of all decay rates from state 2}} = \frac{A_{21}}{\sum A_{2j} + k_2}$$

$$\Delta\nu_n = \frac{1}{2\pi}\left\{A_2 + A_1\right\}$$

Homogeneous Broadening of Spectral Lines

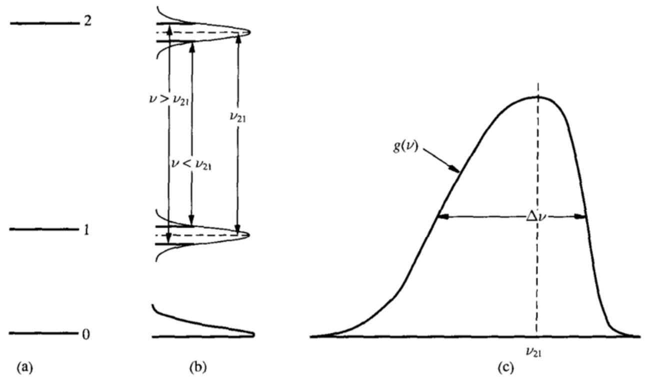


FIGURE 7.6. The evolution of the energy level diagram in (a) emitting a zero width line to broadened levels in (b) yielding the spectral line shape shown in (c).

Collision broadening (also known as *pressure broadening* in a gas). Probably the most important homogeneous broadening mechanism is that described by the following reaction equation:

$$[N_{2,1}] + [M] \rightarrow [N_{2,1}] + [M] \tag{7.6.10}$$

Yes, it does appear that nothing is happening since the left-hand side is the same as the right, and, no, it is not a misprint! Equation (7.6.10) describes an "elastic" collision of the atomic states (2,1) with something whose density or concentration is $[M]$. During the "collision," the energy levels $E_{2,1}$ can be considered as functions of time with some of the potential energy being converted to a kinetic variety due to the attractive or repulsive forces between the states and $[M]$. But since it is an elastic collision, the atom emerges with a final energy equal to initial value. Now the time scale for this collision is extremely short, roughly (diameter of the atoms) $\approx 1.0\text{Å} \div$ [thermal velocity of the atoms $\approx 3 \times 10^4$ cm/sec] $\sim 3 \times 10^{-13}$ sec—which is much shorter than the radiative lifetime. This will be discussed in greater detail in Chapter 14. But for now, a simple picture is that such collisions interrupt

$$\Delta\nu = \frac{1}{2\pi}\left[(A_2 + k_2) + (A_1 + k_1) + 2\nu_{col}\right]$$

$$\nu_{col} = N_m \sigma \left[\frac{8kT}{\pi}\left(\frac{1}{M_m} + \frac{1}{M_n}\right)\right]^{1/2}$$

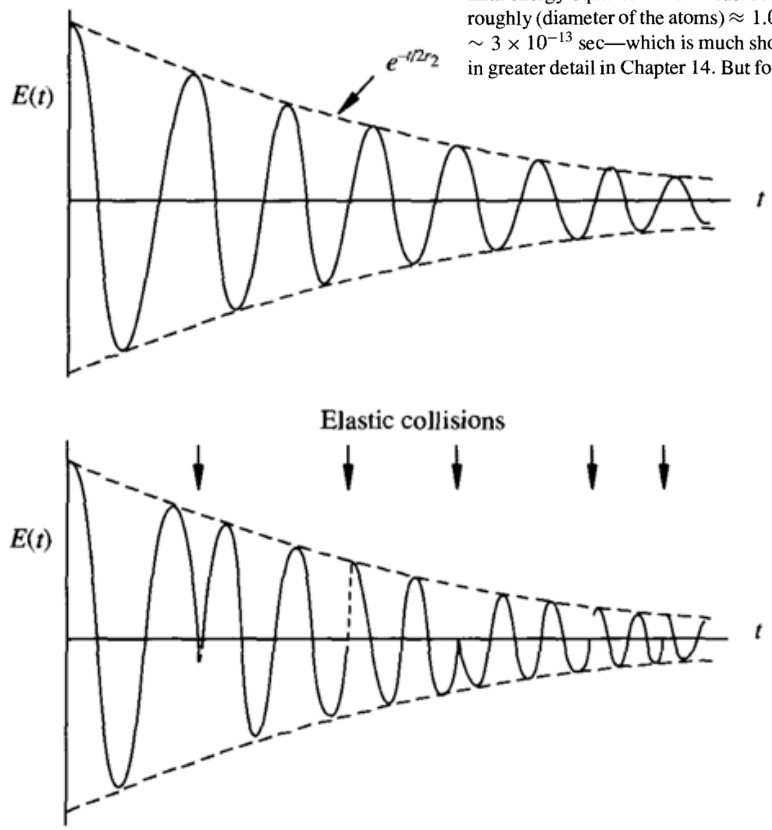


FIGURE 7.8. Classical picture of the effect of (b) elastic collisions on the emission radiation shown in (a).

Homogeneous Broadening of Spectral Lines

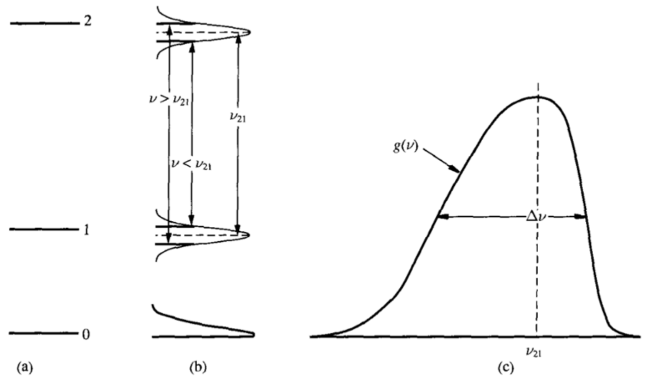


FIGURE 7.6. The evolution of the energy level diagram in (a) emitting a zero width line to broadened levels in (b) yielding the spectral line shape shown in (c).

$$\Delta\nu = \frac{1}{2\pi}\left[(A_2 + k_2) + (A_1 + k_1) + 2\nu_{\text{col}}\right]$$

$$\nu_{\text{col}} = N_m \sigma \left[\frac{8kT}{\pi}\left(\frac{1}{M_m} + \frac{1}{M_n}\right)\right]^{1/2}$$

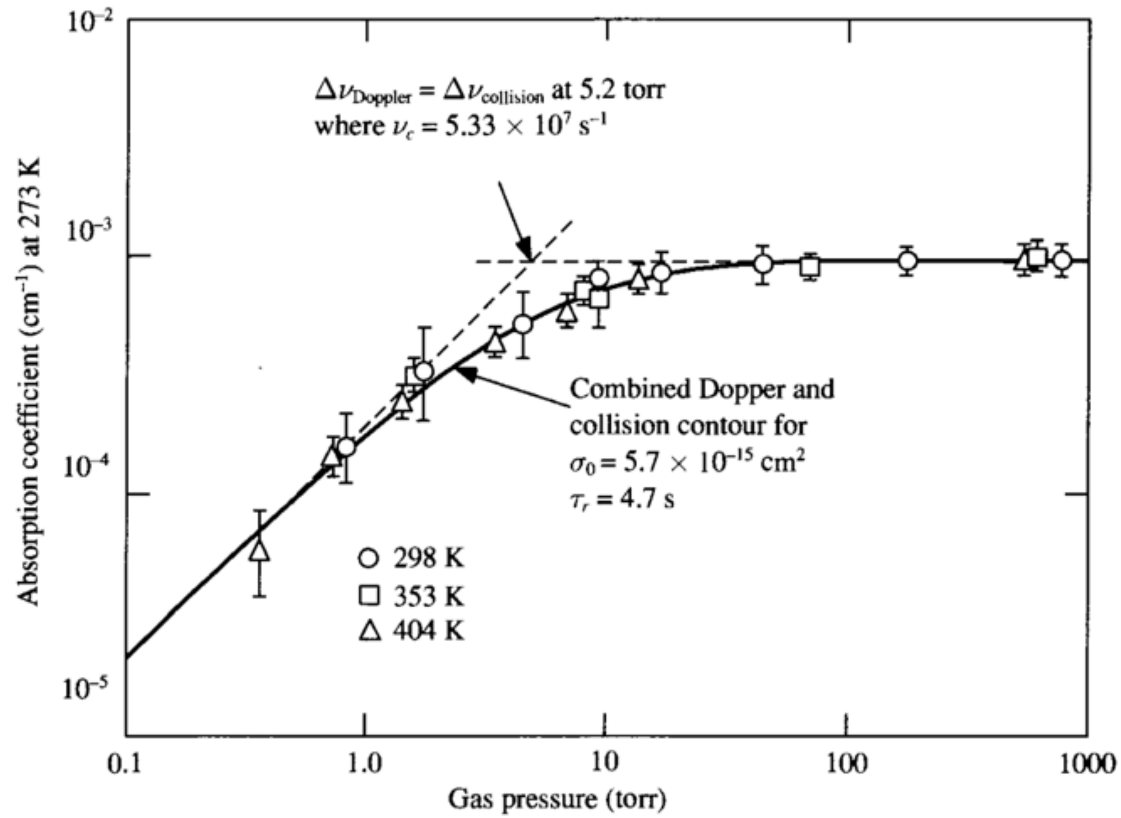


FIGURE 7.9. Absorption coefficient in CO_2 at 10.6 μm as a function of CO_2 pressure. (After E. T. Gerry and D. A. Leonard, *Appl. Phys. Lett.* 8, 227, 1966.)

Homogeneous Broadening of Spectral Lines

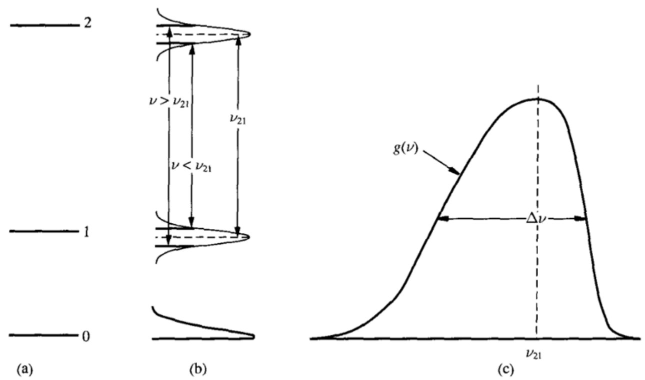


FIGURE 7.6. The evolution of the energy level diagram in (a) emitting a zero width line to broadened levels in (b) yielding the spectral line shape shown in (c).

In all the cases studied, the line shape $g(\nu)$ has the same functional form: the Lorentzian.

$$g_h(\nu) = \frac{\Delta\nu_h}{2\pi\left[(\nu_0 - \nu)^2 + (\Delta\nu_h/2)^2\right]} \tag{7.6.6}$$

where

$$\Delta\nu_h = \frac{1}{2\pi}\left[(A_2 + k_2) + (A_1 + k_1) + 2\nu_{col}\right] \tag{7.6.11}$$

In most practical cases, the last term of (7.6.11) dominates, and the width of the homogeneously broadened line becomes

$$\Delta\nu_h \simeq \frac{\Delta\nu_{col}}{\pi} = \frac{1}{\pi T_2} \tag{7.6.12}$$

where T_2 is the mean time between phase interrupting collisions. If we can distinguish between different groups of atoms under special circumstances, a different functional form of the line shape results.

$$\Delta\nu = \frac{1}{2\pi}\left[(A_2 + k_2) + (A_1 + k_1) + 2\nu_{col}\right]$$

$$\nu_{col} = N_m\sigma\left[\frac{8kT}{\pi}\left(\frac{1}{M_m} + \frac{1}{M_n}\right)\right]^{1/2}$$

Inhomogeneous Broadening of Spectral Lines

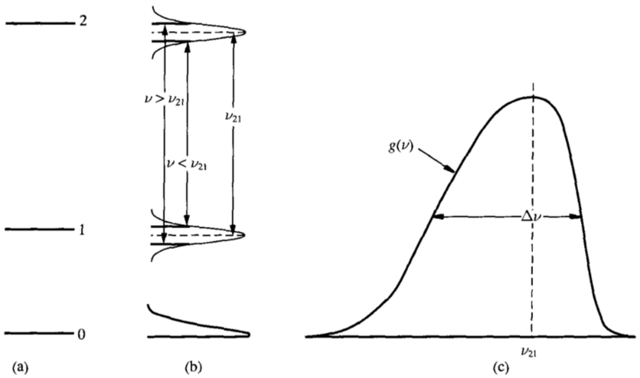


FIGURE 7.6. The evolution of the energy level diagram in (a) emitting a zero width line to broadened levels in (b) yielding the spectral line shape shown in (c).

$$g(\nu) = \left(\frac{4\ln 2}{\pi}\right)^{1/2} \frac{1}{\Delta\nu_D} \exp\left[-4\ln 2\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right]$$

$$(\nu_+ - \nu_-) = \Delta\nu_D = \left(\frac{8kT\ln 2}{Mc^2}\right)^{1/2} \nu_0$$

7.6.2 Inhomogeneous Broadening

$$\nu_0' = \nu_0\left(1 + \frac{v_z}{c}\right)$$

Thus the homogeneous line width radiated by that particular group identified by their velocity is

$$g(v_z, \nu) = \frac{\Delta\nu_h}{2\pi\left[(\nu - \nu_0 - \nu_0 v_z/c)^2 + (\Delta\nu_h/2)^2\right]} \tag{7.6.13}$$

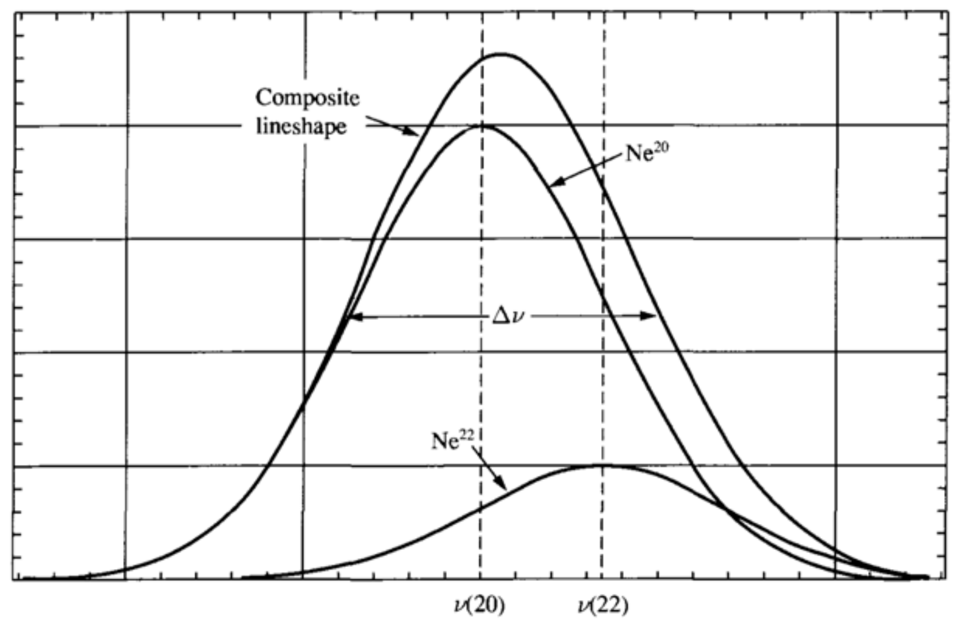


FIGURE 7.10. Inhomogeneous broadening in neon owing to the isotope effect. Each component line is symmetrically broadened owing to the Doppler effect and other homogeneous causes, but the composite line shape is slightly asymmetrical.

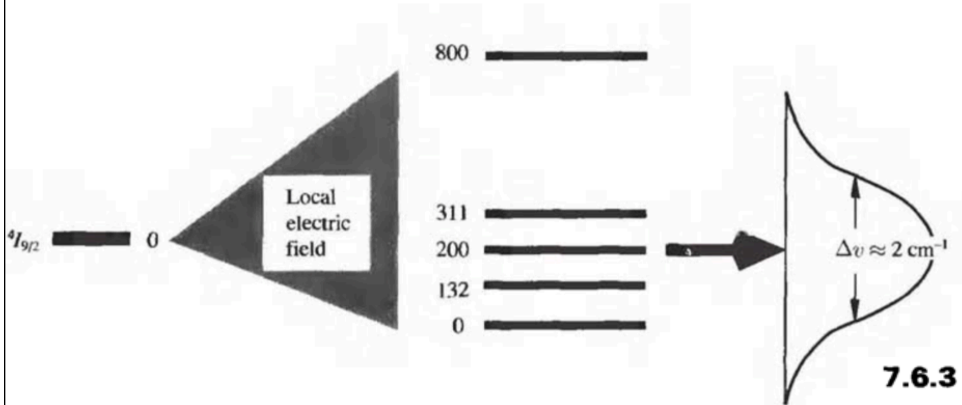


FIGURE 7.11. The Stark splitting of the $^4I_{9/2}$ level of neodymium in YAG (After Kaminski [17]). The numerical values refer to the energies of the levels in cm^{-1}. More detail will be discussed in Chapter 10.

7.6.3 General Comments on the Line Shape

The reader should be cautioned against assuming a direct relationship between the amount of mathematics expended here on a broadening mechanism and its relative importance. Although Doppler and pressure-broadening mechanisms are important, they do not overwhelm all other types (indeed, they do not even apply in a solid). In fact, only the *central portion* of some transitions in a gas is adequately described by the theory presented here.

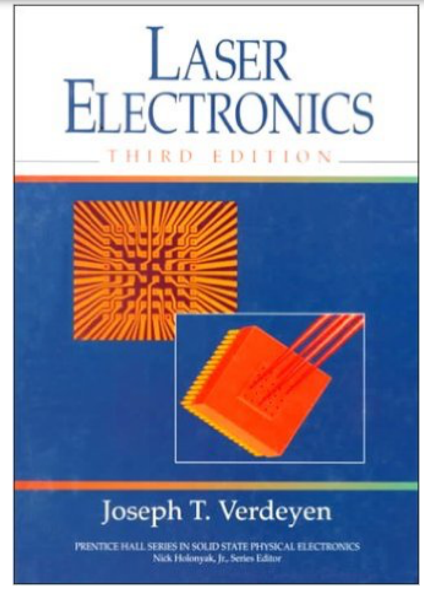
However, the idea of a line shape is most important, quite general, and independent of the maze of mathematics surrounding its development. The *line-shape function*, $g(\nu)\,d\nu$, is the relative probability that

1. A photon emitted by a *spontaneous* transition will appear between ν and $\nu + d\nu$.
2. Radiation in the frequency interval ν to $\nu + d\nu$ can be *absorbed* by atoms in state 1.
3. Radiation in this interval will *stimulate* atoms in state 2 to give up their internal energy.

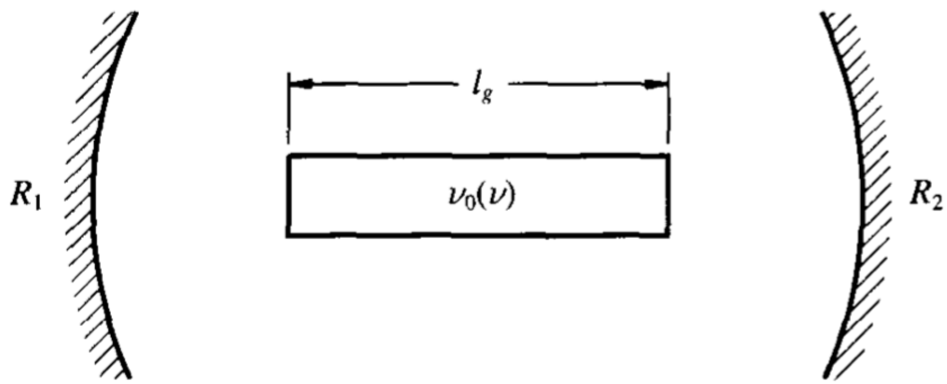
Obviously, the first applies to spontaneous emission, the second to absorption, and the third to stimulated emission. However, the same line-shape function applies to all three processes.

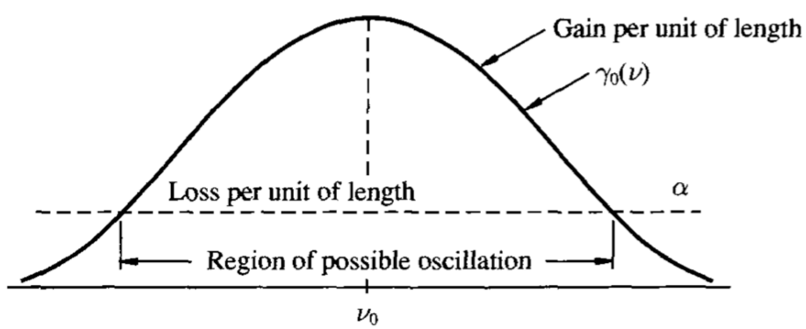
Many of the real-life line-shape functions are asymmetric and mathematically intractable. However, the atoms have no knowledge of and no trouble with *our* arithmetic. In response, we must be prepared to tolerate and use a real-life line-shape function about which we have imperfect information.

Course Outline



Laser Oscillation and Amplification





$$\gamma_0(\nu) = A_{21}\frac{\lambda_0^2}{8\pi n^2}g(\nu)\left(N_2 - \frac{g_2}{g_1}N_1\right)$$

$$G_0 = \exp\left[\gamma_0(\nu) l_g\right]$$

$$R_1 R_2 e^{2\gamma_0(\nu) l_g} \geq 1$$

$$\boxed{\gamma_0(\nu) \geq \frac{1}{2l_g}\ln\left(\frac{1}{R_1 R_2}\right) = \alpha}$$

$$\gamma_0(\nu) = \gamma_0(\nu_0)\frac{(\Delta\nu/2)^2}{(\nu_0 - \nu)^2 + (\Delta\nu/2)^2}$$

$$\left.\frac{dN_2}{dt}\right|_{\text{stimulated emission}} = -B_{21}N_2\rho_\nu g(\nu) = -\frac{\sigma(\nu) I_\nu}{h\nu}N_2$$

Laser Oscillation and Amplification

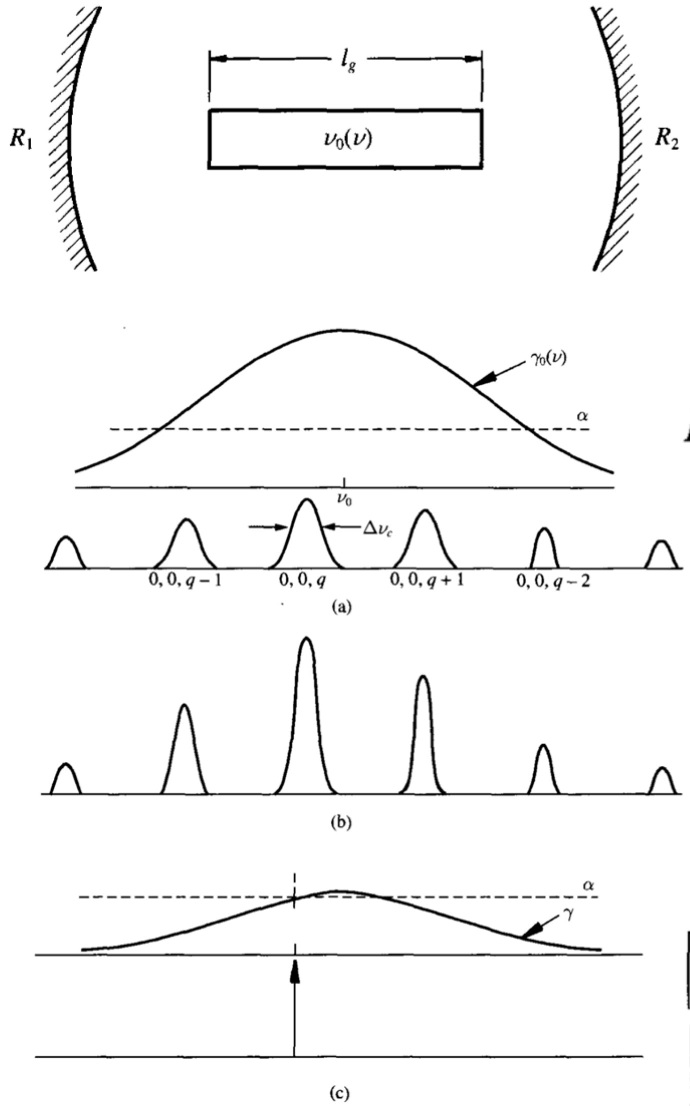


FIGURE 8.3. Evolution of laser oscillation from spontaneous emission: (a) initial; (b) intermediate; and (c) final.

$$\left.\frac{dN_2}{dt}\right|_{\text{stimulated emission}} = -B_{21} N_2 \rho_\nu g(\nu) = -\frac{\sigma(\nu) I_\nu}{h\nu} N_2$$

$$B_{21}\rho_\nu g(\nu) = \frac{c/n_g}{h\nu}\cdot\left\{A_{21}\frac{\lambda_0^2}{8\pi n^2} g(\nu)\right\}\cdot\frac{I_\nu}{c/n_g} = \frac{\sigma(\nu) I_\nu}{h\nu}$$

Stimulated emission can always be neglected for threshold calculations.

Stimulated emission can never be neglected in the dynamics of the laser.

Stimulated vs Spontaneous Emission

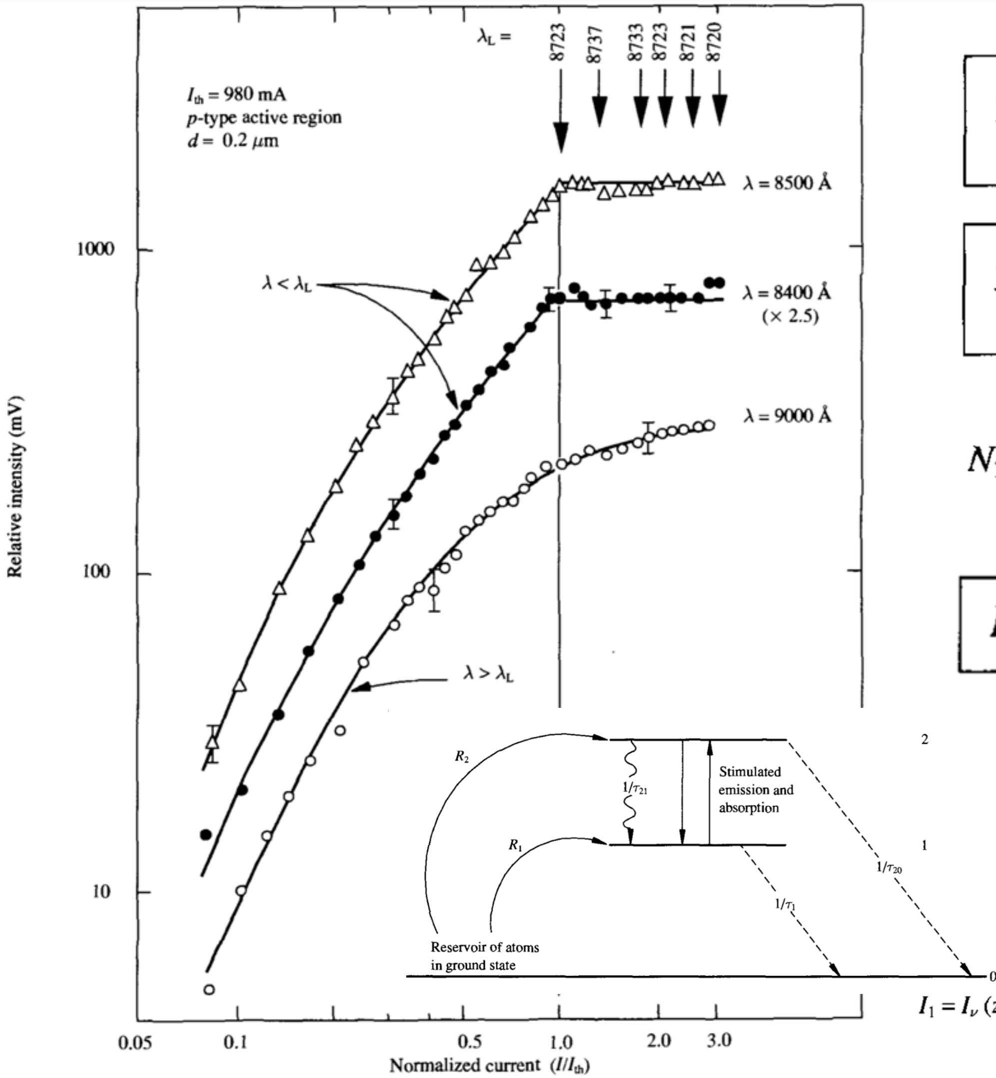


FIGURE 8.6. Variation of the spontaneous emission from the side of a semiconductor diode as the pumping current is increased. Once the diode starts lasing, stimulated emission uses the carriers as fast as they are injected into the junction. Hence the inversion is clamped at threshold. (Data from T. Paoli. *IEEE J. Quant. Electr. QE-9*, 267, 1973.)

$$\boxed{\frac{dN_2}{dt} = R_2(t) - \frac{N_2}{\tau_2} - \frac{\sigma(\nu)I_\nu}{h\nu}[N_2 - N_1]}$$

$$\boxed{\frac{dN_1}{dt} = R_1(t) + \frac{N_2}{\tau_{21}} + \frac{\sigma(\nu)I_\nu}{h\nu}[N_2 - N_1] - \frac{N_1}{\tau_1}}$$

$$N_2(t) = \frac{R_{20}\tau_2}{(1 + I_\nu/I_s)}\left\{1 - \exp\left[-\frac{t}{\tau_2}\left(1 + \frac{I_\nu}{I_s}\right)\right]\right\}$$

$$\boxed{I_s = h\nu/\sigma\tau_2 = \text{saturation intensity}}$$

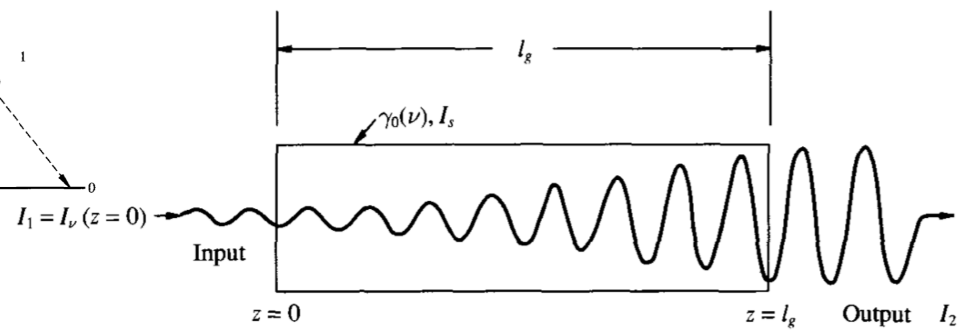
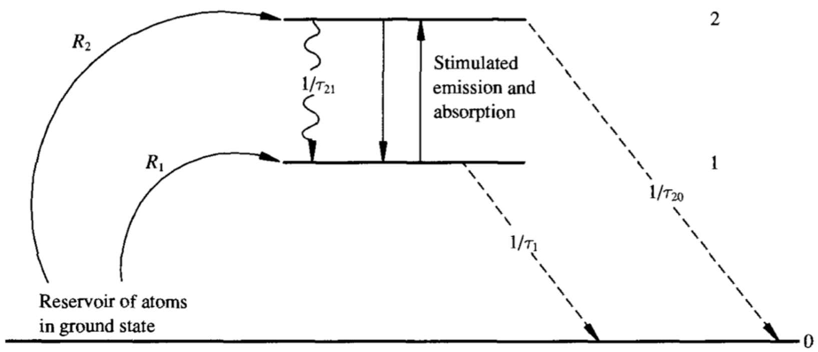


FIGURE 8.7. An optical amplifier.

Optical Gain Saturation



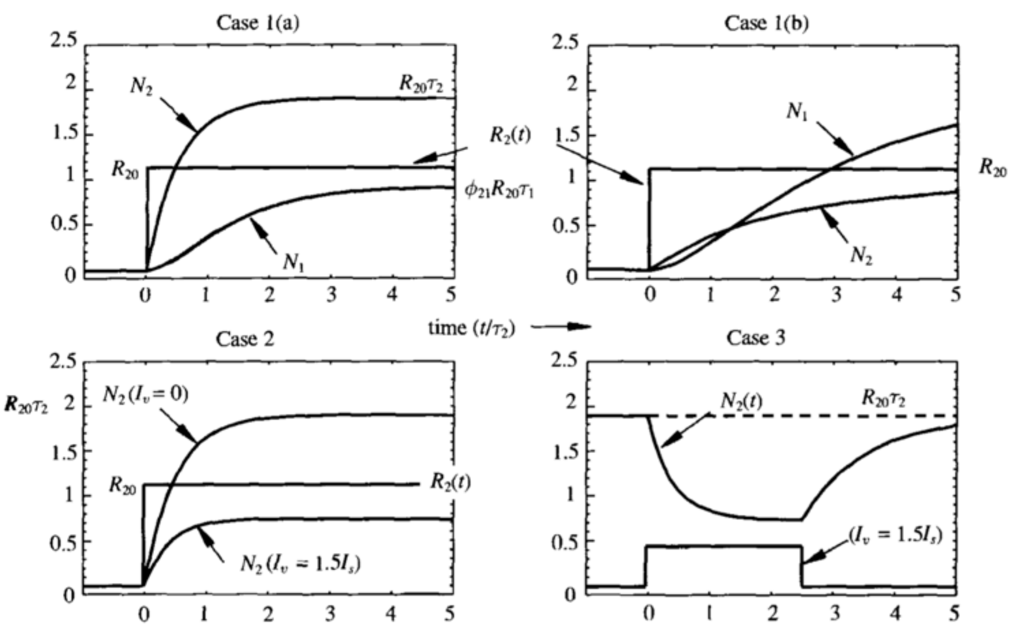


FIGURE 8.5. The variation of the populations with time (t/τ_2) for the three time dependent examples. For case 1a, the lifetime ratio $\tau_2/\tau_1 = 2$, whereas the ratio was 0.5 for 1b, and $\tau_1 = 0$ for cases 2 and 3.

$$\boxed{\frac{dN_2}{dt} = R_2(t) - \frac{N_2}{\tau_2} - \frac{\sigma(\nu)I_\nu}{h\nu}[N_2 - N_1]}$$

$$\boxed{\frac{dN_1}{dt} = R_1(t) + \frac{N_2}{\tau_{21}} + \frac{\sigma(\nu)I_\nu}{h\nu}[N_2 - N_1] - \frac{N_1}{\tau_1}}$$

$$N_2(t) = \frac{R_{20}\tau_2}{(1 + I_\nu/I_s)}\left\{1 - \exp\left[-\frac{t}{\tau_2}\left(1 + \frac{I_\nu}{I_s}\right)\right]\right\}$$

$$I_s = \frac{h\nu}{\sigma(\nu)\tau_2}\cdot\frac{1}{1 + \frac{\tau_1}{\tau_2}\left(1 - \frac{\tau_2}{\tau_{21}}\right)}$$

$$\boxed{I_s = h\nu/\sigma\tau_2 = \text{saturation intensity}}$$

$$\frac{1}{I_\nu}\frac{dI_\nu}{dz} \triangleq \frac{\gamma_0(\nu)}{1 + I_\nu/I_s(\nu)}$$

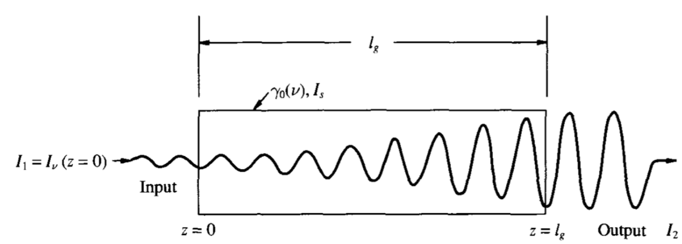


FIGURE 8.7. An optical amplifier.

Amplified Spontaneous Emission

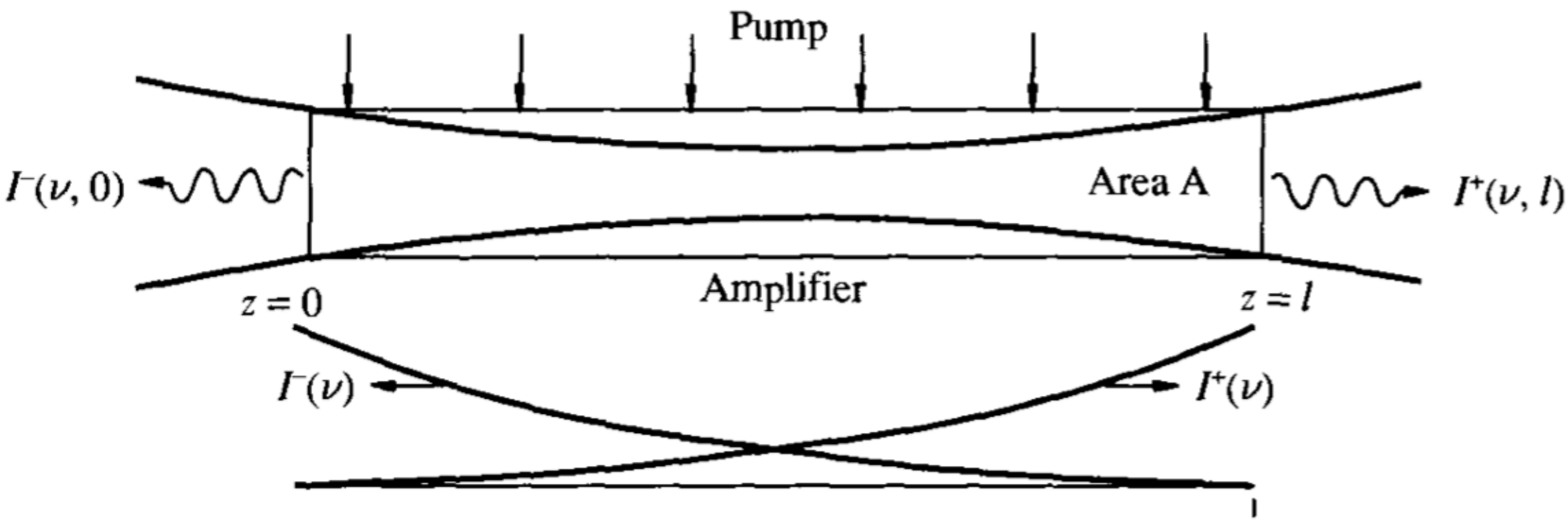


FIGURE 8.17. Optical amplifier generating broad-band incoherent radiation.

$$\frac{d}{dz}\left[I^+(\nu, z)\,d\nu\right] = \gamma_0(\nu) I^+(\nu, z)\,d\nu + h\nu A_{21} N_2 g(\nu)\,d\nu\,\frac{d\Omega}{4\pi}$$

$$\boxed{I^+(\nu, z = l_g) = \frac{h\nu A_{21} N_2 g(\nu)}{\gamma_0(\nu)}\left(e^{\gamma_0(\nu) l_g} - 1\right)\frac{d\Omega}{4\pi}}$$

Amplified Spontaneous Emission

$$I^{+}(\nu, z = l_g) = \frac{h\nu A_{21} N_2 g(\nu)}{\gamma_0(\nu)} \left(e^{\gamma_0(\nu) l_g} - 1\right) \frac{d\Omega}{4\pi}$$

$$\boxed{I^{+}(\nu, l_g) = \frac{8\pi n^2 h\nu^3}{c^2} \frac{N_2}{N_2 - (g_2/g_1)N_1} \left[G_0(\nu) - 1\right] \frac{d\Omega}{4\pi}}$$

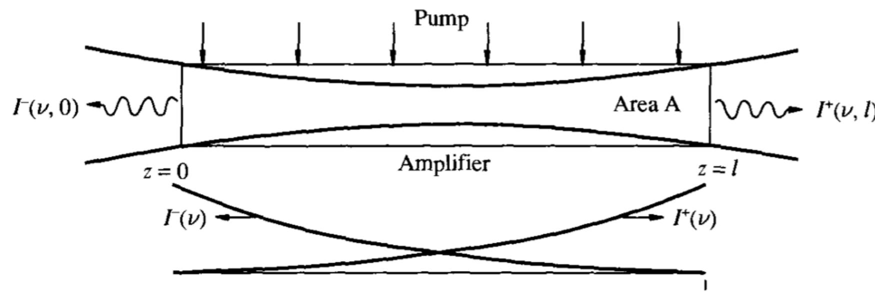
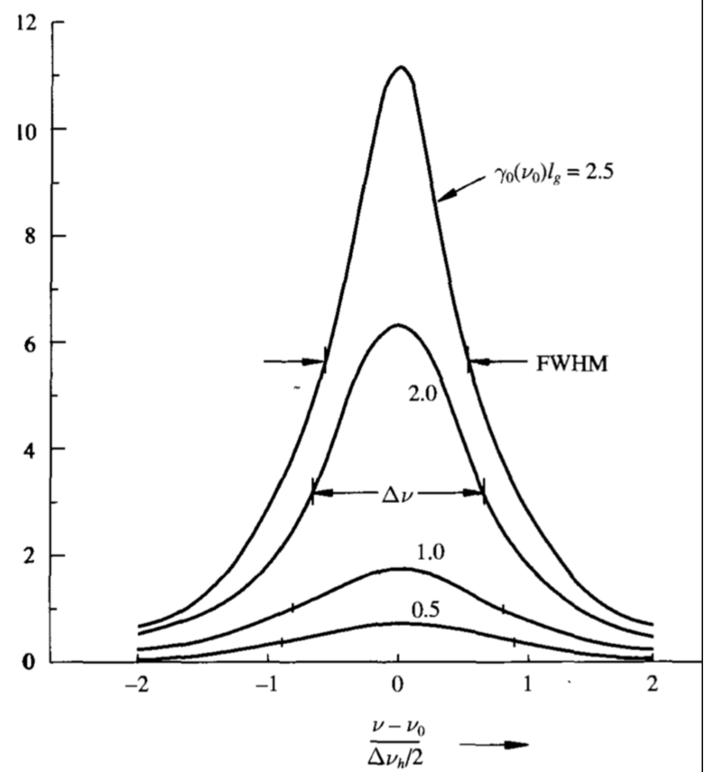


FIGURE 8.17. Optical amplifier generating broad-band incoherent radiation.

Case A: An Optically "Thin" Amplifier or Attenuator. If $G_0(\nu)$ is very close to 1, the amplifier (or attenuator) is said to be optically thin, and thus $\gamma_0(\nu)l_g$ is small. Therefore the Taylor series expansion of $\exp(\gamma_0 l_g) - 1$ yields $\gamma_0(\nu)l_g$, and we obtain a most logical result:

$$\boxed{I^{+}(\nu, l_g) = A_{21} h\nu N_2 l_g g(\nu) \frac{d\Omega}{4\pi} \quad \text{(optically thin)}} \tag{8.7.3b}$$

This states that the power from $N_2 l_g$ atoms radiating into $d\Omega/4\pi$ as $g(\nu)$ add their radiation, a result that would be guessed from the start. In other words, each element dz along z contributes an equal amount to the power.



Case B: A Thermal Population. If the atomic populations are such that $N_2 < (g_2/g_1)N_1$, the amplifier is an attenuator and $G_0 < 1$. Furthermore, if N_2/N_1 can be related to a "temperature" by a Boltzmann relation,

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left(-\frac{h\nu}{kT}\right)$$

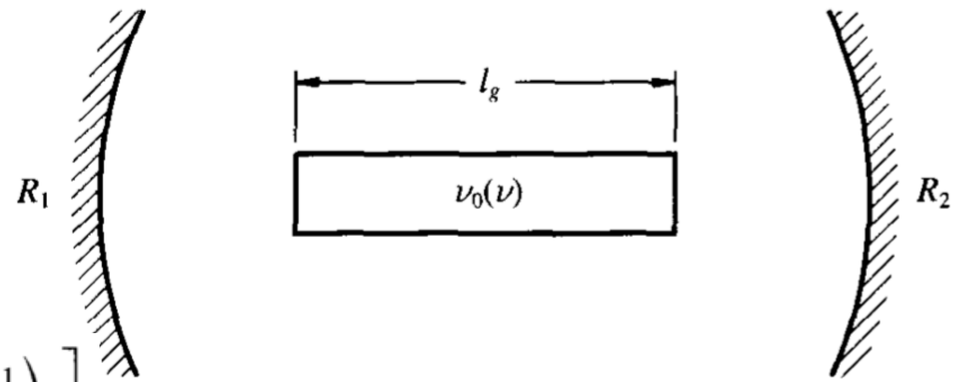
then (8.7.3a) becomes

$$\boxed{I^{+}(\nu, l) = \left[\frac{8\pi n^2 h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}\right] \frac{d\Omega}{4\pi} \left(1 - e^{-|\gamma_0(\nu)| l_g}\right)} \tag{8.7.3c}$$

Laser Oscillation Physics in One Slide

$$\left.\frac{dN_p}{dt}\right|_{\text{spont.}} = (A_{21}N_2V)\left[g(\nu)\frac{c}{2nd}\right]\frac{1 \text{ mode}}{\text{no. modes} = (8\pi n^2\nu^2/c^3)(c/2nd)V} = N_2c\left[A_{21}\frac{\lambda^2}{8\pi}g(\nu)\right] = N_2c\sigma_{\text{SE}}$$

$$\left.\frac{dN_p}{dt}\right|_{\text{cavity with gain}} = \frac{G^2R_1R_2 - 1}{2nd/c}N_p$$



$$\boxed{\frac{dN_p}{dt} = \frac{G^2R_1R_2 - 1}{2nd/c}N_p + N_2c\sigma_{\text{SE}}}$$

$$N_p(t) \doteq N_p(0)\exp\left[+\left(\frac{G^2R_1R_2 - 1}{2nd/c}\right)t\right]$$

$$\frac{N_p}{2nd/c} = \frac{P_{\text{out.}}}{h\nu}\frac{1}{1 - R_1R_2}$$

$$1 - G_s^2R_1R_2 = \frac{h\nu}{P_0}(1 - R_1R_2)N_2^{(s)}c\sigma_{\text{SE}}$$

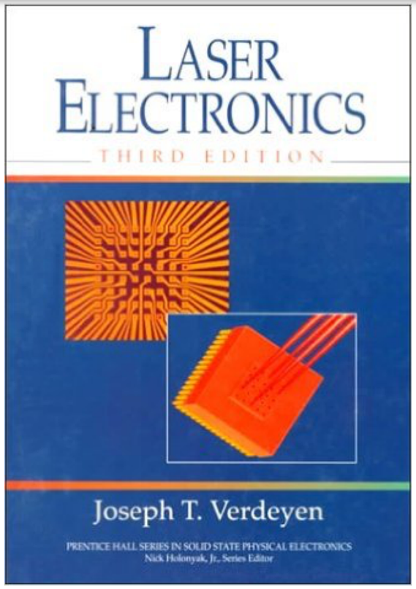
$$P(\nu) = \frac{K}{\left[1 - G_s(R_1R_2)^{1/2}\right]^2 + 4G_s(R_1R_2)^{1/2}\sin^2\left[2\pi(\nu - \nu_q)\,d/c\right]}$$

Steady State

Linewidth: depends on power & inversion

$$\Delta\nu_{\text{osc.}} = \frac{1 - G_s(R_1R_2)^{1/2}}{\pi\left[G_s(R_1R_2)^{1/2}\right]^{1/2}}\frac{c}{2d} \longleftrightarrow \boxed{\Delta\nu_{\text{osc.}} = 2\pi\frac{h\nu}{P_{\text{out.}}}(\Delta\nu_{1/2})^2\left(1 - \frac{g_2}{g_1}\frac{N_1^{(s)}}{N_2^{(s)}}\right)^{-1}}$$

Course Outline



Laser "Wall-Plug" Efficiency

$$\boxed{\eta = \eta_{qe} \cdot \eta_{cpl} \cdot \eta_{\text{pump}}} \tag{9.1.1}$$

η_{qe}: : *Quantum efficiency*. This depends solely on the position of the energy levels of the laser and cannot be "designed." It is the upper limit for the performance of any laser.

η_{cpl}: : *Coupling efficiency*. This is an electromagnetic or cavity problem affected by the stimulated emission issue. Optical components are not perfect, and thus we must choose the arrangement of cavity components to maximize the stimulated emission and the output (in a desired direction) while minimizing the useless conversion of photons into heat.

η_{pump}: : *Pumping efficiency*. This is the fraction of the total pump power that is useful in creating the population inversion and thus contributes to the output. It is the most complicated of all topics but is the most essential part of any laser.

Quantum Efficiency

$$\eta_a = 1 \qquad n_b = \frac{(E_2 - E_1)}{E_2} \qquad \eta_c = \frac{(E_2 - E_1)}{(E_3 - E_1)} \qquad \eta_d = \frac{(E_2 - E_1)}{(E_3 - E_0)}$$

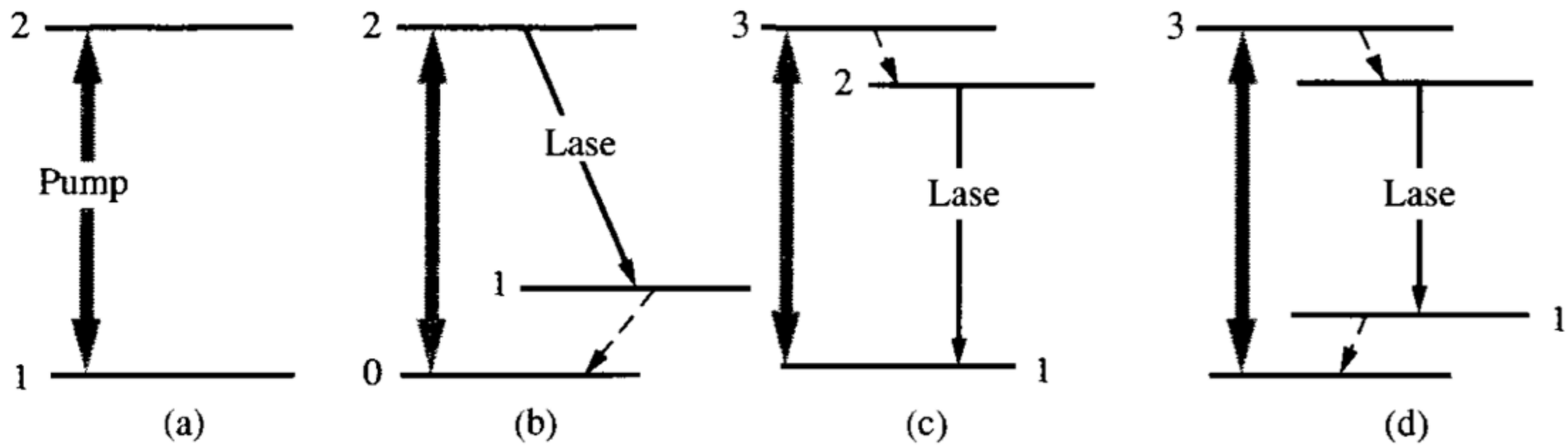


FIGURE 9.1. Possible arrangement of the energy levels of a laser: (a) represents a two level system, (b) and (c) are three level lasers, and (d) is a four-level laser. All double-headed arrows represent the pumping route, the dashed single-headed arrows represent relaxation by any cause, and the solid arrows between 2 and 1 represent stimulated emission by the laser radiation.

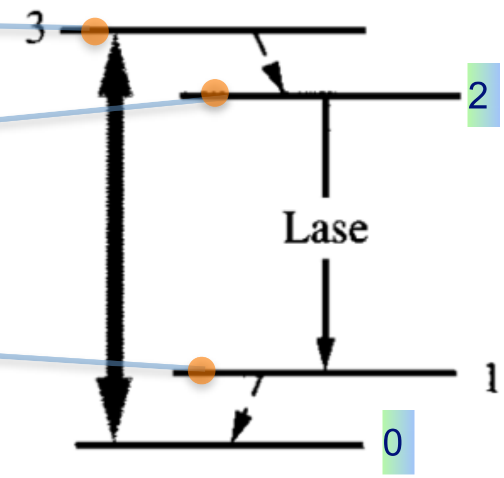
Pumping Efficiency

$$\frac{dN_3}{dt} = \frac{\sigma_{30} I_p}{h\nu_{30}} \left(\frac{g_3}{g_0} N_0 - N_3 \right) - \frac{N_3}{\tau_3}$$

$$\frac{dN_2}{dt} = \phi_{32} \frac{N_3}{\tau_3} - \frac{N_2}{\tau_2} - \frac{\sigma_{21} I_1}{h\nu_{21}} \left(N_2 - \frac{g_2}{g_1} N_1 \right)$$

$$\frac{dN_1}{dt} = \phi_{31} \frac{N_3}{\tau_3} + \phi_{21} \frac{N_2}{\tau_1} - \frac{N_2}{\tau_1} + \frac{\sigma_{21} I_1}{h\nu_{21}} \left(N_2 - \frac{g_2}{g_1} N_1 \right)$$

$$[N] = N_0 + N_1 + N_2 + N_3 \qquad \text{(conservation of atoms)}$$



$$I_{sp} = h\nu_p / \sigma_{30} \tau_3$$

$$N_3 = \frac{g_3}{g_0} \cdot \frac{I_p / I_{sp}}{1 + I_p / I_{sp}} \cdot N_0$$

$$N_2 = \phi_{32} \frac{\tau_2}{\tau_3} N_3 = \phi_{32} \frac{\tau_2}{\tau_3} \cdot \left[\frac{g_3}{g_0} \frac{I_p / I_{sp}}{1 + I_p / I_{sp}} \right] \cdot N_0$$

$$N_1 = \phi_{31} \cdot \frac{\tau_1}{\tau_3} \cdot N_3 + \phi_{21} \frac{\tau_1}{\tau_2} \cdot N_2 = (\phi_{31} + \phi_{32}\phi_{21}) \cdot \frac{\tau_1}{\tau_3} \cdot N_3$$

Small-Signal Gain

$$\frac{\gamma_0}{\sigma_{21}} = \left(N_2 - \frac{g_2}{g_1} N_1 \right)_0 = \left[\phi_{32} \frac{\tau_2}{\tau_3} - \frac{g_2}{g_1} (\phi_{31} + \phi_{32}\phi_{21}) \frac{\tau_1}{\tau_3} \right] \cdot N_3$$

Intensity Profile in a Continuous-Wave (CW) Laser

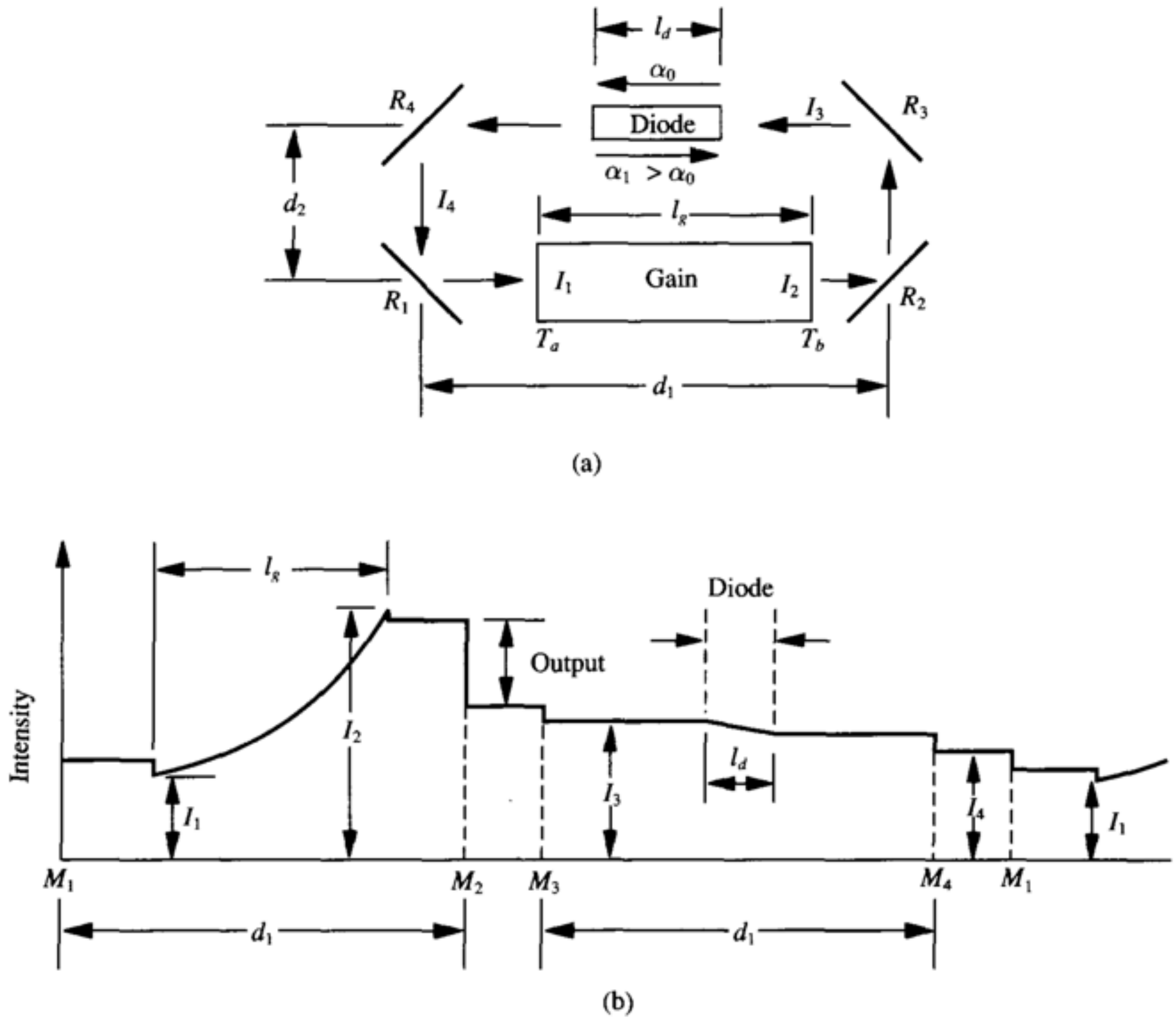
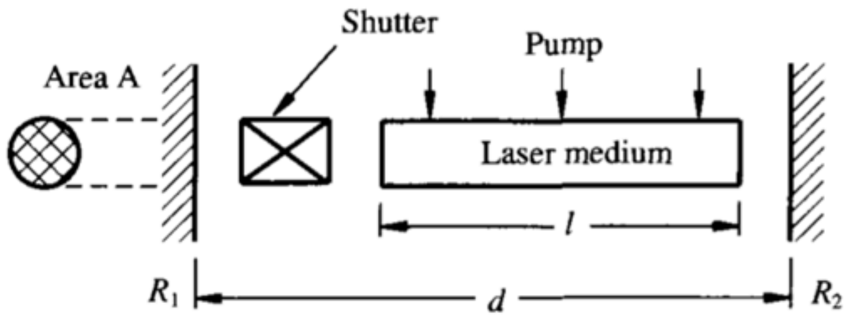
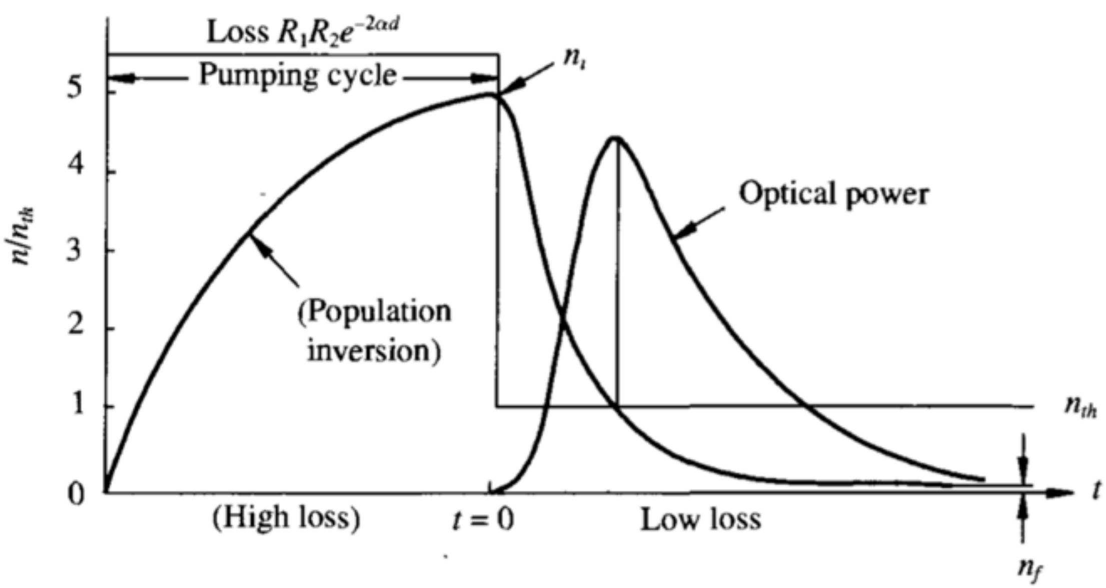


FIGURE 9.2. Unidirectional traveling wave laser. (a) The geometry. (b) A self-consistent variation of the intensity inside the laser cavity.

Q-Switching in Lasers





Method to obtain a large photon Intensity in short pulses.

But control of pulse shape and repetition rate by Q-switching is limited.

Mode Locked Lasers

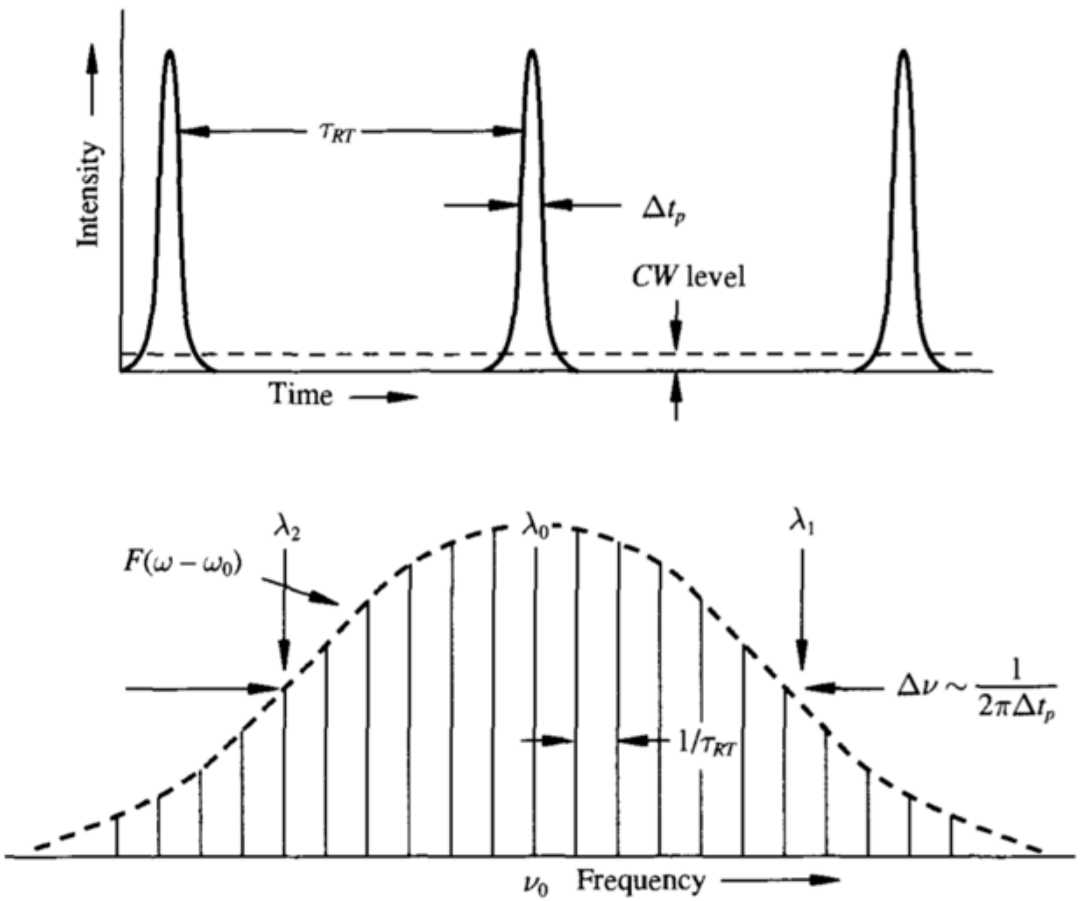


FIGURE 9.15. The (a) time and (b) frequency domain representation of a mode-locked laser.

$$e(t) = \sum_{-(N-1)/2}^{+(N-1)/2} E_n(t) \exp\left[j(\omega_0 + n\omega_c)t + \phi_n(t)\right]$$

$$\frac{e(t)}{E_0} = \sum_{-(N-1)/2}^{+(N-1)/2} e^{j\omega_0 t} e^{jn\omega_c t} = e^{j\omega_0 t} \sum x^n$$

$$e(t) = E_0 e^{j\omega_0 t} \left\{ \frac{\sin N\omega_c t/2}{\sin \omega_c t/2} \right\}$$

$$I(t) = \frac{e(t)\, e^*(t)}{2\eta_0} = \frac{E_0^2}{2\eta_0} \left[\frac{\sin(N\omega_c t/2)}{\sin(\omega_c t/2)} \right]^2$$

$$P_{\text{peak}} = N \times P_{\text{ave}}$$

$$P(\text{peak})\Delta t_p \approx \langle P_{\text{ave}} \rangle \tau_{RT}$$

$$\Delta t_p \approx \frac{\tau_{RT}}{N} \qquad N \sim \Delta\nu/(c/2d) = \Delta\nu\tau_{RT}$$

$$\Delta t_p \sim \frac{\tau_{RT}}{N} \sim \frac{1}{\Delta\nu}$$

Mode Locked Lasers

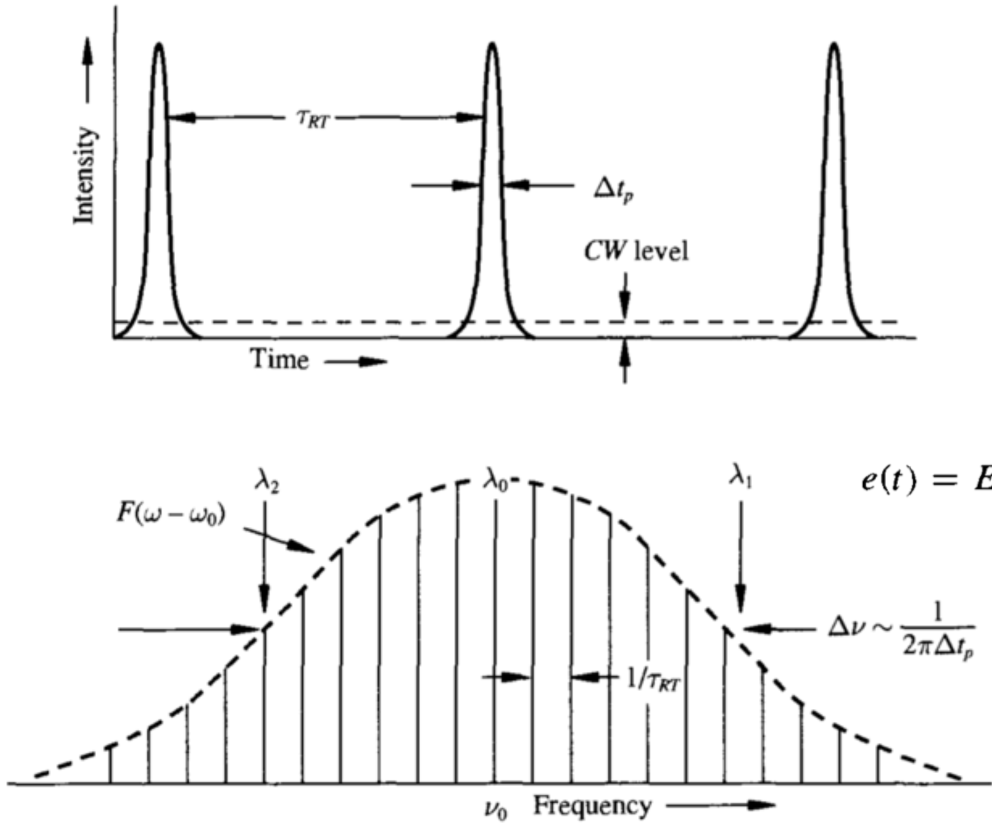


FIGURE 9.15. The (a) time and (b) frequency domain representation of a mode-locked laser.

$$I_n = I_0 \exp\left[-4(\ln 2)\left(\frac{n\omega_c}{\Delta\omega}\right)^2\right]$$

$$E_n = E_0 \left\{\exp\left[-2(\ln 2)\left(\frac{n\omega_c}{\Delta\omega}\right)^2\right]\right\}$$

$$e(t) = E_0\, e^{j\omega_0 t} \sum_{n=-(N-1)/2}^{+(N-1)/2} \left\{\exp\left[-2(\ln 2)\left(\frac{n\omega_c}{\Delta\omega}\right)^2\right]\right\} \cdot (1) \cdot e^{jn\omega_c t}$$

$$e(t) = E_0\, e^{j\omega_0 t} \left\{\left(\frac{\pi}{2\ln 2}\right)^{1/2}\left(\frac{\Delta\omega}{\omega_c}\right) \cdot \exp\left[-\left(\frac{\Delta\omega t}{2(\ln 2)^{1/2}}\right)^2\right]\right\}$$

$$\frac{p(t)}{P_0} = \left(\frac{\pi}{2\ln 2}\right)\left(\frac{\Delta\omega}{\omega_c}\right)^2 \exp\left[-\left(\frac{\Delta\omega t}{2(\ln 2)^{1/2}}\right)^2\right]$$

Mode Locked Lasers

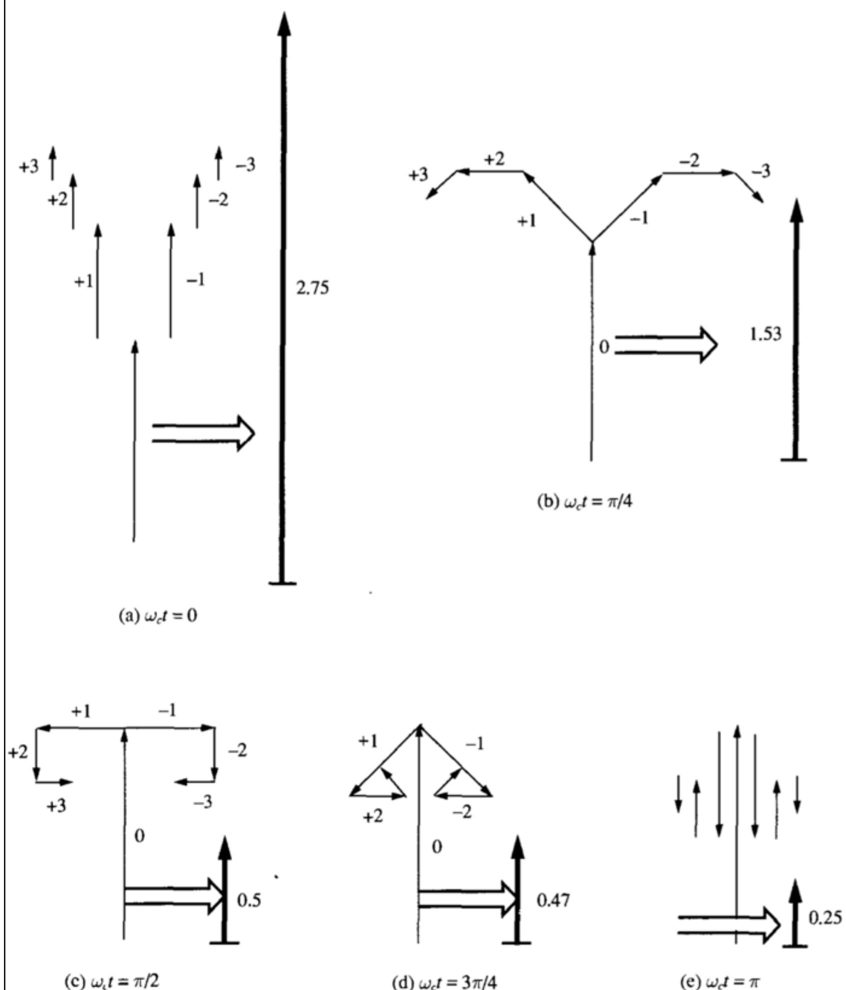
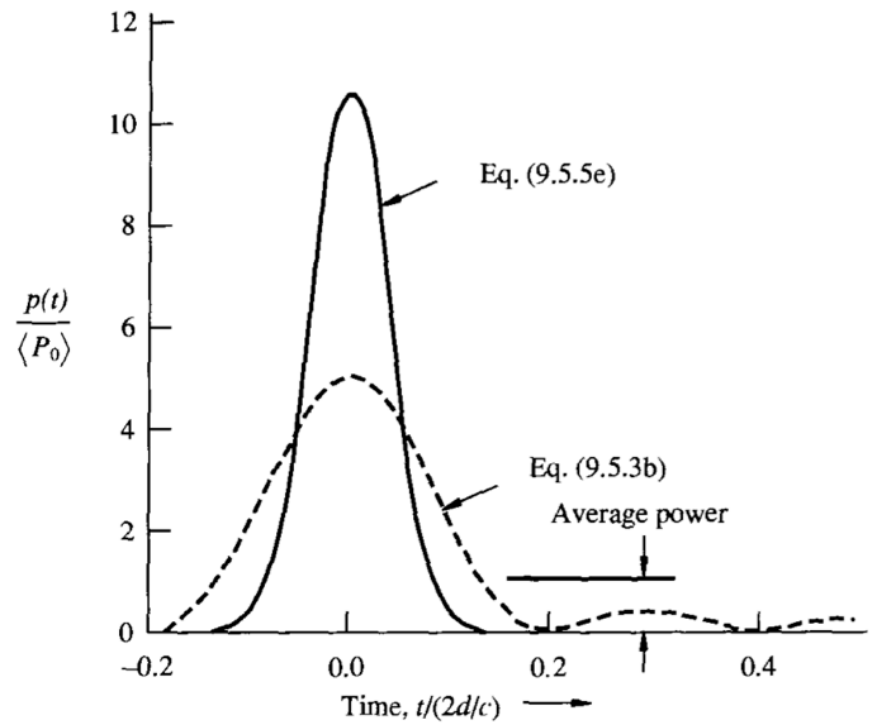


FIGURE 9.16. Phasor addition of the fields of a mode-locked laser. The mode amplitudes were chosen according to a proportional relation: 1 : 0.5 : 0.25 : 0.125.



Active Mode Locking

$$T_g(\omega) = \exp\left\{-jkl_g + \frac{|\gamma(\omega_0)l_g|}{2}\left[1 - j\left(\frac{\omega-\omega_0}{\Delta\omega/2}\right) - \left(\frac{\omega-\omega_0}{\Delta\omega/2}\right)^2\right]\right\}$$

$$\frac{\gamma(\omega)}{|\gamma(\omega_0)|} = 1 - j\left(\frac{\omega-\omega_0}{\Delta\omega/2}\right) - \left(\frac{\omega-\omega_0}{\Delta\omega/2}\right)^2$$

Gain Medium Transfer Function

$$\frac{\gamma(\omega)}{|\gamma(\omega_0)|} = \frac{1}{1 + j\left(\dfrac{\omega-\omega_0}{\Delta\omega/2}\right)}$$

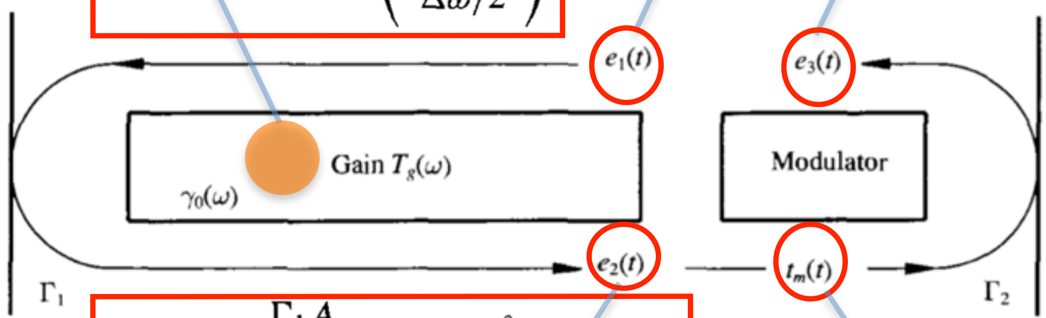
$$E_1(\omega) = A\left[\left(\frac{\pi}{a}\right)^{1/2} e^{-[(\omega-\omega_0)^2/4a]}\right]$$

$$e(t) = A\exp\left[-2(\ln 2)\left(\frac{t}{\Delta t_p}\right)^2\right] e^{j\omega_0 t} = A\, e^{-at^2}\, e^{j\omega_0 t}$$

Should be equal!

$$e_3(t) = \Gamma_1\Gamma_2\frac{A}{2}\frac{e^{g_0}}{\sqrt{qa}} e^{j\omega_0 t}\exp\left\{-\left[\frac{1}{4q} + 2\delta^2\left(\frac{\omega_m}{2}\right)^2\right]t^2\right\}$$

$$2g_0 \triangleq |\gamma\omega_0)2l_g| = \ln\frac{1}{R_1R_2} + \ln\left[1 + \frac{(8\ln 2)g_0}{(\Delta\omega\Delta t_p/2)^2}\right]$$

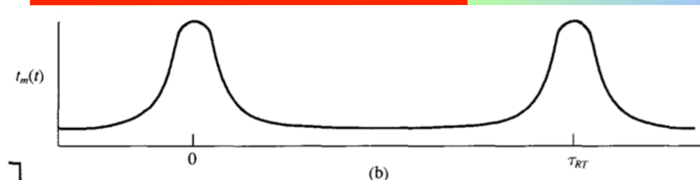


$$e_2(t) = \frac{\Gamma_1 A}{2\sqrt{qa}}(e^{g_0})(e^{-(t^2/4q)})(e^{j\omega_0 t})$$

$$E_2(\omega) = \Gamma_1 T_g^2(\omega)E_1(\omega)$$

$$t_m(t) = \exp\left[-\delta^2\sin^2\left(\frac{\omega_m t}{2}\right)\right]$$

Amplitude Modulation



$$q = \frac{1}{4a} + \frac{g_0}{(\Delta\omega/2)^2}$$

$$f_m = m\left[\frac{1}{\dfrac{2d}{c'} + \dfrac{g_0}{\Delta\omega/2}}\right] = \left[\frac{m\Delta\nu_c}{\left(1 + \dfrac{g_0\Delta\nu_c}{\pi\Delta\nu}\right)}\right]$$

$$\omega_m\tau_{RT} = 2m\pi$$

Mode Locked Lasers

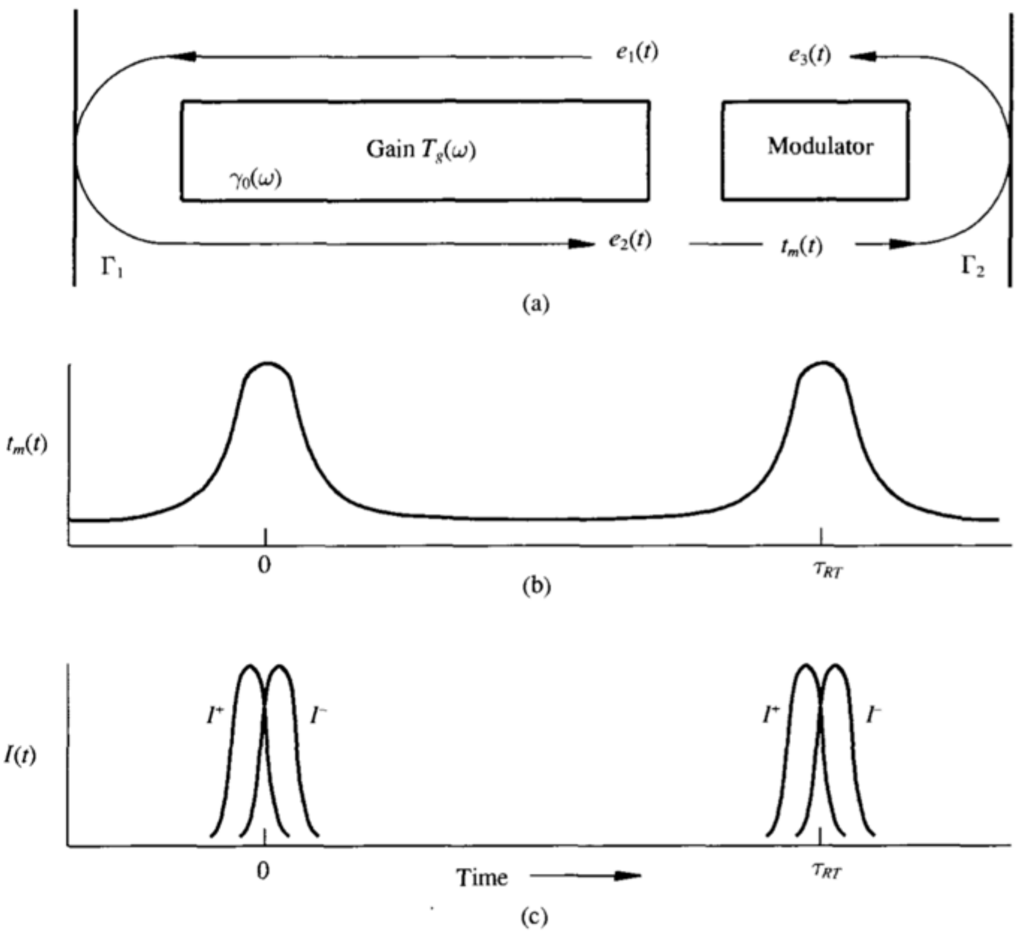
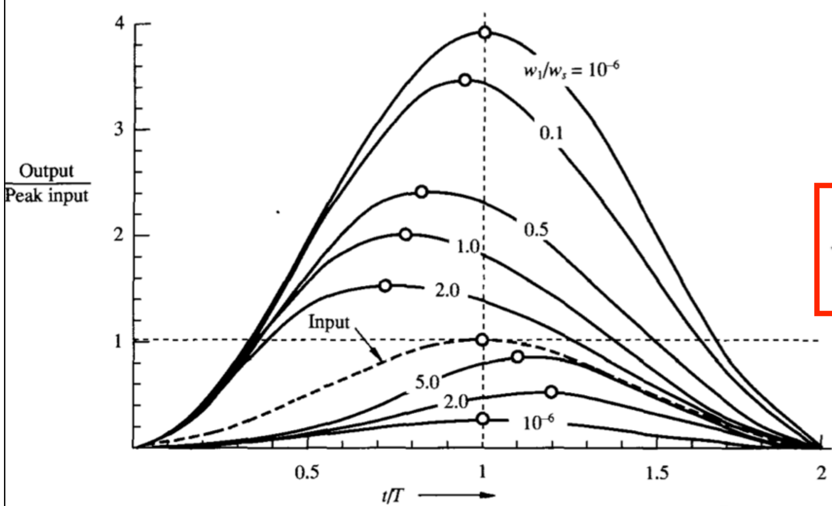


FIGURE 9.18. Mode locking of a laser. (a) Geometry showing the external modulator and the fields inside the laser cavity. (b) The transmission coefficient of the modulator. (c) The intensities arriving at the modulator.

Saturable Amplifier and Saturable Absorber



$$\frac{\partial I}{\partial z'} + \frac{1}{v_g}\frac{\partial I}{\partial t'} = \Delta N(z',t')\sigma I(z,t') - \alpha_0 I(z',t')$$

$$\boxed{\frac{\partial I(z,t)}{\partial z} = [N_2(z,t) - N_1(z,t)]\sigma I(z,t) - \alpha_0 I(z,t)}$$

$$\frac{\partial N_2}{\partial t} + \frac{2\sigma I}{h\nu}N_2 = \frac{\sigma I}{h\nu}N = \frac{N}{2}\frac{2\sigma I}{h\nu} \qquad w_s = \frac{h\nu}{2\sigma}$$

$$w(z,t) = \int_{-\infty}^{t} I(z,t)\,dt \qquad I(z,t) = \dot{w}(z,t) \qquad \boxed{u(z,t) = \frac{w(z,t)}{w_s}}$$

$$\boxed{\frac{\partial}{\partial z}u(z,t) = \gamma_0\left[1 - e^{-u(z,t)}\right] - \alpha_0 u}$$

$$\frac{\partial N_2}{\partial t} + \dot{u}N_2 = \frac{N}{2}\dot{u} \qquad \frac{\partial}{\partial t}\left[N_2\, e^{u(z,t)}\right] = \frac{N}{2}\left\{\dot{u}\, e^{u} = \frac{\partial}{\partial t}e^{u}\right\}$$

$$\boxed{e^{u_2(t)} = 1 + G_0\left(e^{u_1(t)} - 1\right)}$$

$$N_2(z,t) = \frac{N}{2} + K\, e^{-u(z,t)} \qquad N_2(t) = \frac{N}{2} + \frac{\Delta N^0}{2}e^{-u(z,t)}$$

$$\boxed{I_2(t) = I_1(t)\frac{G_0\, e^{u_1(t)}}{1 + G_0\left[e^{u_1(t)} - 1\right]}}$$

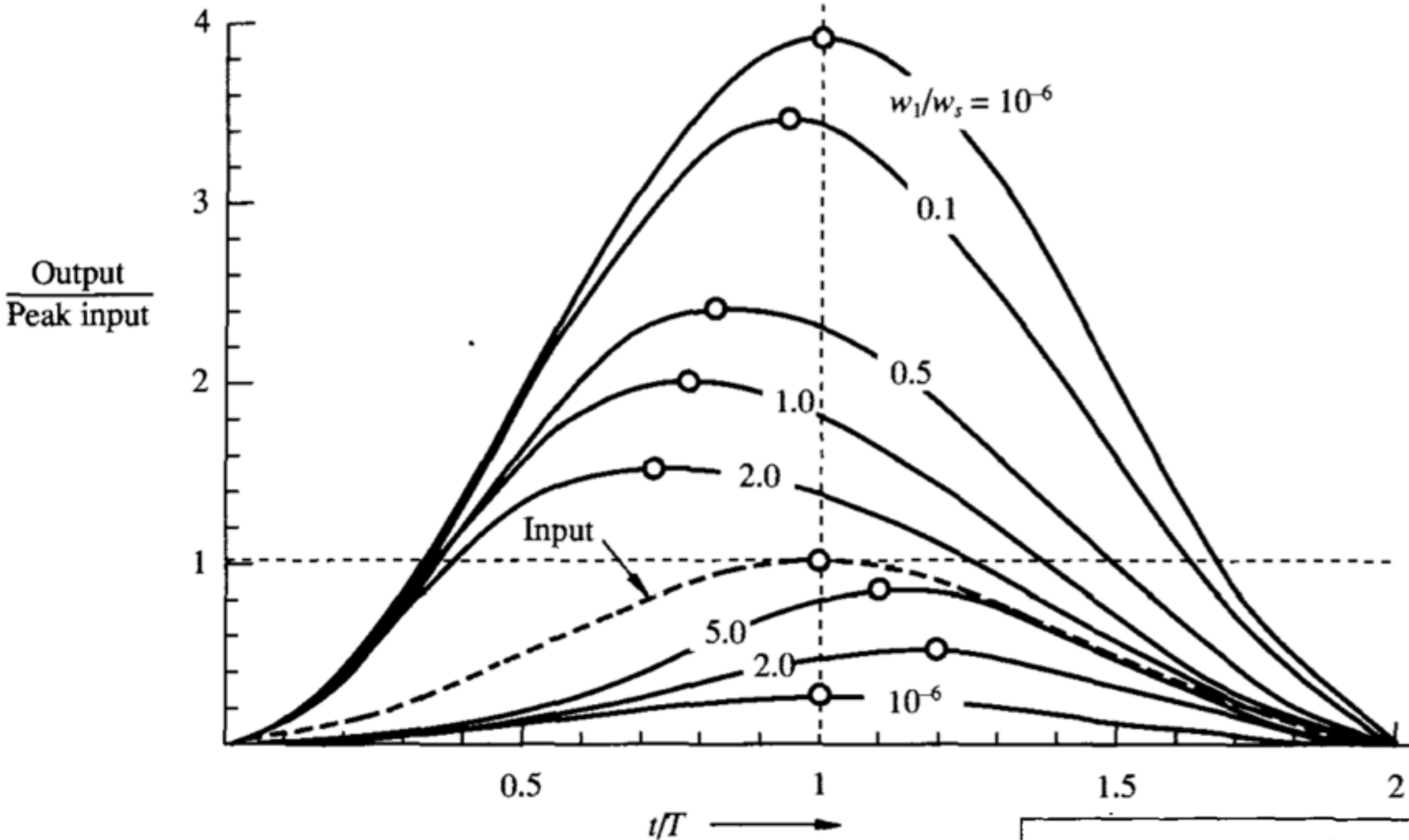
$$N_1(t) = \frac{N}{2} - \frac{\Delta N^0}{2}e^{-u(z,t)}$$

$$w_2 - w_1 = h\nu\left(\frac{\Delta N^0}{2}\right)$$

$$\boxed{\frac{\partial}{\partial z}\dot{u}(z,t) = \gamma_0\dot{u}(z,t)\, e^{-u(z,t)} - \alpha_0\dot{u}(z,t)}$$

$$N_2(t) - N_1(t) = \Delta N(t) = \Delta N^0\, e^{-u(t)}$$

Saturable Amplifier and Saturable Absorber



$$e^{u_2(t)} = 1 + G_0\left(e^{u_1(t)} - 1\right)$$

$$I_2(t) = I_1(t)\,\frac{G_0\, e^{u_1(t)}}{1 + G_0\left[e^{u_1(t)} - 1\right]}$$

$$w_2 - w_1 = h\nu\left(\frac{\Delta N^0}{2}\right)$$

Mode-Locked Laser

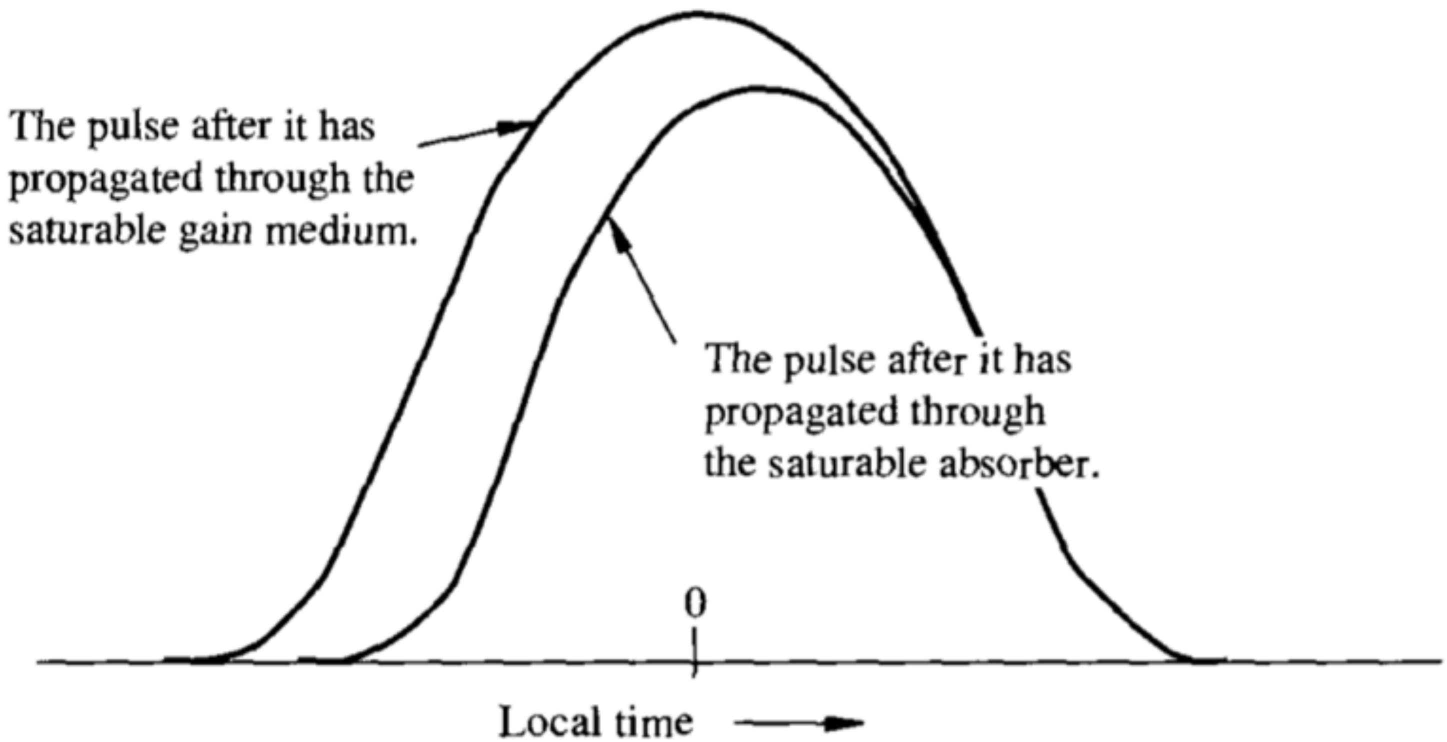


FIGURE 9.23. The transmission of a pulse through a saturable absorber or amplifying medium. The larger pulse should be considered as the input to the absorber whose output is the smaller pulse which, in turn, is the input to the amplifier.

Colliding Pulse Mode-Locked (CPM) Laser

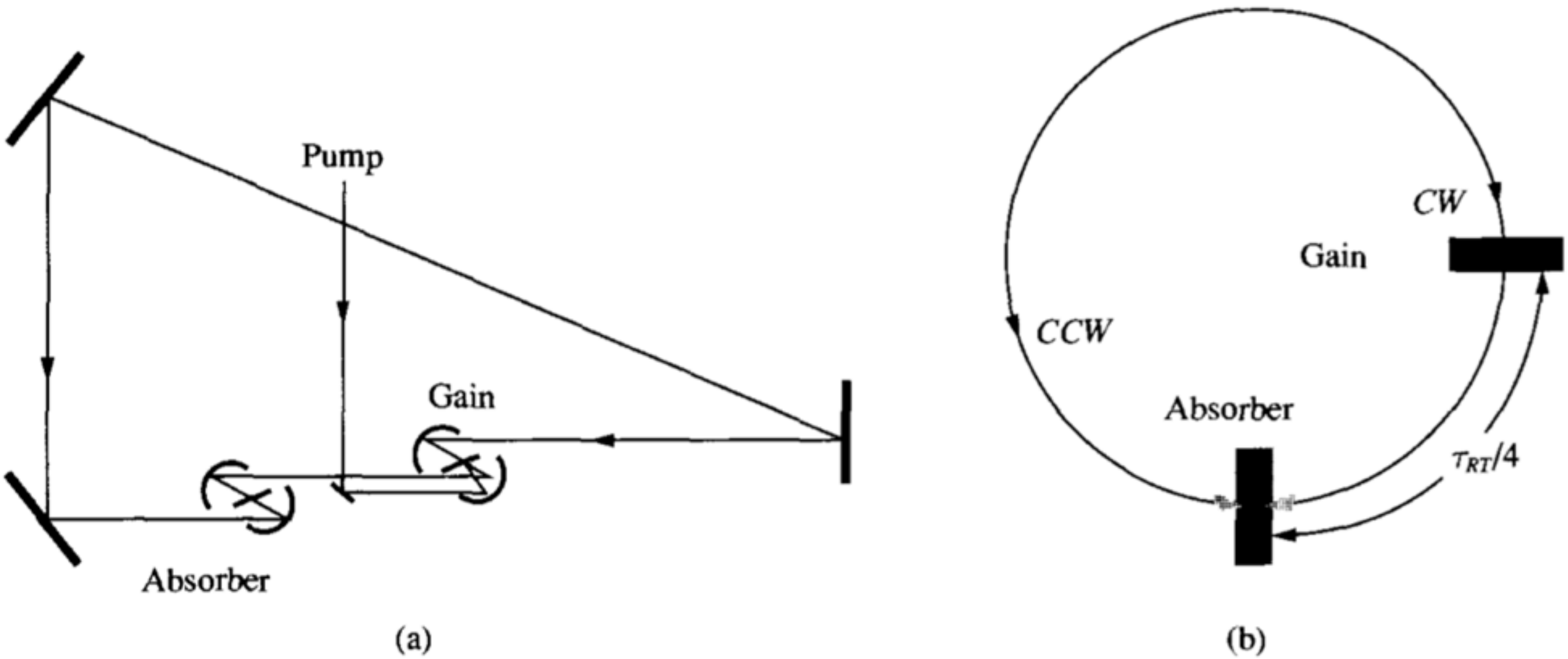


FIGURE 9.21. (a) A typical geometry of a CPM laser. (b) The circular schematic of the optical path showing the timing of the collision of the two circulating pulses in the absorber. (Adaptation of Fig. 1 of [21] and Fig. 3 of [22].

Colliding Pulse Mode-Locked (CPM) Laser

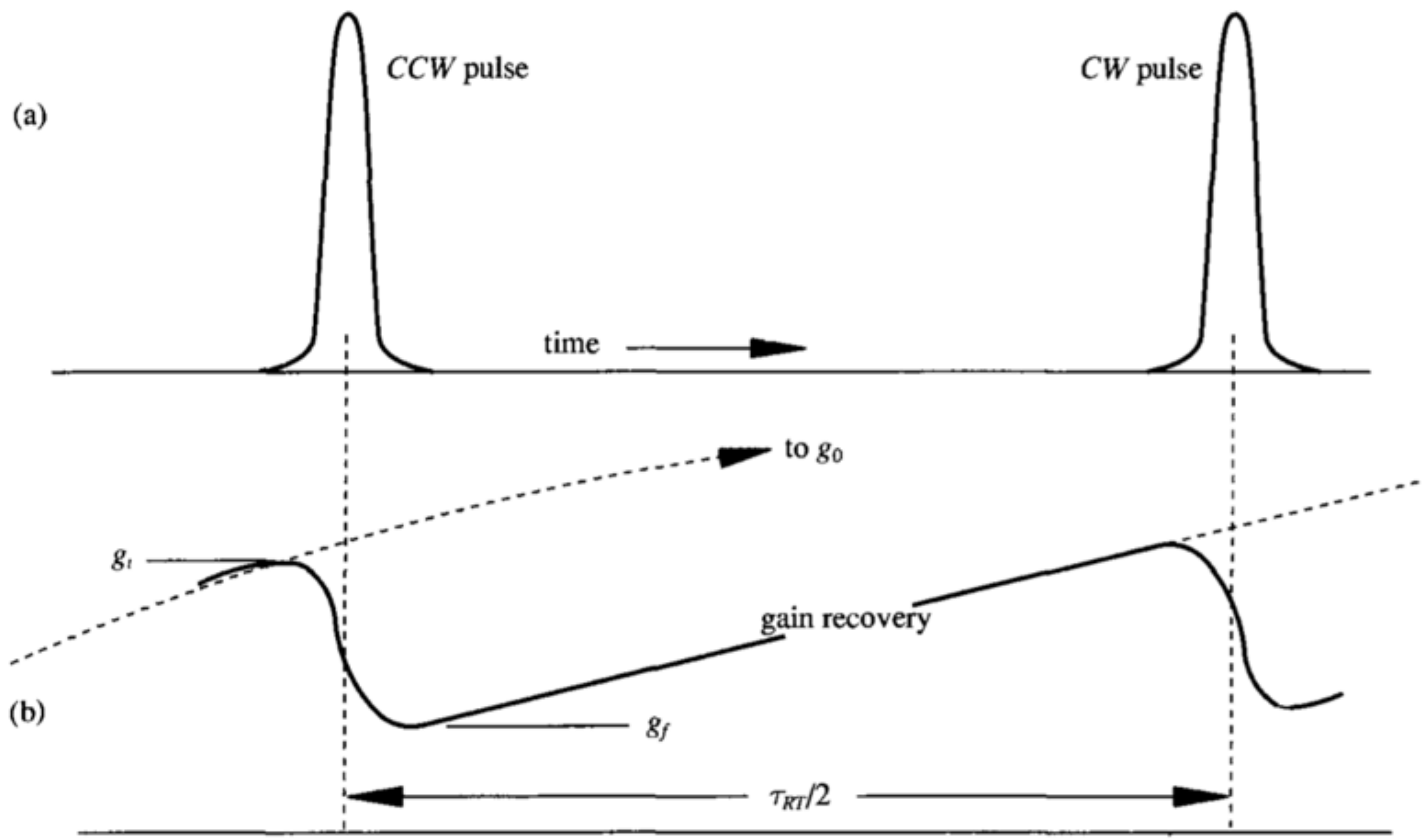


FIGURE 9.22. The interaction of the counter-propagating pulses with the gain medium. While the gain recovers between interrogations, it does not recover to the small signal value.

Additive-Pulse Mode-Locked Laser

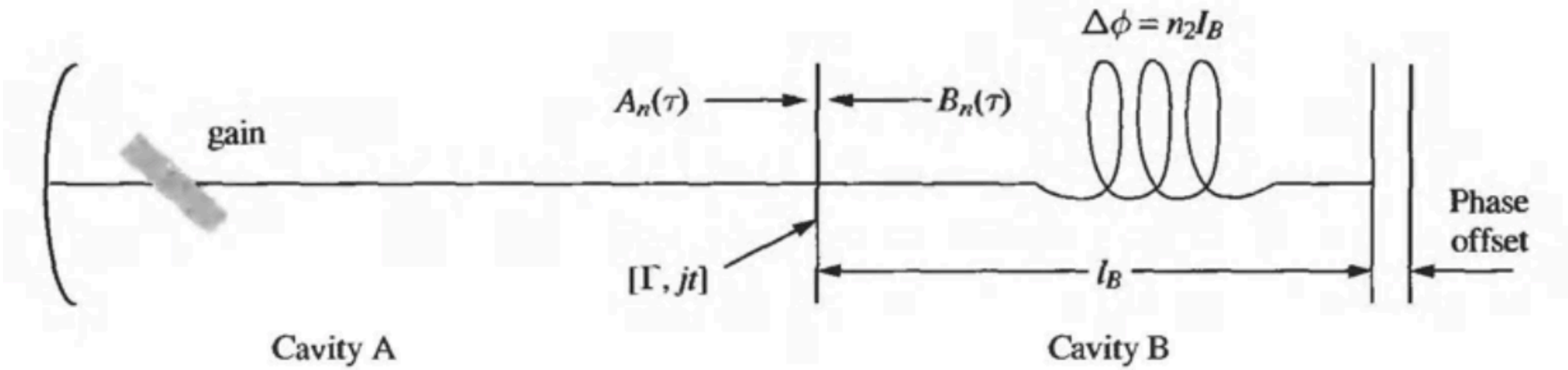
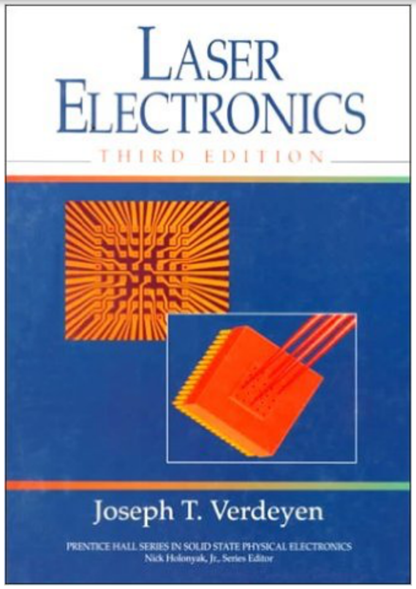
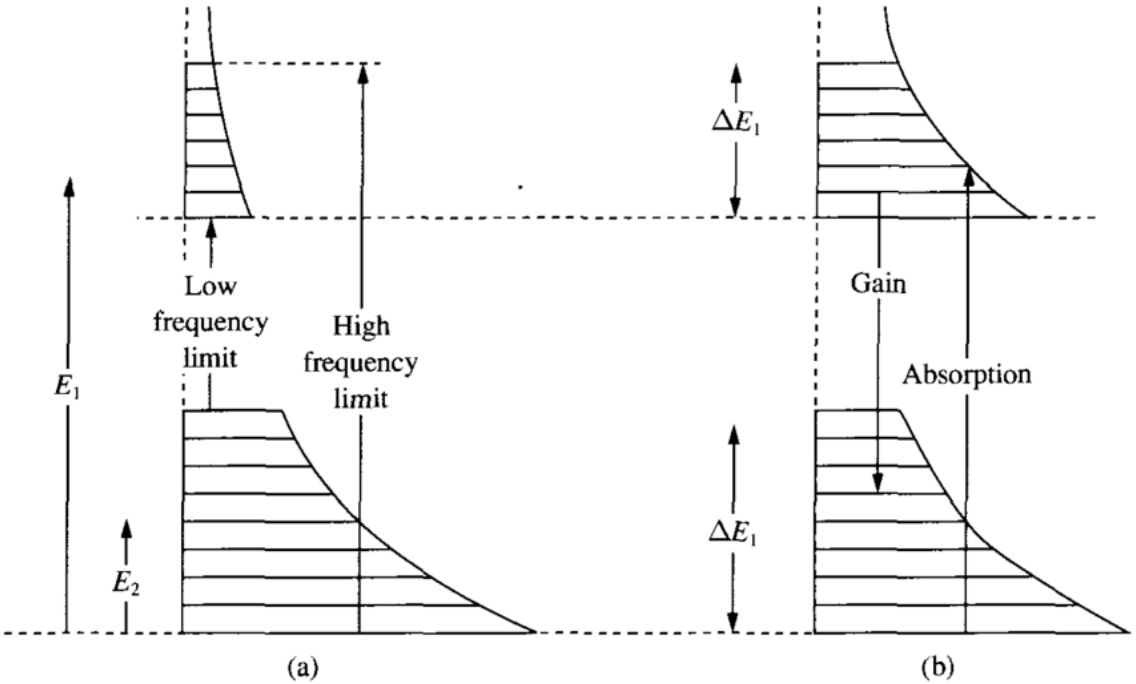


FIGURE 9.25. The geometry used by Wang [34] for the analysis of passive additive-pulse mode locking. The element common to both cavities is characterized by a field reflection coefficient Γ and a field transmission coefficient jt such that $|\Gamma|^2 + |t|^2 = 1$.

Course Outline



Broadband Optical Gain



$$B_{12}(E_2, E_1) = \left[\frac{g_2/\Delta E_2}{g_1/\Delta E_1}\right] B_{21}(E_2, E_1)$$

$$\frac{A_{21}(E_2, E_1)}{B_{21}(E_2, E_1)} = \frac{8\pi n^2 n_g}{c^3} h\nu^3$$

$$\gamma(\nu) = [N_2\sigma_{em}(\nu) - N_1\sigma_{ab}(\nu)]$$

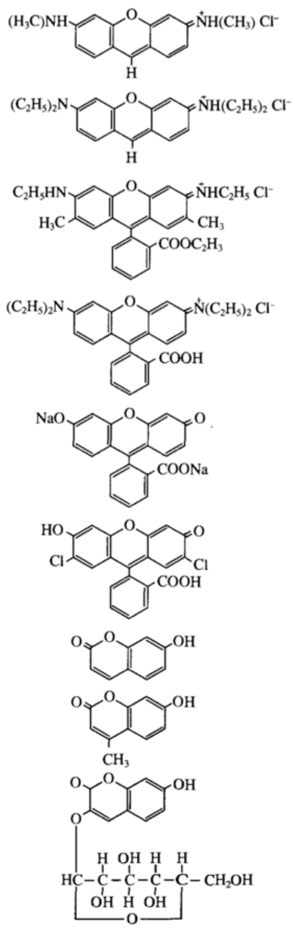
$$A_{21}\left[\frac{g_2/\Delta E_2}{g_1/\Delta E_1}\right] e^{-h\nu/kT} + B_{21}\left[\frac{g_2/\Delta E_2}{g_1/\Delta E_1}\right] e^{-h\nu/kT}\rho_{eq}(\nu) - B_{12}\rho_{eq}(\nu) \equiv 0$$

$$\boxed{\sigma_{ab}(\nu) = e^{(h\nu-\epsilon)/kT}\sigma_{em}(\nu)}$$

$$\sigma_{em}(\nu) \triangleq A_{21}g(\nu)\frac{\lambda_0^2}{8\pi n^2}$$

$$N_2\sigma_{em}(\nu) = \left\{\frac{[A_{21}N_2 g(\nu)]}{2\cdot 4\pi}\right\}\cdot\frac{\lambda_0^2}{n^2}$$

Tunable Lasers I: Dye Lasers



Dye	Solvent	Wavelength
Acridine red	EtOH	Red 600–630 nm
Puronin B	MeOH H_2O	Yellow
Rhodamine 6G	EtOH MeOH H_2O DMSO Polymethyl-methacrylate	Yellow 570–610 nm
Rhodamine B	EtOH MeOH Polymethyl-methacrylate	Red 605–635 nm
Na-fluorescein	EtOH H_2O	Green 530–560 nm
2,7-Dichloro-fluorescein	EtOH	Green 530–560 nm
7-Hydroxy-coumarin	H_2O (pH ~ 9)	Blue 450–470 nm
4-Methylem-belliferone	H_2O (pH ~ 9)	Blue 450–470 nm
Esculin	H_2O (pH ~ 9)	Blue 450–470 nm

FIGURE 10.17. Molecular structure, laser wavelength, and solvents for some laser dyes. (Data from Snavely [8].)

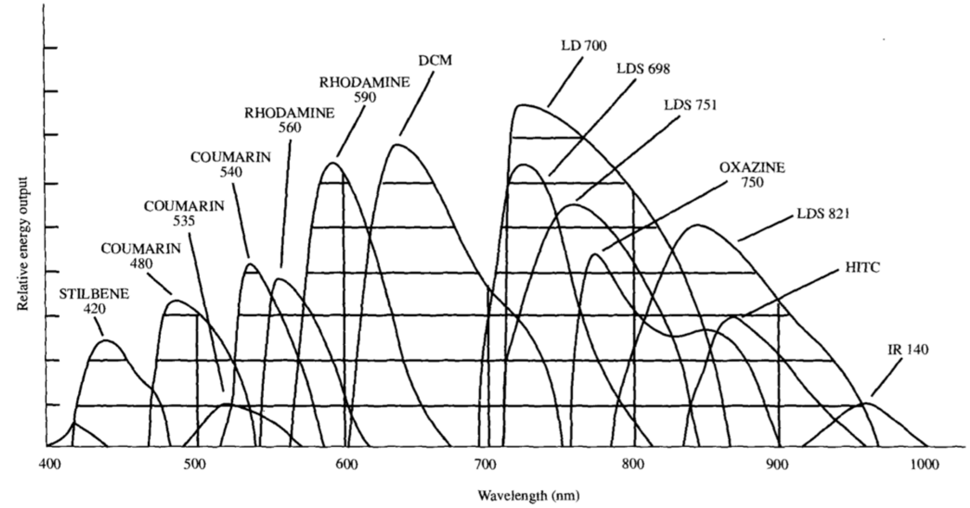
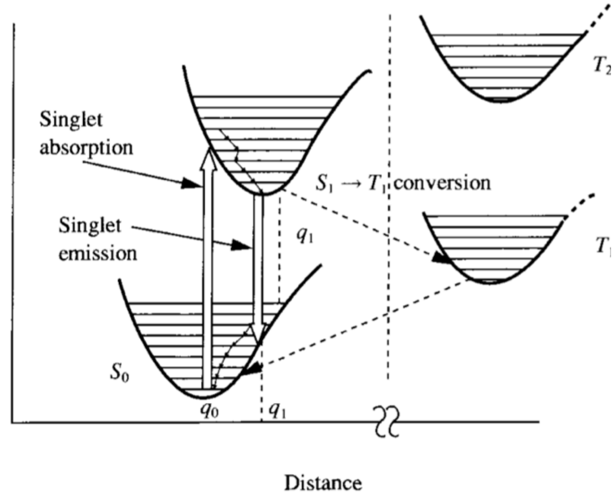
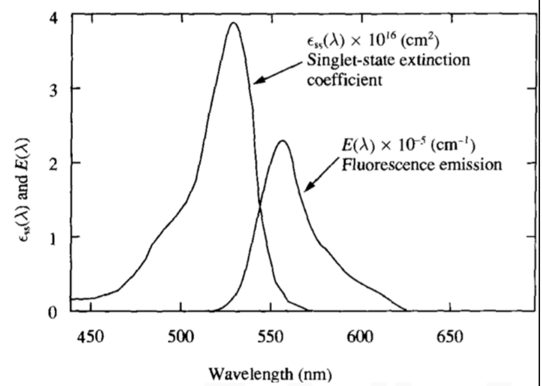
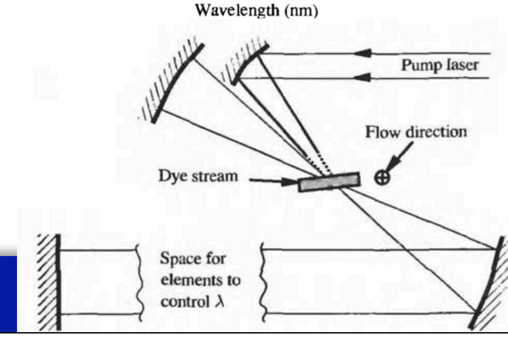


FIGURE 10.18. Performance of various dyes when pumped with an argon-ion or Krypton-ion laser (Data from Spectra-Physics and advertised in Exuton, Inc. catalog, p.4 [61].)







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Tunable Lasers II: Tunable Solid-State (Ti:Sapphire)

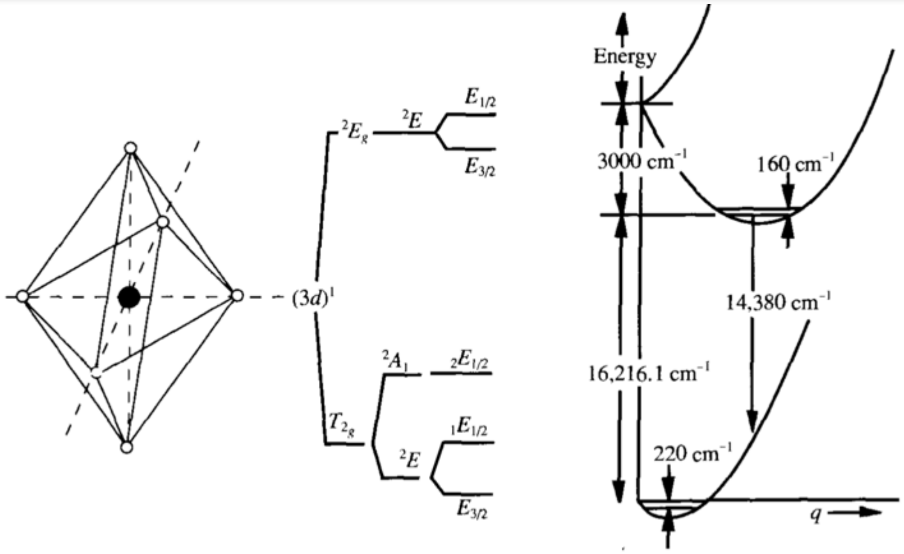


FIGURE 10.22. (a) The octahedral site for the titanium ion (solid) surrounded by the six oxygen atoms in sapphire. (b) Term splitting by the crystalline fields. (c) Simplified schematic of the potential energy curves for the $^2T_{2g}$ and the ^2E_g states of Ti^{3+} in sapphire. (Same as Fig. 1 of Byvik and Buoncristiani [79]. Numerical values for (c) from Fig. 3 of Gächter and Koningstein [85].)

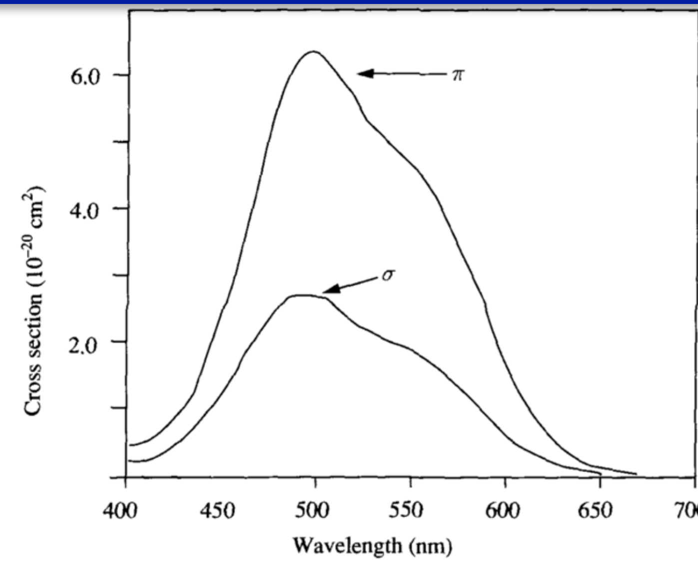


FIGURE 10.23. The polarized absorption cross section for the $^2T_{2g} \rightarrow {}^2E$ transition in $Ti:Al_2O_3$ (Data from Fig. 1 of Moulton [73].)

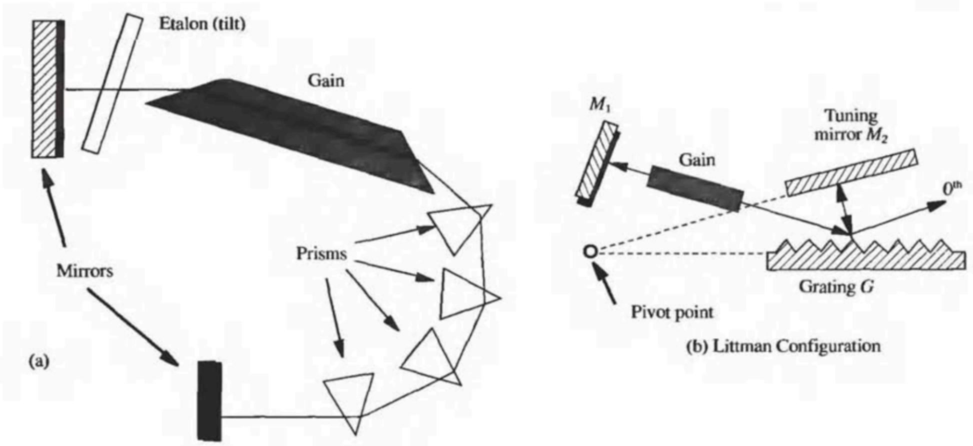


FIGURE 10.27. Various methods of tuning a laser.

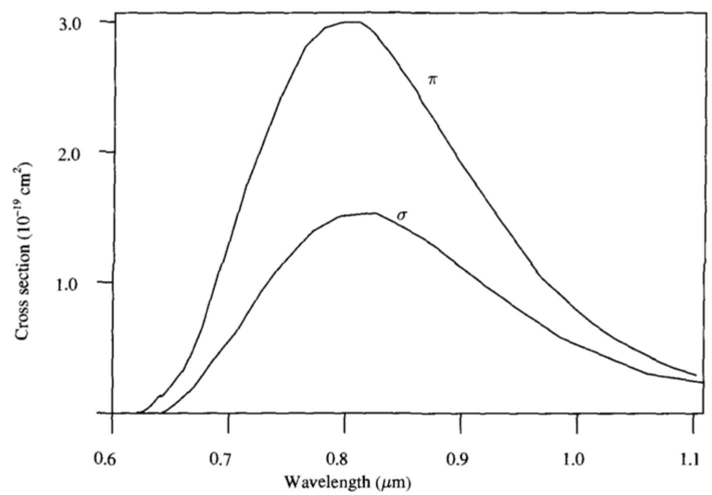
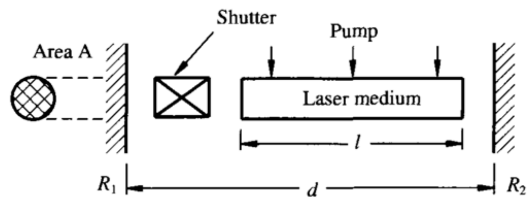


FIGURE 10.24. The polarized fluorescence spectra and relative gain cross section for $Ti:Al_2O_3$ (Data from Fig. 2 of Eggleston et al., [66].)

Nd: YAG Laser for Q-Switching



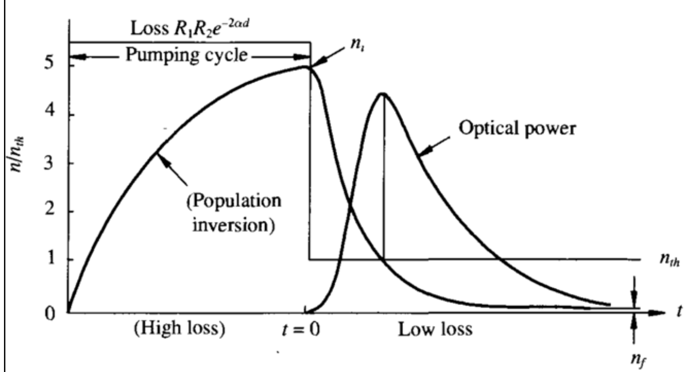
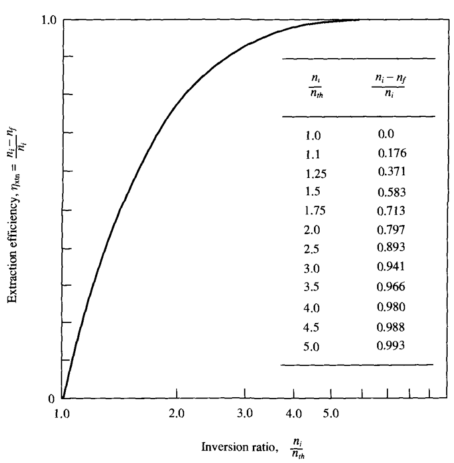


FIGURE 9.11. Guess at the sequence of events during a Q switch.



$\frac{n_i}{n_{th}}$	$\frac{n_i - n_f}{n_i}$
1.0	0.0
1.1	0.176
1.25	0.371
1.5	0.583
1.75	0.713
2.0	0.797
2.5	0.893
3.0	0.941
3.5	0.966
4.0	0.980
4.5	0.988
5.0	0.993

FIGURE 9.13. The energy extraction efficiency for a Q switched pulse.

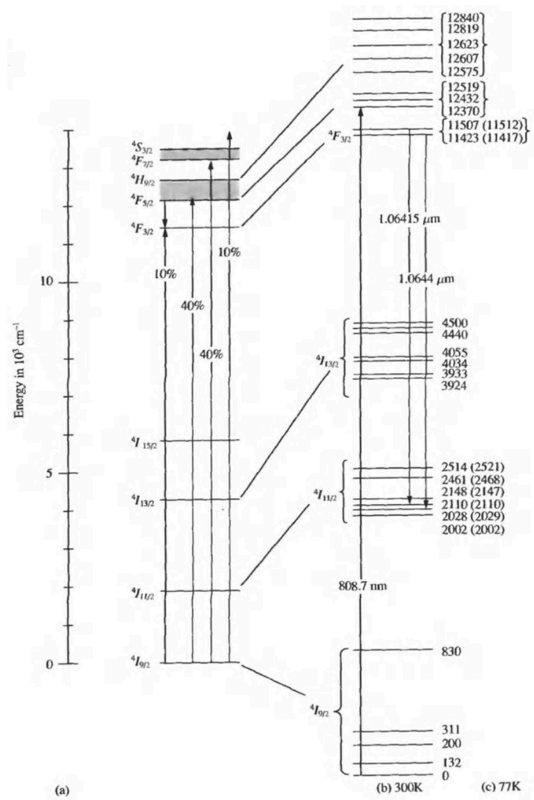
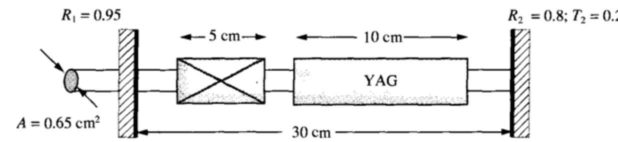


FIGURE 10.5. Energy level for neodymium in YAG. (a) Structure of YAG showing the pumping routes with the percentages referring to a pump with a broad spectral output. (b) Details of the manifold at 300 K showing the dominant transitions, the semiconductor laser pumping route is also shown. (c) Energy levels at 77 K. [Data from Kaminski [25]. See also Koechner [24].]

$$\frac{n_i}{n_f} = \exp\left(\frac{n_i - n_f}{n_{th}}\right)$$

10.25. The YAG laser shown below is pumped to three times threshold before the electro-optic shutter is opened for Q switching. Assume (1) the characteristics of the YAG rod are those specified in Tables 10.2 and 10.3 but ignore the scattering loss, (2) the transmission at all air-device interfaces is 0.99, (3) the index of refraction of the electro-optic switch is 2.3, (4) lasing at 1.0615 μm (where there is no significant overlap with any other transition), (5) the populations in the $^4F_{3/2}$ states are always distributed according to the Boltzmann relation with $kT = 208$ cm^{-1}, even within the Q-switched pulse.



(a) Evaluate the following parameters to be used in the calculation: photon lifetime of passive cavity in ns, A_{21} coefficient for the transition in sec^{-1}, stimulated emission cross section in cm^2, initial density of atoms in $^4F_{3/2}$ manifold in cm^{-3}, and final density of atoms in $^4F_{3/2}$ manifold in cm^{-3}.

(b) Compute the peak power in watts, the energy in joules, and determine the pulse width using the theory of Sec. 9.4.

(c) The theory of Sec. 9.4 is not quite applicable to this problem since the lifetime of the lower state is only 30 ns and the time scale for the establishment of a Boltzmann factor among the levels of the $^4I_{11/2}$ manifold is even shorter. Redo the calculations of (a) assuming that $N_I = 0$, which is a bit different from the analysis of Sec. 9.4.

Gas Discharge Lasers: CO_2 Laser

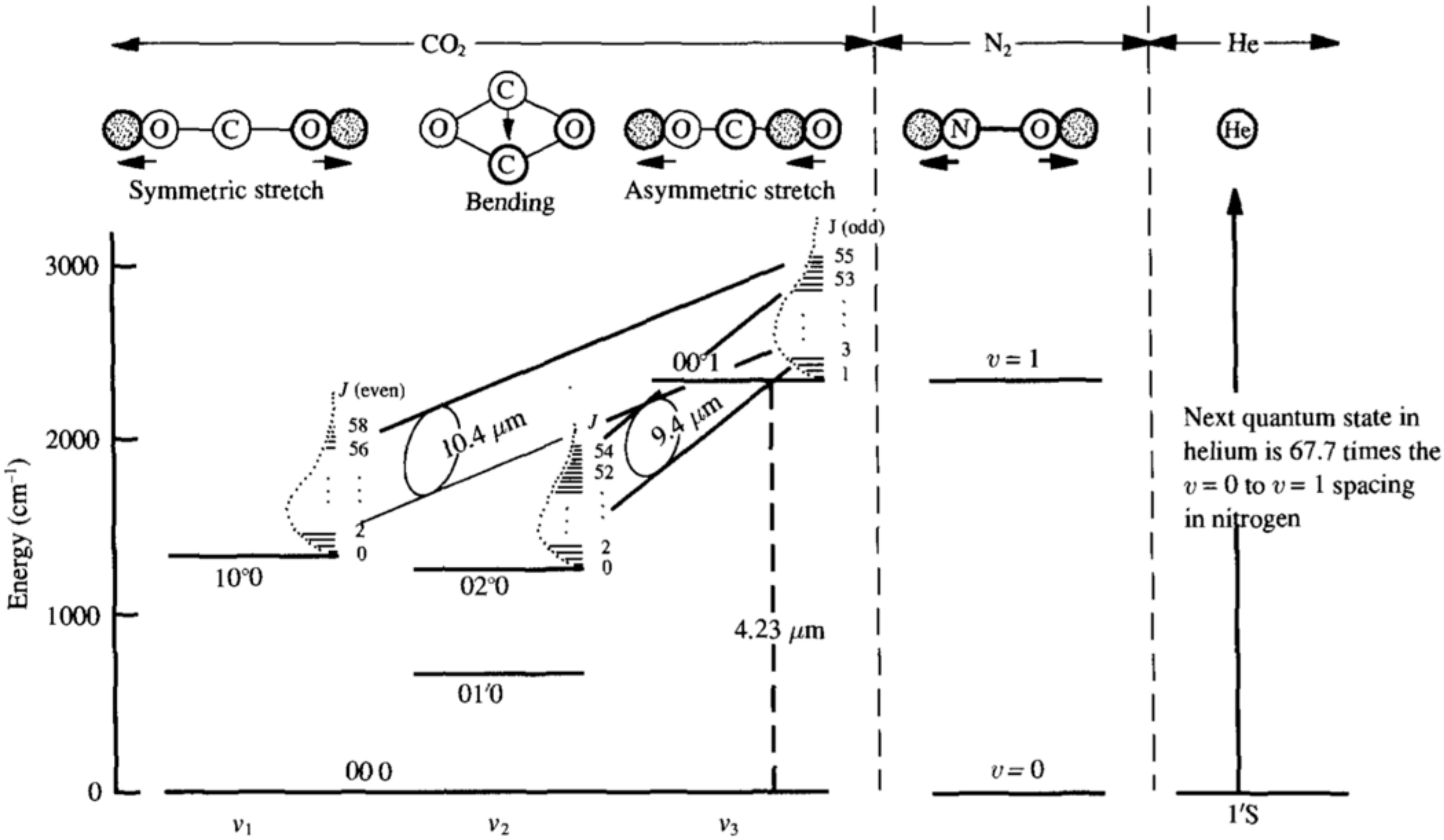


FIGURE 10.31. Energy-level diagram of the CO_2-N_2-He laser.

Gas Discharge Lasers: CO_2 Laser

1. The electrical power is transferred to the electrons (as is the case in *all* discharges) by the electric field.
2. The electrons transfer this power by collisions to the neutral gas atoms. This power is apportioned to the gas in three different categories:
   a. *Gas heating:* This is caused by the "elastic" collisions of the very light electrons with the more massive neutral atoms. Although these collisions are mostly elastic, there are many such collisions, and some energy is expended in raising the kinetic temperature of the gas.
   b. *Vibration excitation:* This is an inelastic collision process represented by the following chemical equations:
      1. For the upper state:

$$e + N_2(v = 0) \longrightarrow N_2(v = n \leq 8) + (e - KE) \tag{10.7.12}$$

followed by

$$\begin{aligned} &N_2(v = n) + CO_2(000) \\ &\quad \longrightarrow N_2(v = n - 1) + CO_2(001) \end{aligned} \tag{10.7.13}$$

or

$$e + CO_2(000) \longrightarrow CO_2(001) + (e - KE) \tag{10.7.14}$$

      2. For the lower state:

$$\begin{aligned} e + CO_2(000) \longrightarrow\ & CO_2(010) \\ & \text{or } CO_2(020) \\ & \text{or } CO_2(100) + (e - KE) \end{aligned} \tag{10.7.15}$$

   c. *Electronic excitation and ionization:* Although ionization is essential to maintain an active discharge, the fraction of the electrical power used to do so is usually insignificant in discharges in molecular gases.
3. Theory and experiment show that 60% of the electrical power can be funneled into pumping the upper laser level (see Chapter 17).

Excimer Lasers

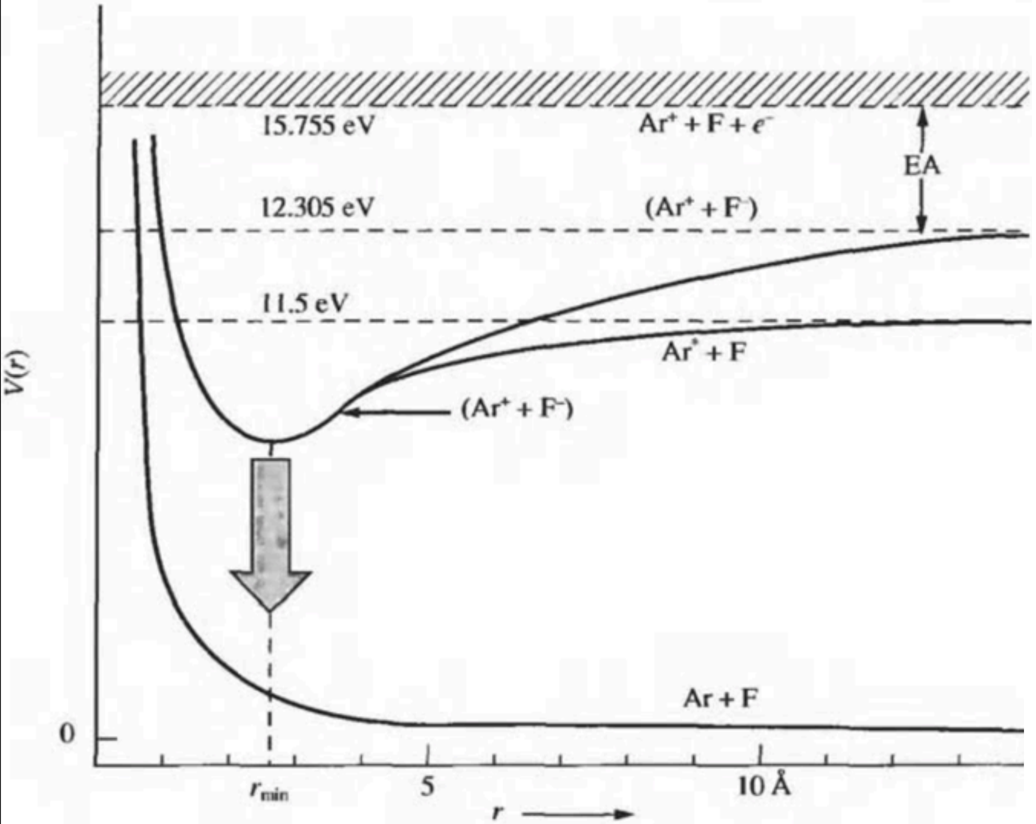


FIGURE 10.33. Energy-level diagram associated with the formation of the (Ar^+F^-) excimer.

$$Ar^* + F_2 \longrightarrow (Ar^+F^-)^* + F$$

TABLE 10.9 Data on Rare Gas–Halide Laser Systems

Excimer	r (Å)	ω_e (cm^{-1})	σ (10^{-16} cm^2)	τ (ns)	λ (nm)
XeBr	3.1	120	2.2	12–17.5	282
XeCl	2.9	194	4.5	11	308
XeF	2.4	309	5.3	12–18.8	351
KrCl	2.8	210	—	—	222
KrF	2.3	310	2.5	6.7–9	249
ArCl	2.7	(280)	—	—	175
ArF	2.2	(430)	2.9	4.2	193

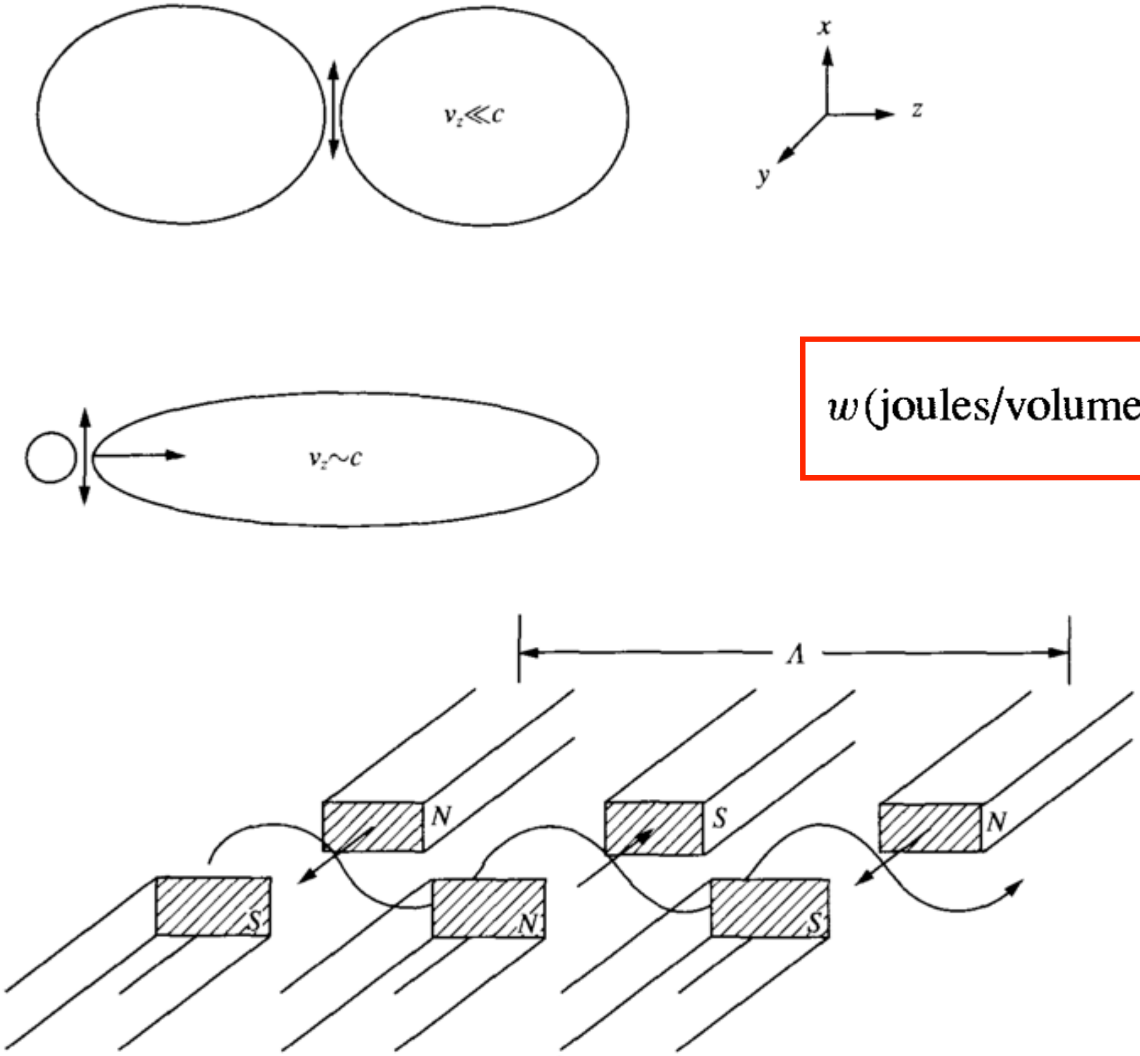
r_e, minimum of the lowest ionically bound excimer state; ω_e, vibration constant representative; σ, stimulated emission cross section; τ, radiative lifetime; λ, dominant laser wavelength. These lasers hold a commanding lead as far as efficiency in the production of UV and near-UV power. Values in parentheses are estimates. Data from Brau [31].

TABLE 10.8 Rare Gas–Halide Wavelengths (nm)

Halogen	EA		Rare Gas: Neon	Argon	Krypton	Xenon
		IP (eV)	21.56	15.755	13.996	12.127
		M (eV)	16.6	11.55	9.92	8.31
			nm	nm	nm	nm
Fluorine	(3.45)		108	**193**	**249**	**351**
Chlorine	(3.61)		—	**175**	**222**	**308**
Bromine	(3.36)			161	**206**	**282**
Iodine	(3.06)				185	253

IP, ionization potential; M, metastable level; EA, electron affinity. Wavelengths in boldface type refer to the peak of the laser; those in lightface type refer to the fluorescence assignable to an excimer (see text) and have not yet lased.
Data from Brau [31].

Free Electron Lasers



$$\boxed{w(\text{joules/volume}) = \int \mathbf{E}_{\text{opt}} \cdot [\mathbf{i}_b = n_{\text{beam}}(-e)\mathbf{v}_{\text{beam}}]\, dt}$$

FIGURE 10.35. The radiation by an accelerated electron. (a) The pattern when the electron velocity $v_z \ll c$, (b) $v_z \sim c$, and (c) the electron trajectory in a wiggler.

Free Electron Lasers

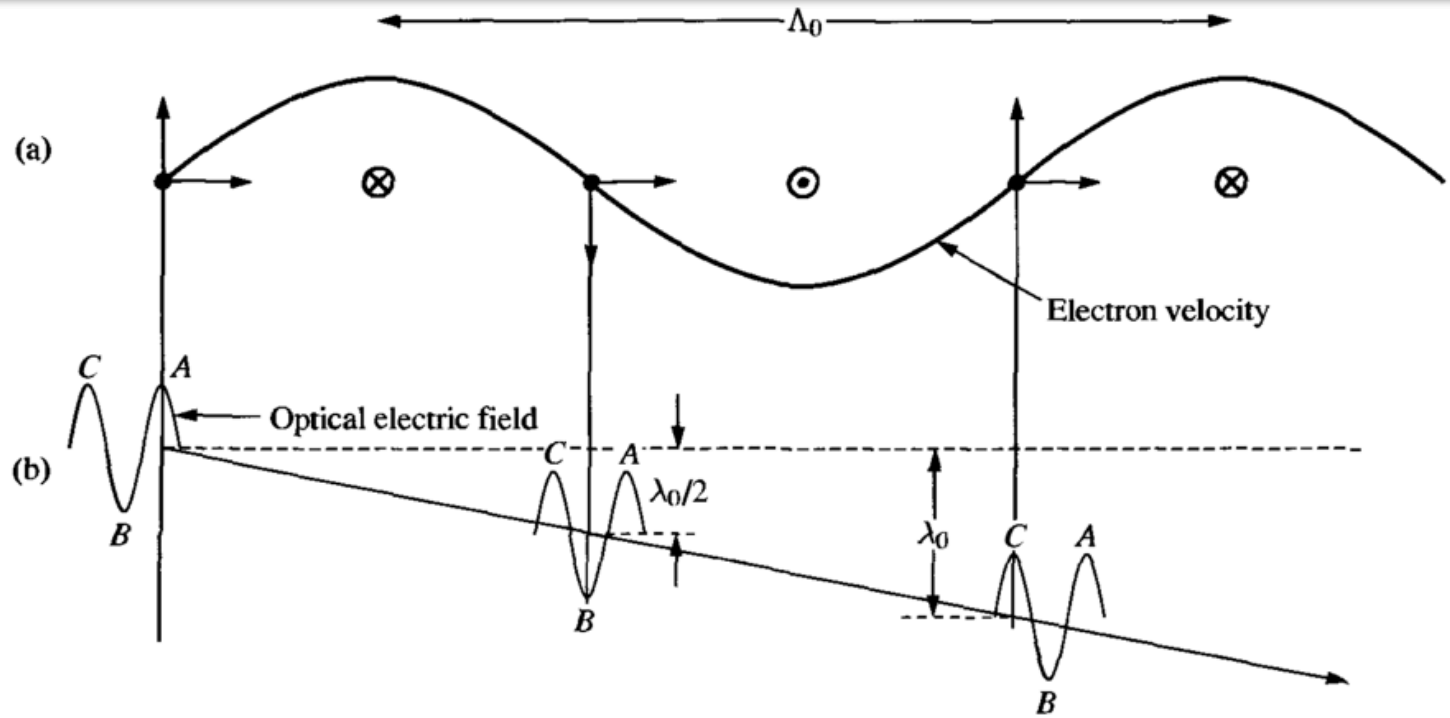


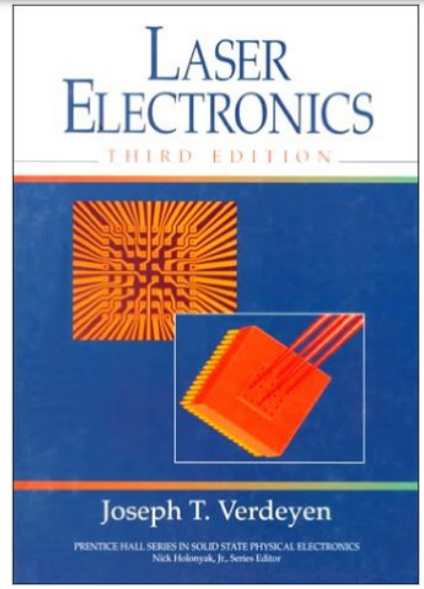
FIGURE 10.36. The accumulated interaction of the wiggled beam and the optical field. (Adaptation of Fig. 13.1 of Yario [17b].)

$$w(\text{joules/volume}) = \int \mathbf{E}_{\text{opt}} \cdot [\mathbf{i}_b = n_{\text{beam}}(-e)\mathbf{v}_{\text{beam}}]\, dt$$

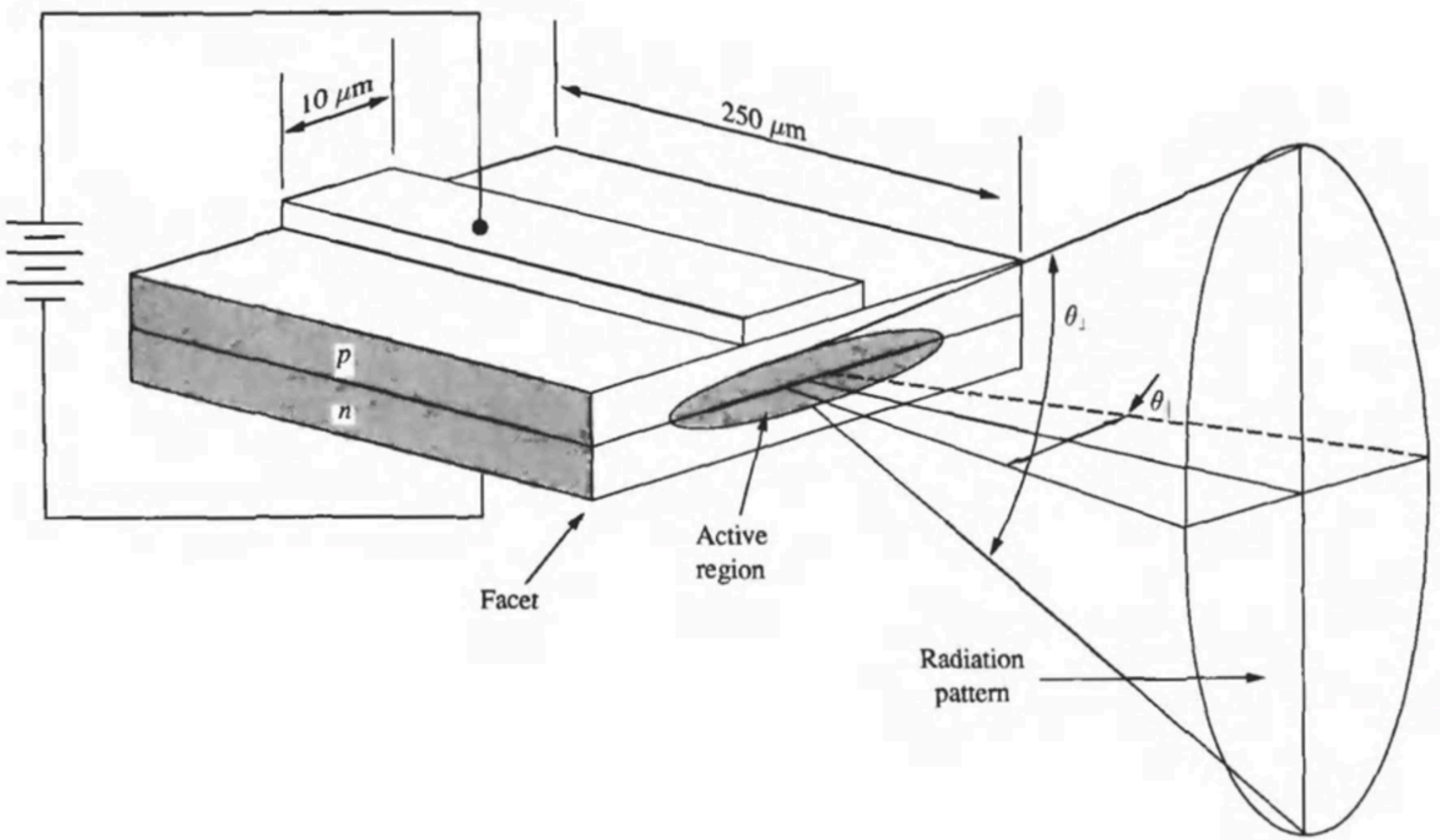
$$\boxed{\frac{d\mathbf{p}}{dt} = \frac{d[\gamma m_0 \mathbf{v}]}{dt} = (-e)[\mathbf{E}_{\text{opt}} + \mathbf{v} \times \mathbf{B}]}$$

$$\boxed{\Lambda_0 = \frac{\lambda_0 v_z}{c - v_z} = \frac{\lambda_0 \beta_z}{1 - \beta_z}}$$

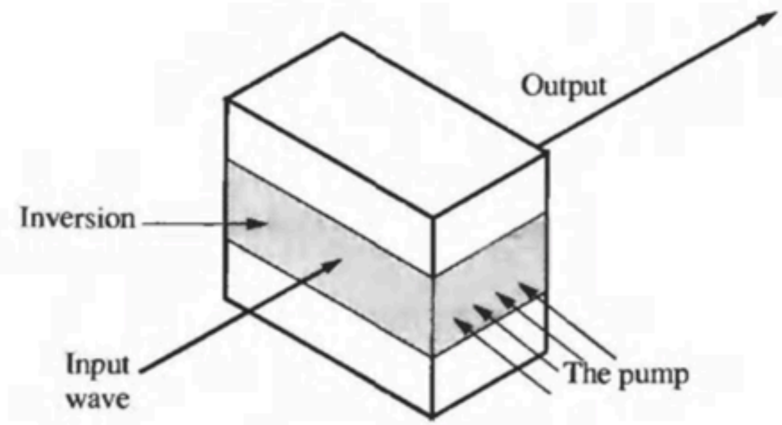
Course Outline



Semiconductor Diode Lasers



Semiconductor Optical Gain and Population Inversion



(a) The experiment

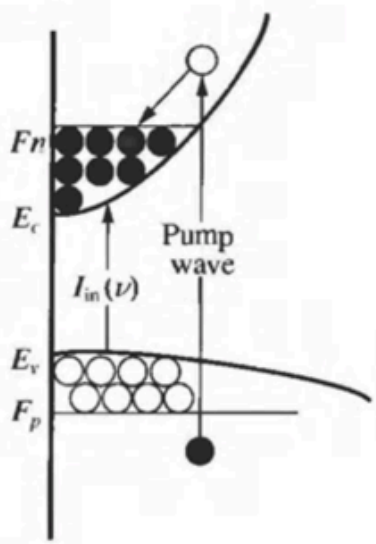
Gain spectrum of atomic systems

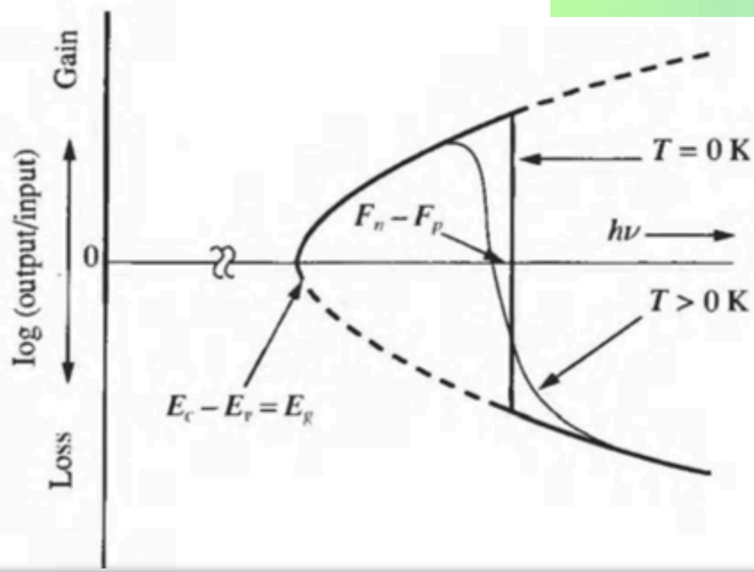
$$\gamma_0(\nu) = A_{21}\frac{\lambda_0^2}{8\pi n^2}g(\nu)\left(N_2 - \frac{g_2}{g_1}N_1\right)$$

Compare

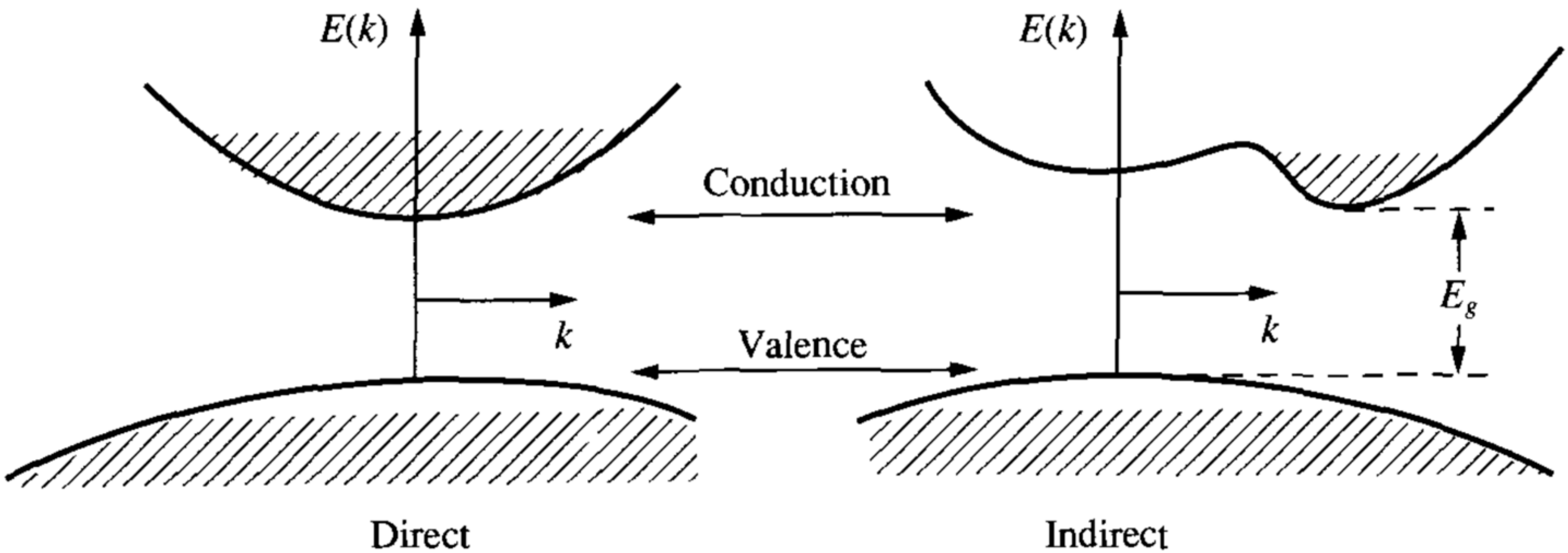
$$\boxed{\gamma(\nu) = A_{21}\frac{\lambda_0^2}{8\pi n^2}\rho_{\rm jnt}(\nu)[f_c(E_2) - f_v(E_1)]}$$

Gain spectrum of a semiconductor

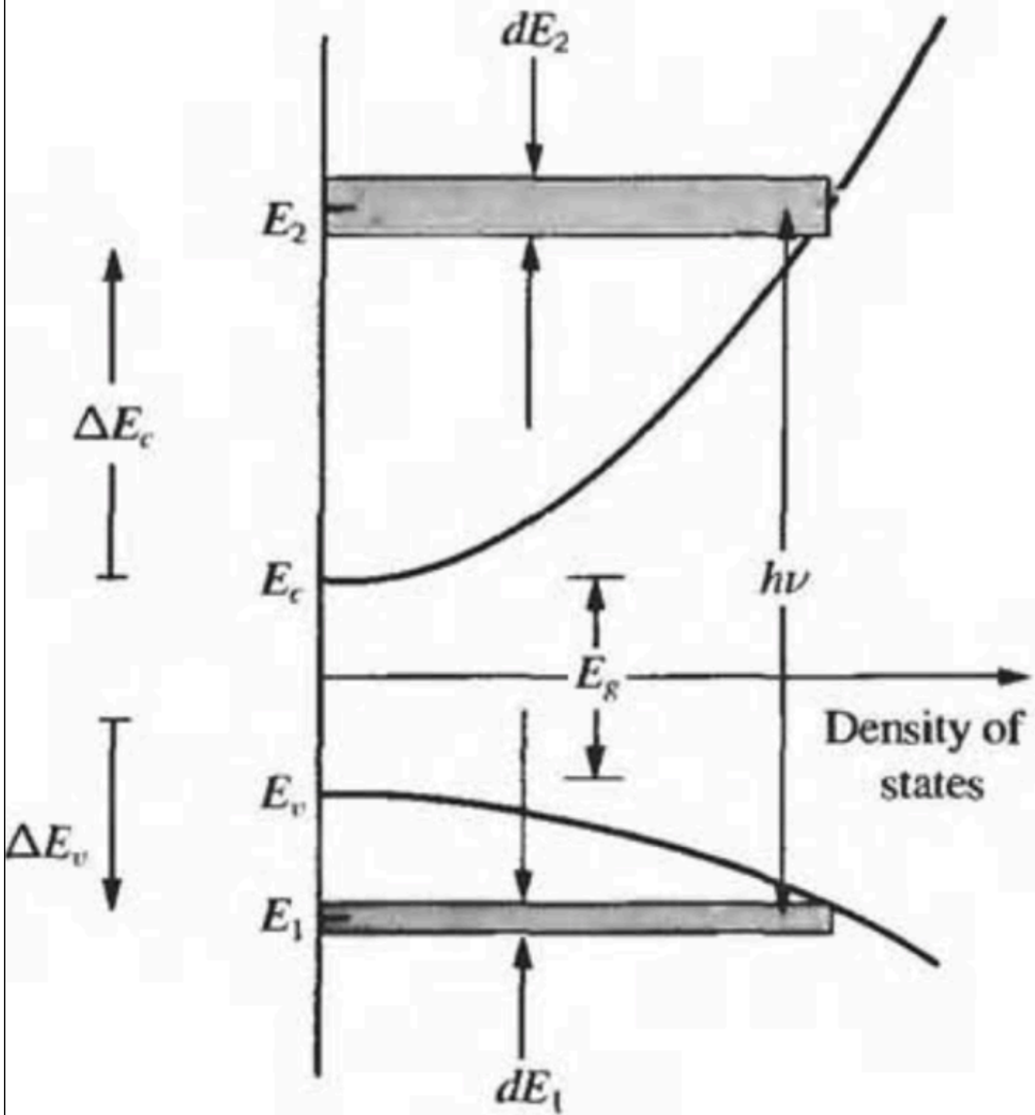




Direct and Indirect Bandgap Semiconductors



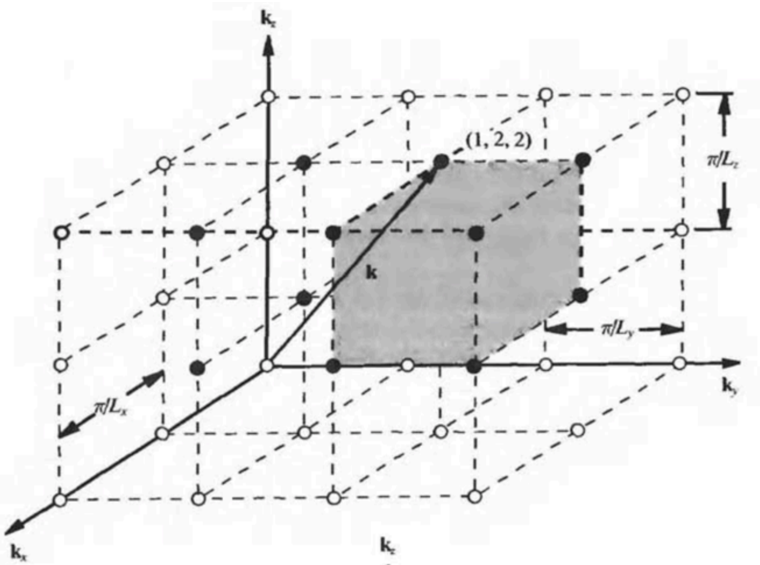
Semiconductor Optical Gain and Population Inversion



$$\rho_{jnt}(\nu) = \frac{1}{2\pi^2}\left(\frac{2m_r^\star}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{h\nu - E_g}$$

Optical Joint Density of States for a 3D Semiconductor

Electron wave modes and Density of States



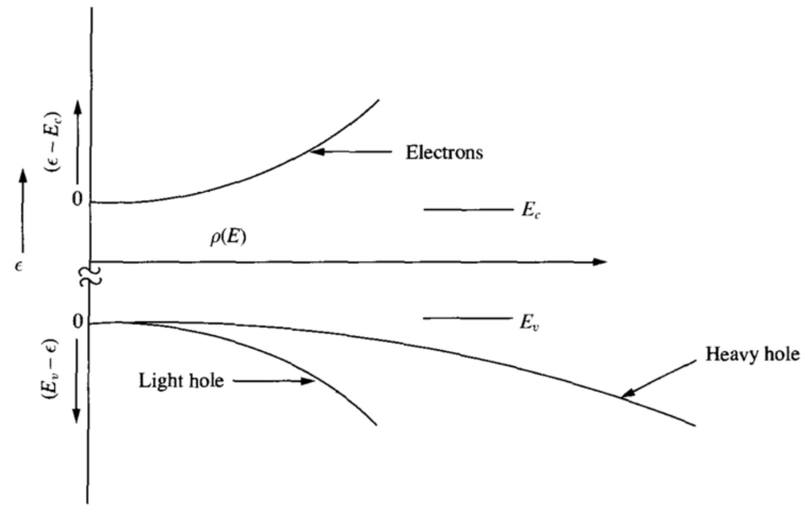
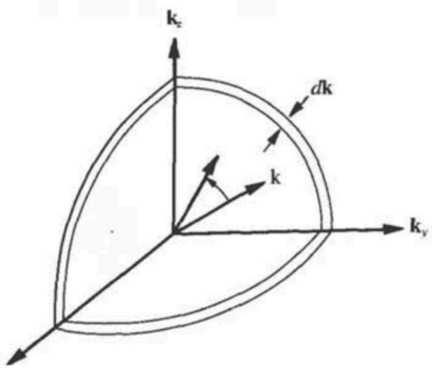
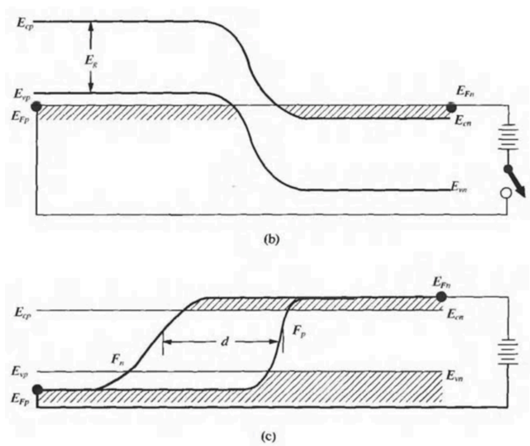
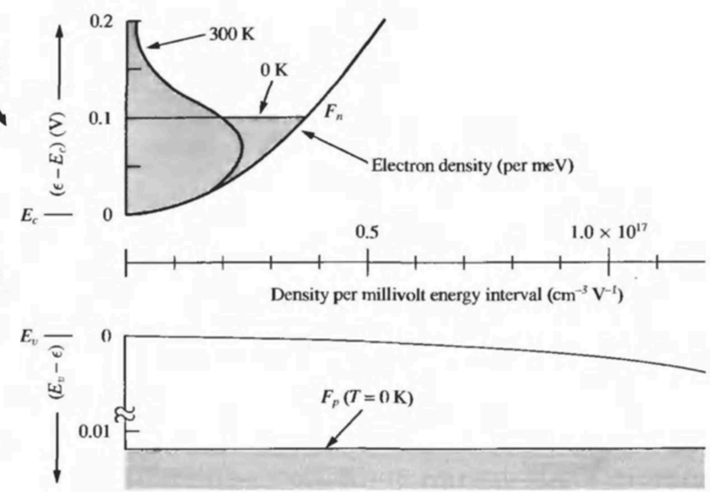


FIGURE 11.4. Density of states in a semiconductor with $m^*_{hh} = 4 \cdot m^*_e$ and $m_{lh} = m^*_e$.



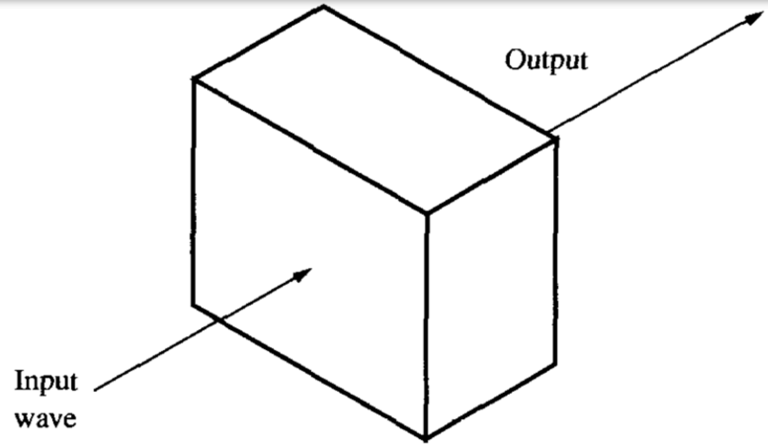


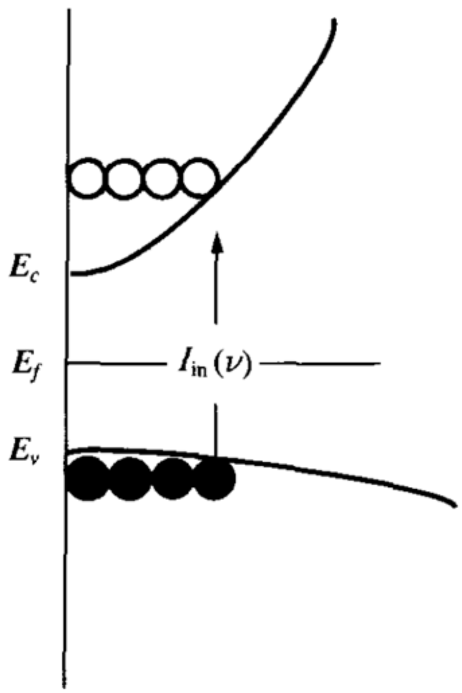


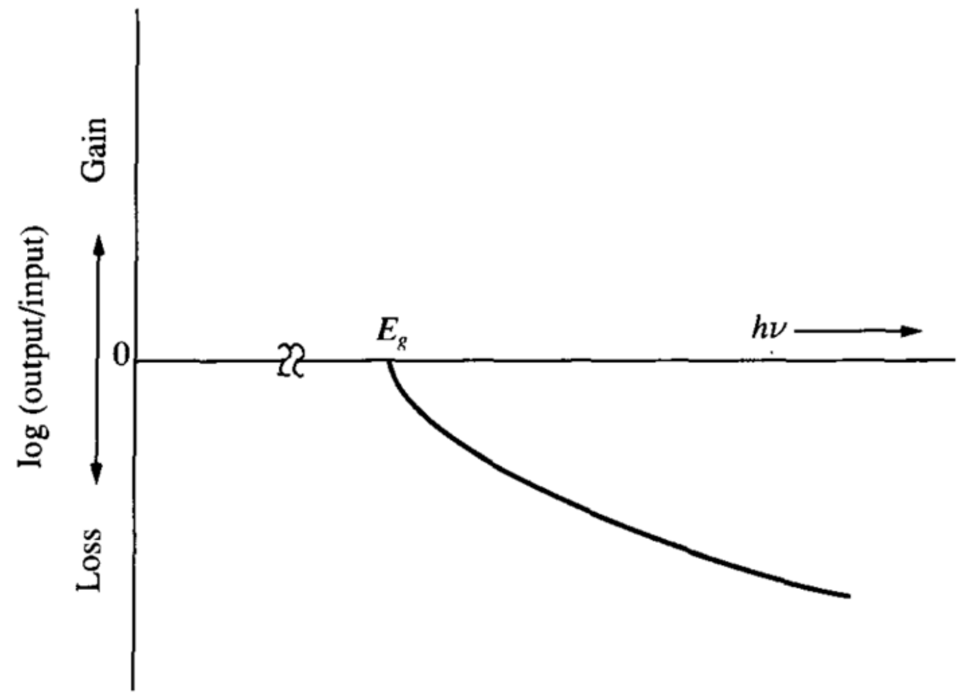
$$f_c(\epsilon) = \frac{1}{\exp[(\epsilon - F_n)/kT] + 1}$$

$$f_v(\epsilon) = \frac{1}{\exp[(\epsilon - F_p)/kT] + 1}$$

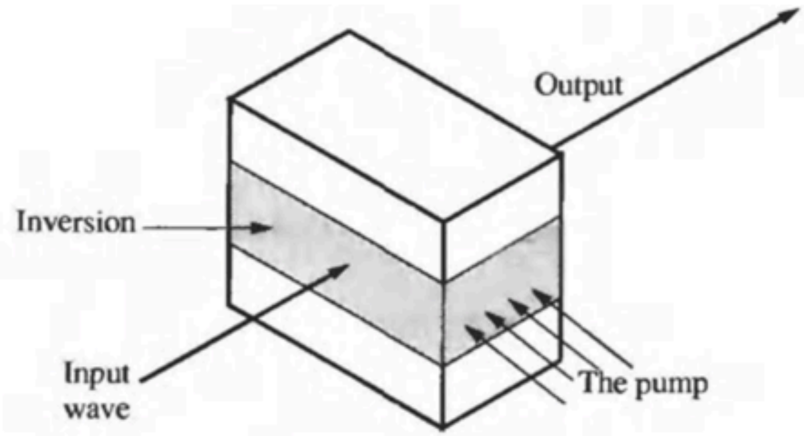
Optical Absorption Coefficient of a Semiconductor



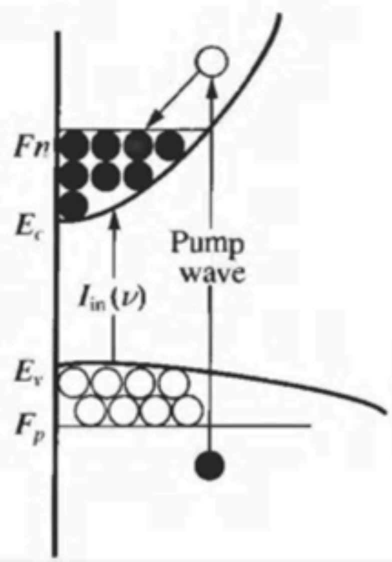


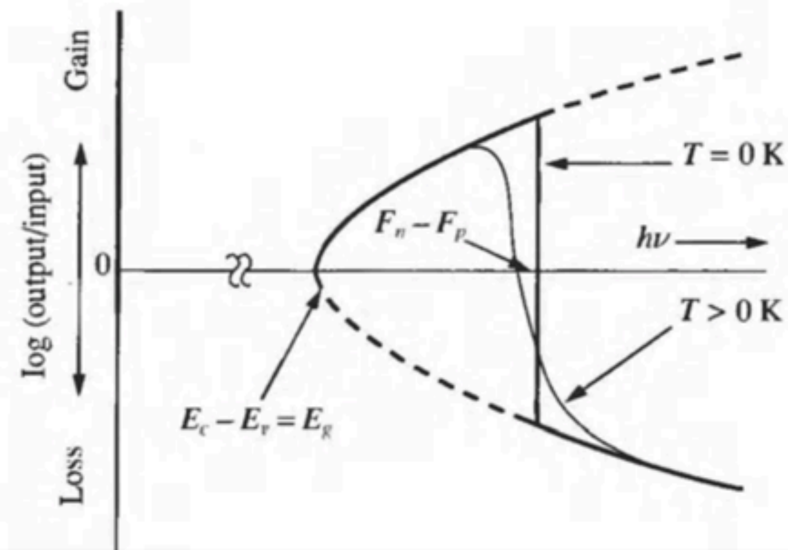


Semiconductor Optical Gain and Population Inversion

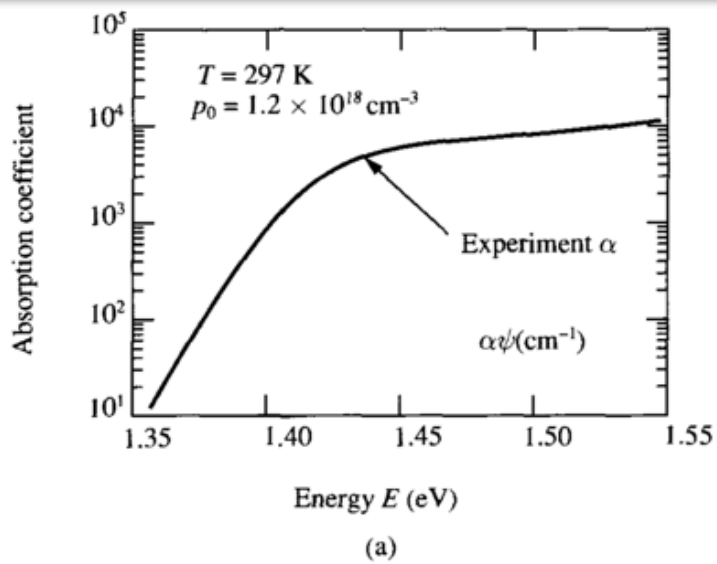


(a) The experiment





Spontaneous Emission Spectrum



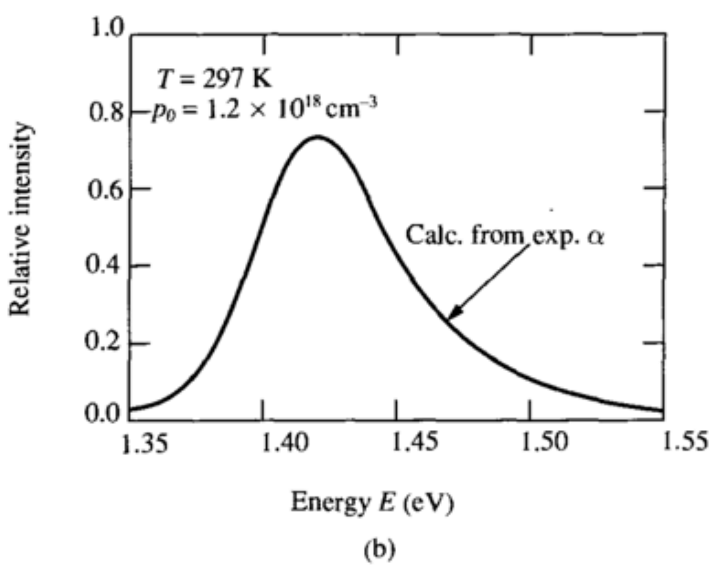


FIGURE 11.9. (a) Measured absorption coefficient in GaAs doped with an acceptor concentration of 1.2×10^{18} cm^3. (b) Spontaneous emission profile calculated from the absorption. (Data from Casey and Stern [19].)

Spontaneous Emission Rate and Spectrum

$$\boxed{R(\nu) = A_{21}[f_2(1 - f_1)]\rho_{\text{jnt}}(\nu)}$$

Carrier Injection to Modulate Quasi Fermi Levels

$$n = \frac{1}{2\pi^2}\left[\frac{2m_e^* kT}{\hbar^2}\right]^{3/2} \cdot \left[\exp - \left(\frac{E_c - F_n}{kT}\right)\right] \int_0^\infty \frac{x^{1/2}\,dx}{e^x + 1/b}$$

$$p = \frac{1}{2\pi^2}\left[\frac{2m_h^* kT}{\hbar^2}\right]^{3/2} \cdot \left[\exp - \left(\frac{F_p - E_v}{kT}\right)\right] \int_0^\infty \frac{x^{1/2}\,dx}{e^x + 1/a}$$

$$a = \exp\left\{[F_p - E_v]/kT\right\} \qquad b = \exp\left\{[E_c - F_n]/kT\right\}$$

TABLE 11.2 Carrier Densities in an Intrinsic Semiconductor as a Function of the Position of the Quasi-Fermi Levels

(a, b)	$(E_c - F_n)/kT$ $(F_p - E_v)/kT$	I_2	n (cm^{-3})	p (cm^{-3})	Comment
10^3	6.91	1.000	4.36^{14}	1.02^{16}	
10^2	4.61	0.996	4.34^{15}	1.02^{17}	
50	3.91	0.993	8.66^{15}	2.04^{17}	
20	3.00	0.983	2.14^{16}	5.04^{17}	
10	2.30	0.967	4.22^{16}	9.92^{17}	
7.75	2.05	0.957	5.40^{16}	$\mathbf{1.27^{18}}$	Fermi
5	1.61	0.936	8.16^{16}	1.92^{18}	levels
2	0.69	0.860	1.87^{17}	4.41^{18}	in gap ↑
1	0.00	0.765	3.34^{17}	7.84^{18}	—
0.5	−0.69	0.641	5.59^{17}	1.31^{19}	Fermi ↓
0.2	−1.61	0.457	9.96^{17}	2.34^{19}	levels
0.129	−2.05	0.373	$\mathbf{1.27^{18}}$	2.99^{19}	within
0.1	−2.30	0.329	1.43^{18}	3.37^{19}	bands

Homojunction Diode Laser

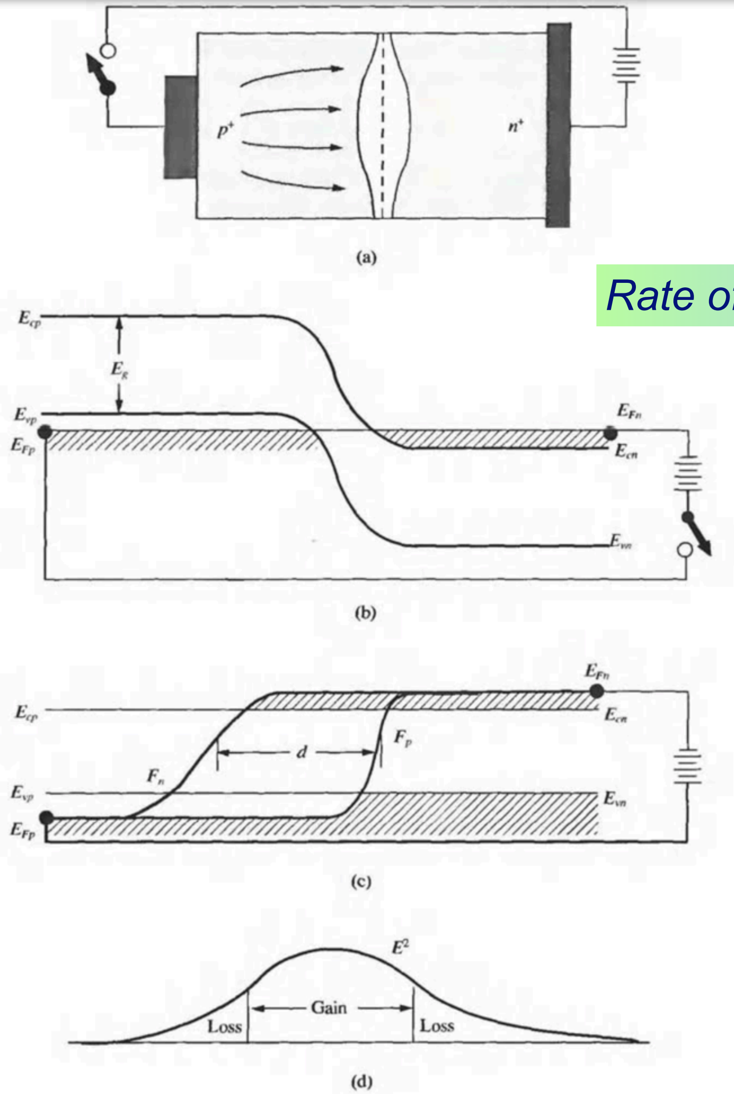


FIGURE 11.10. The homojunction laser: (a) shows a cross section of the junction with the bowed area being due to current spreading; (b) and (c) show the band diagram in equilibrium and with injected current; (d) illustrates the electromagnetic mode experiencing gain and loss.

Rate equation for electrons and holes

$$\boxed{\frac{dn}{dt} = -\beta \cdot n \cdot p + G}$$

Rate of change of electrons

Recombination rate

Generation rate

Estimation of required threshold injection current density

$$G = \beta \cdot n \cdot p = 2 \times 10^{-10} \cdot 1.27 \times 10^{18} \cdot 1.27 \times 10^{18}$$

$$= 3.23 \times 10^{26} \, \frac{e-h \text{ pairs}}{\text{cm}^3 - \text{sec}}$$

$$J = ed \cdot \left[\frac{dn}{dt}\right]_{\text{recomb}} = 1.6 \times 10^{-19} \cdot 1 \times 10^{-4} \cdot 3.23 \times 10^{26}$$

$$= 5.17 \text{ kA/cm}^2$$

$$T^{3/2} \exp -[\Delta E / kT] \quad \Rightarrow \quad J = J_0 \exp\left[\frac{T - T_0}{T_0}\right]$$

T-dependence of carrier density

T-dependence of Laser threshold current density

Heterojunction Diode Laser

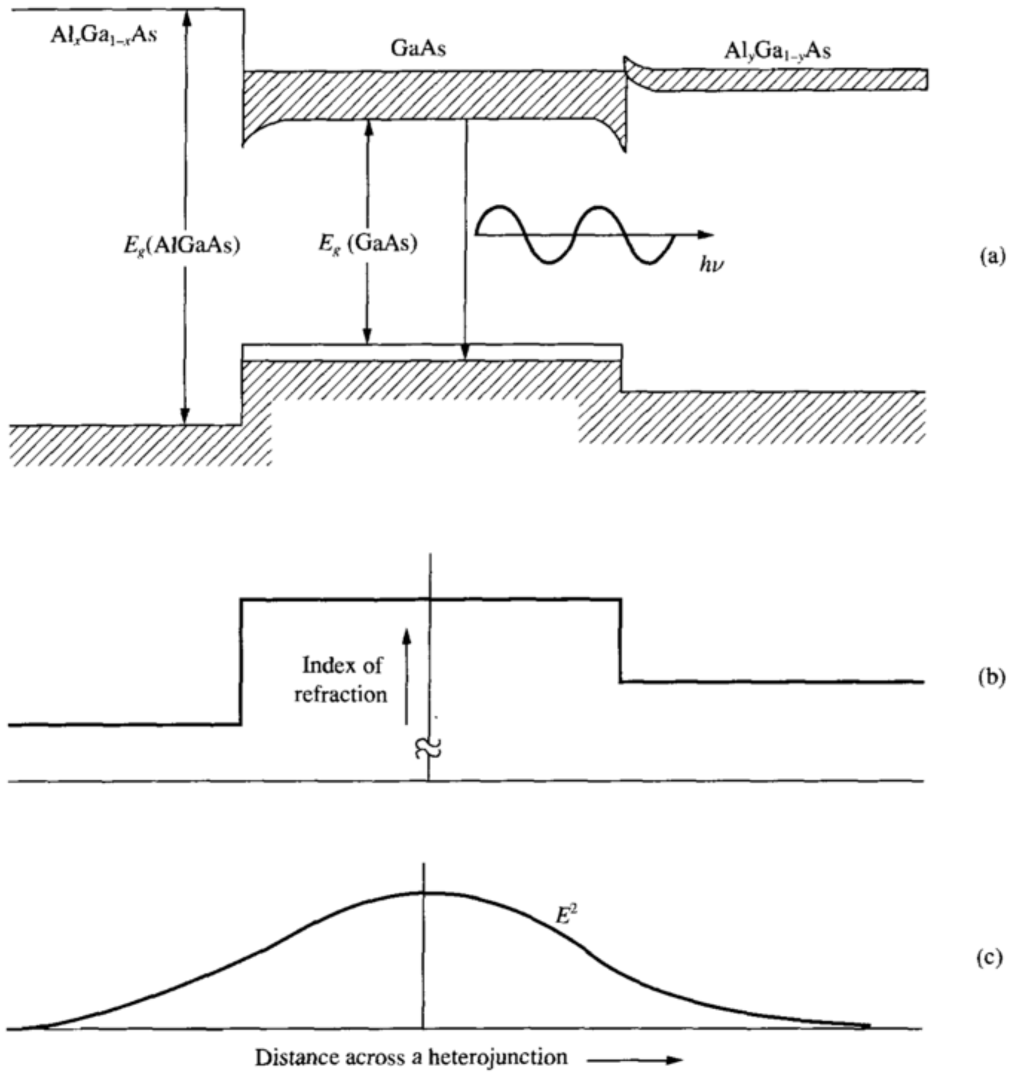
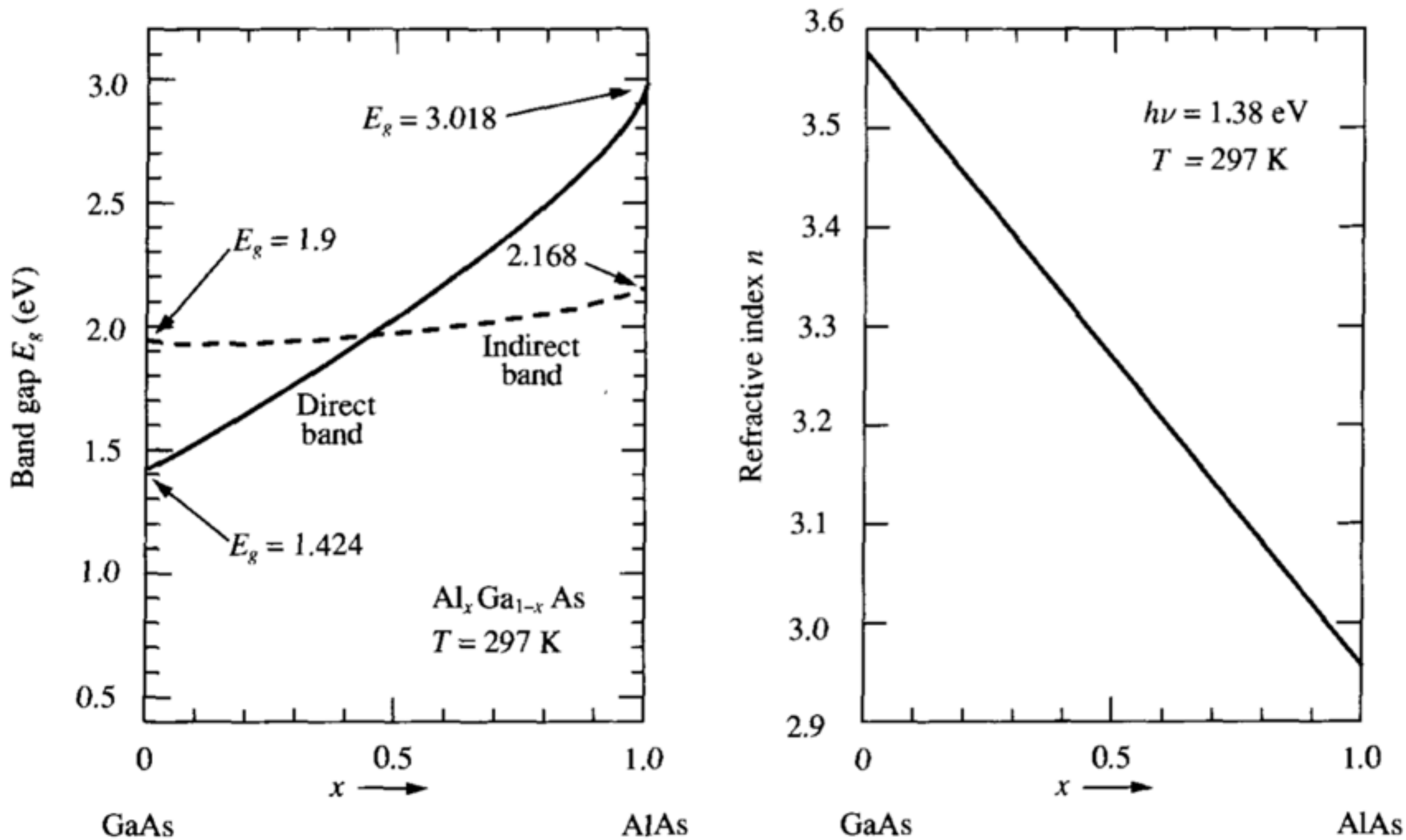


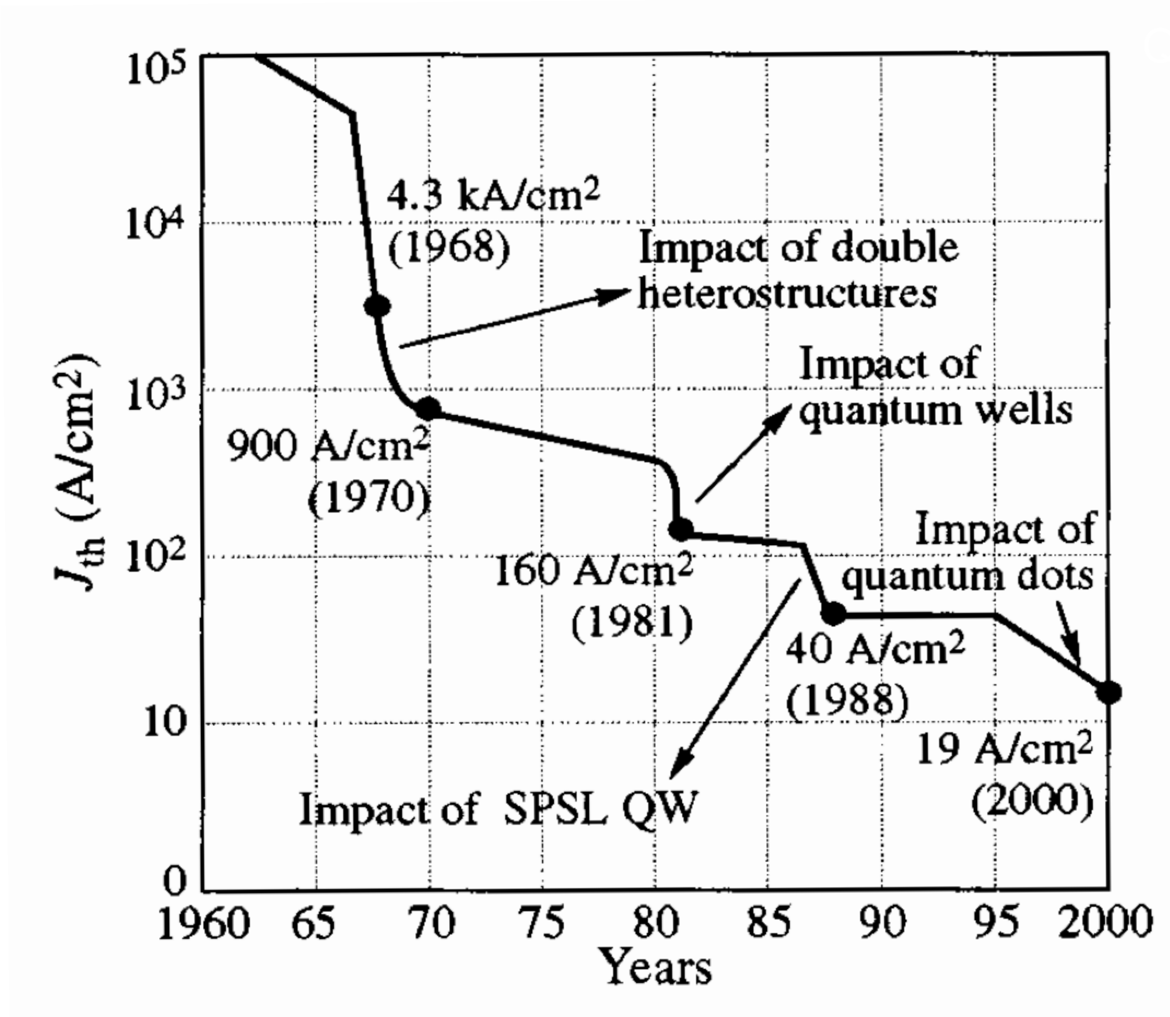
FIGURE 11.13. (a) The band diagram for a forward-biased heterostructure, (b) the refractive index, and (c) a sketch of the light intensity in the vicinity of the active region.

Heterojunction Diode Laser



Refractive Index of Semiconductors Decreases with Increase in the Bandgap

Double Heterostructure Laser



Alferov: 2000 Physics Nobel Lecture

Heterojunction Diode Laser

The Nobel Prize in Physics 2000
Zhores I. Alferov, Herbert Kroemer, Jack S. Kilby

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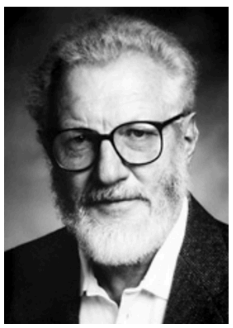
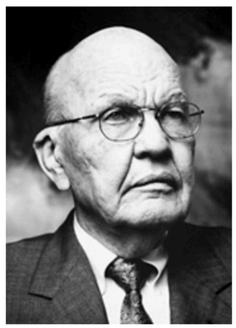
The Nobel Prize in Physics 2000

Zhores I. Alferov
Prize share: 1/4

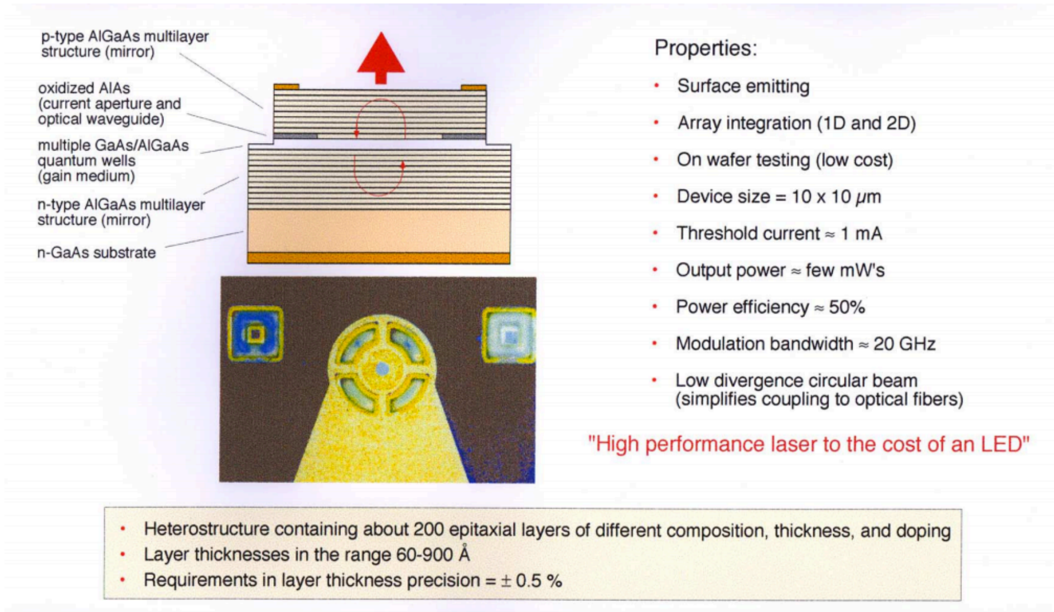


Herbert Kroemer
Prize share: 1/4

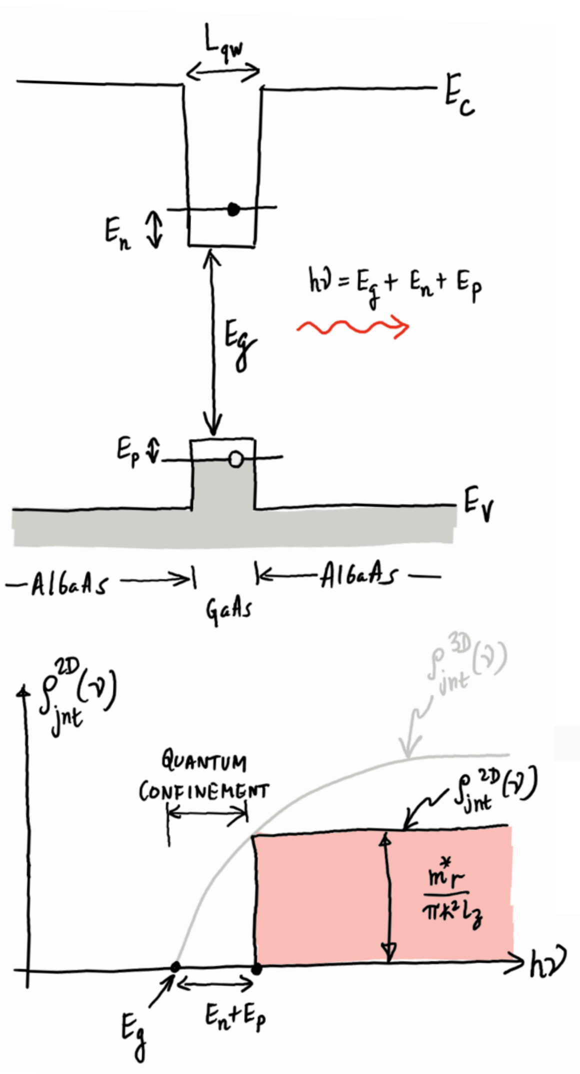


Jack S. Kilby
Prize share: 1/2

The Nobel Prize in Physics 2000 was awarded *"for basic work on information and communication technology"* with one half jointly to Zhores I. Alferov and Herbert Kroemer *"for developing semiconductor heterostructures used in high-speed- and opto-electronics"* and the other half to Jack S. Kilby *"for his part in the invention of the integrated circuit"*.



Quantum Well Laser



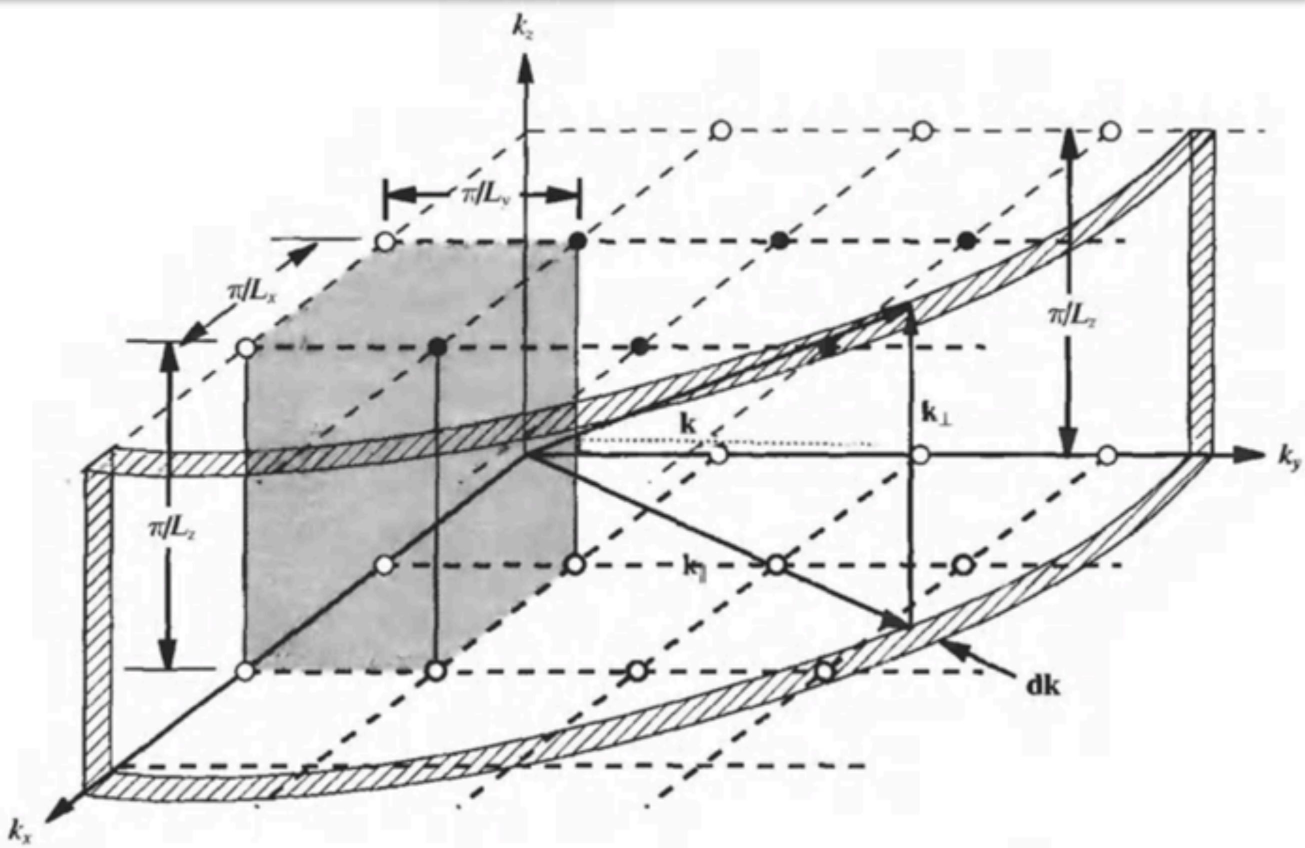


FIGURE 11.14. Allowed momentum vectors in a "thin" (i.e., L_z < 200 Å) semiconductor. The solid dots represent allowed states.

$$\boxed{\rho^{2D}_{jnt}(\nu) = \frac{m_r^\star}{\pi\hbar^2 L_{qw}}\Theta[h\nu - (E_g + E_n + E_p)]}$$

Quantum Well Laser

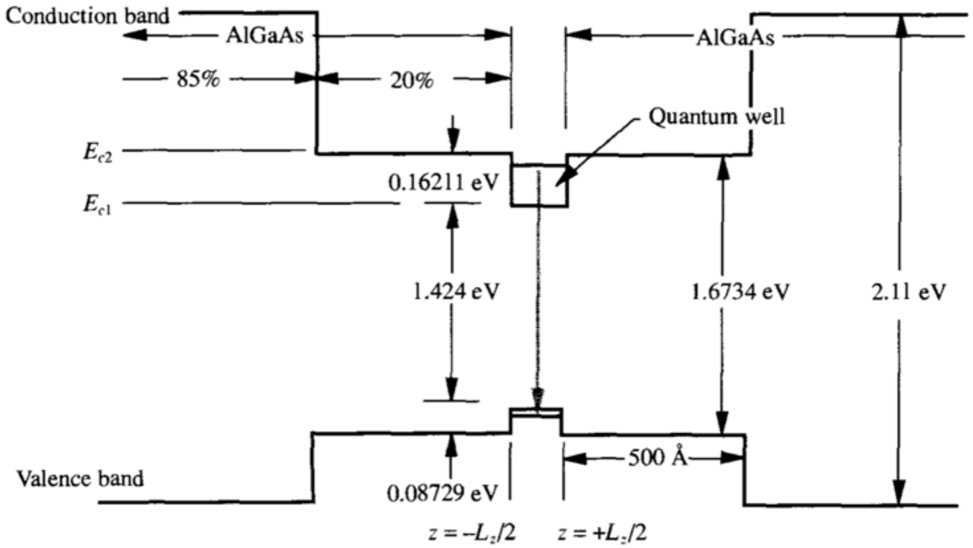


FIGURE 11.18. A 100 Å GaAs layer sandwiched between two 20% AlGaAs barrier layers that are ~ 500 Å thick. The 85% AlGaAs layer is to provide a dielectric waveguide for an electromagnetic mode. The indirect bandgap for $Al_{0.85}Ga_{0.15}As$ is given.

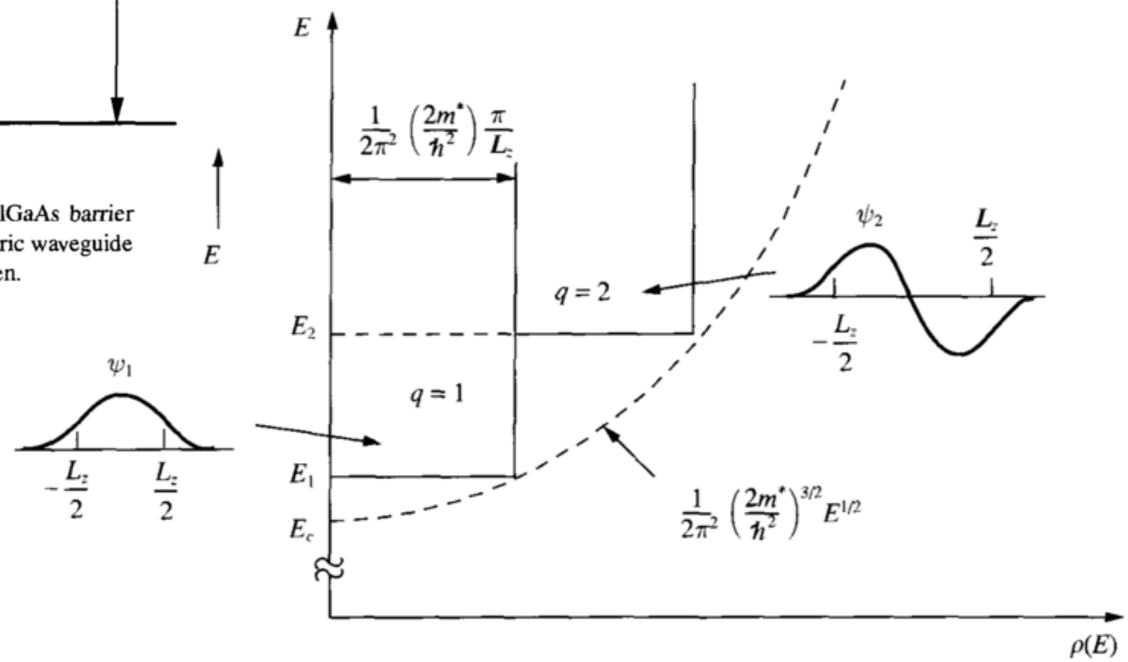
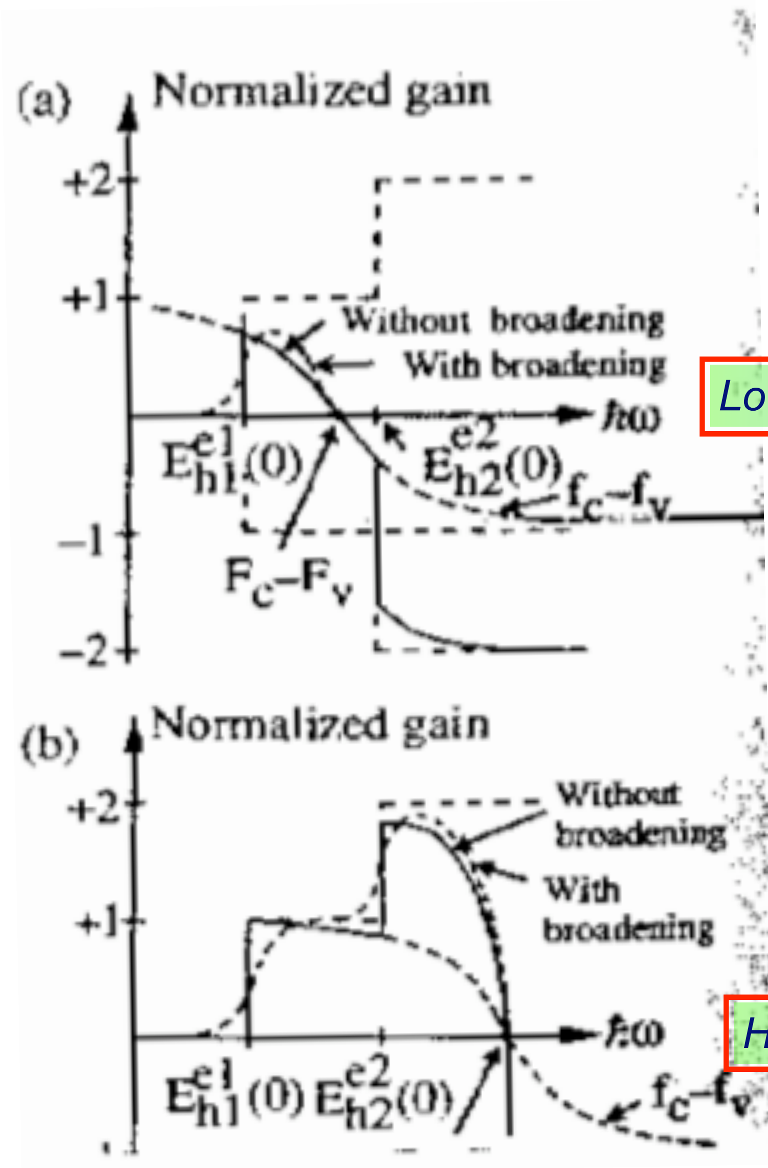
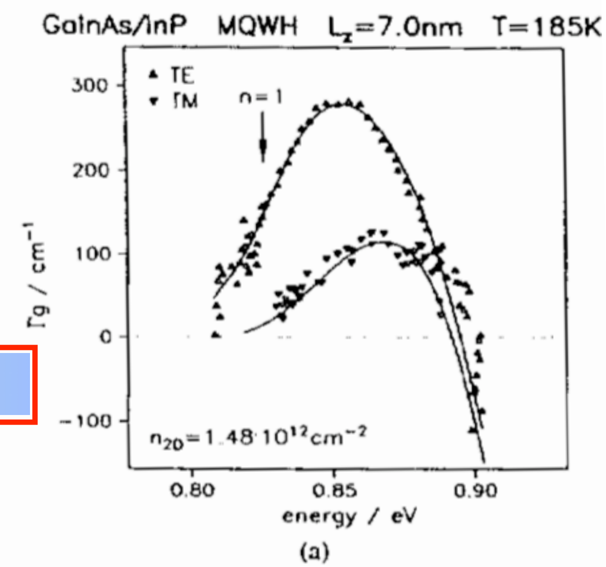


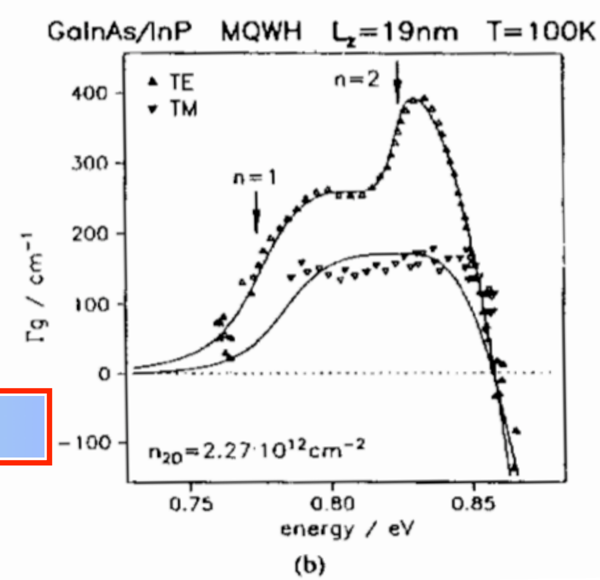
FIGURE 11.15. Density of states in a quantum well of thickness L_z. The lighter dashed curve is the normal density of states given by (11.2.8). The sketch indicates dependence of the wave function along z for the two subbands shown.

Measured Gain Profiles in Quantum Wells



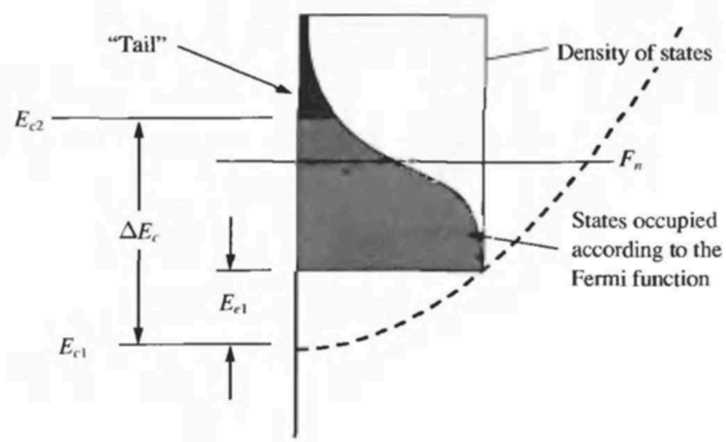
Low Injection Condition





High Injection Condition

Quantum Well Laser



IGURE 11.20. The distribution of electrons in the first bound state of a quantum well.

Mode Confinement Factor for Gain

$$\Gamma = \frac{\int_{QW} E^2(x)\,dx}{\int_{\text{all } x} E^2(x)\,dx}$$

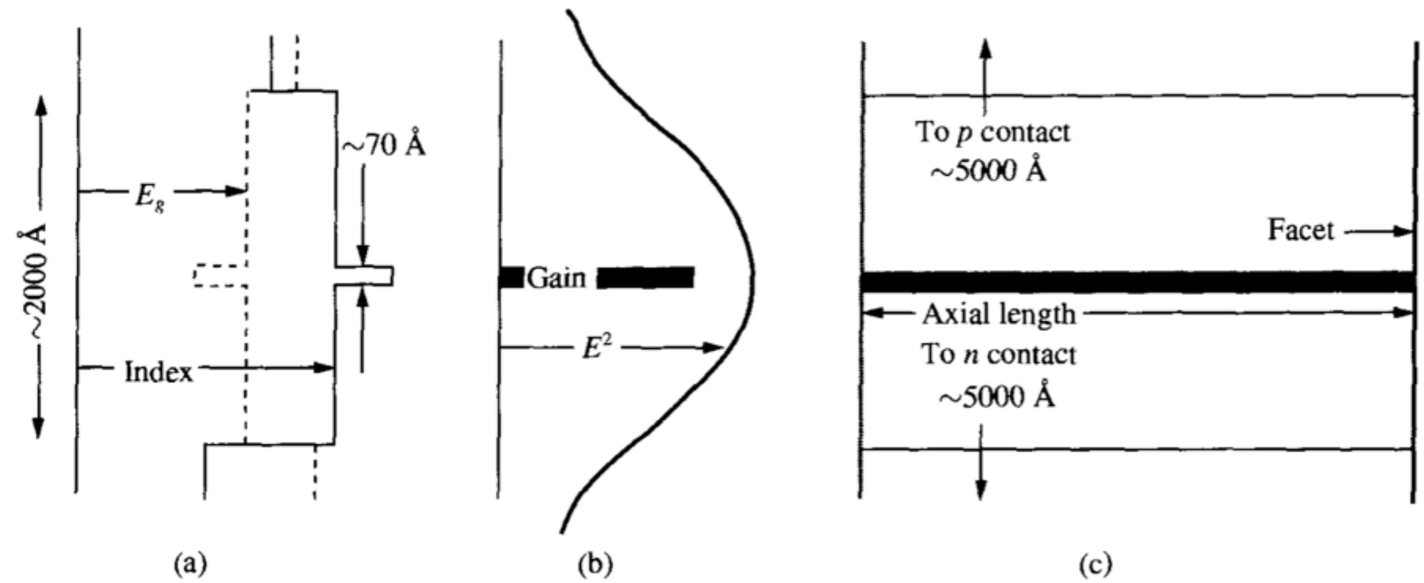
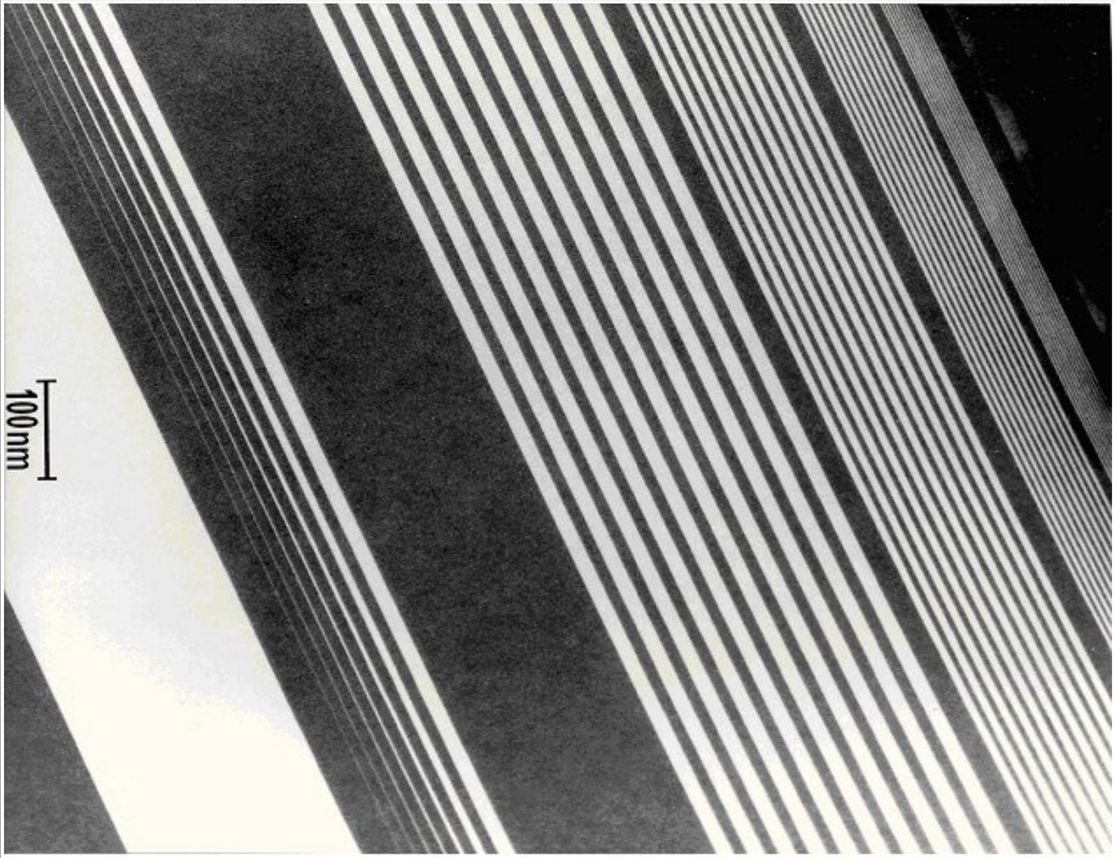
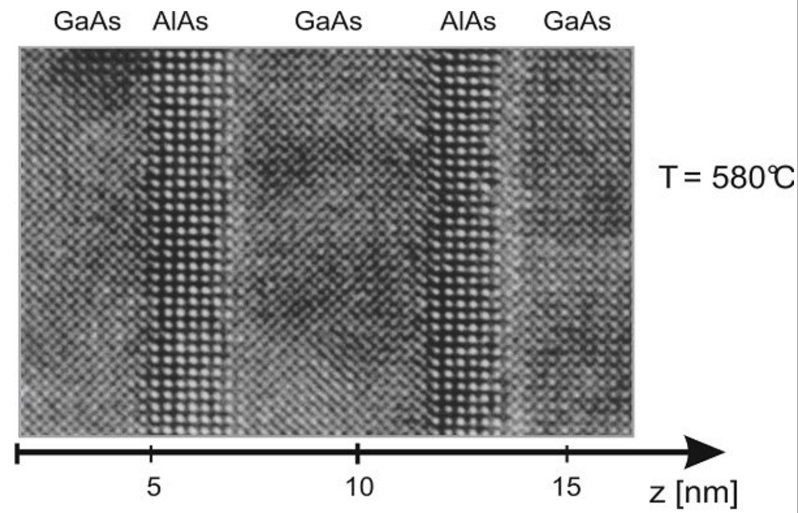


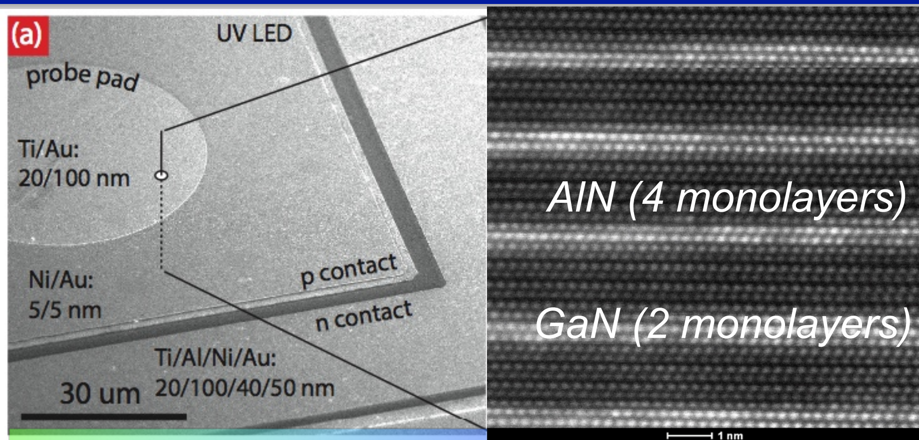
FIGURE 11.21. Typical dimension of a planar quantum well laser.

Quantum Wells: Grown by Epitaxy



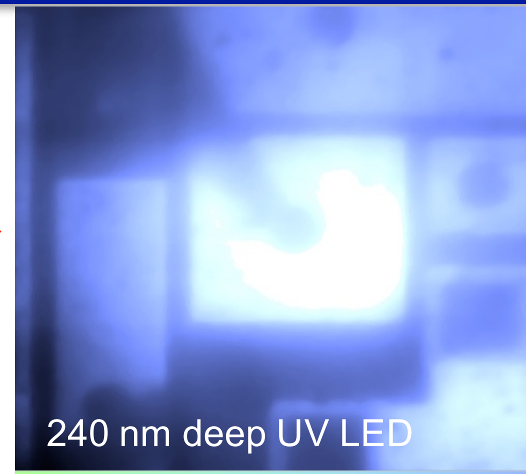


Quantum Well Light-Emitting Diodes (LEDs)

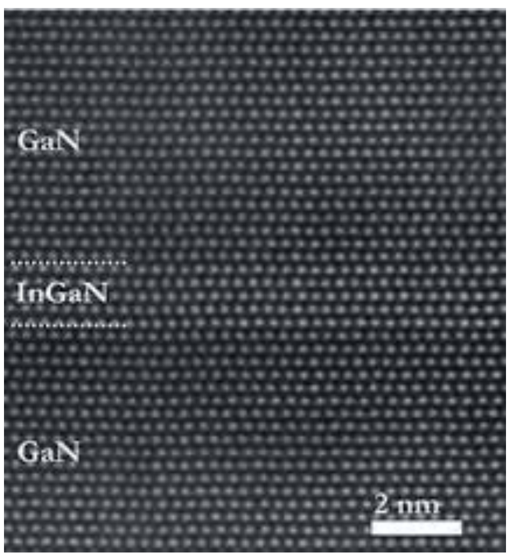


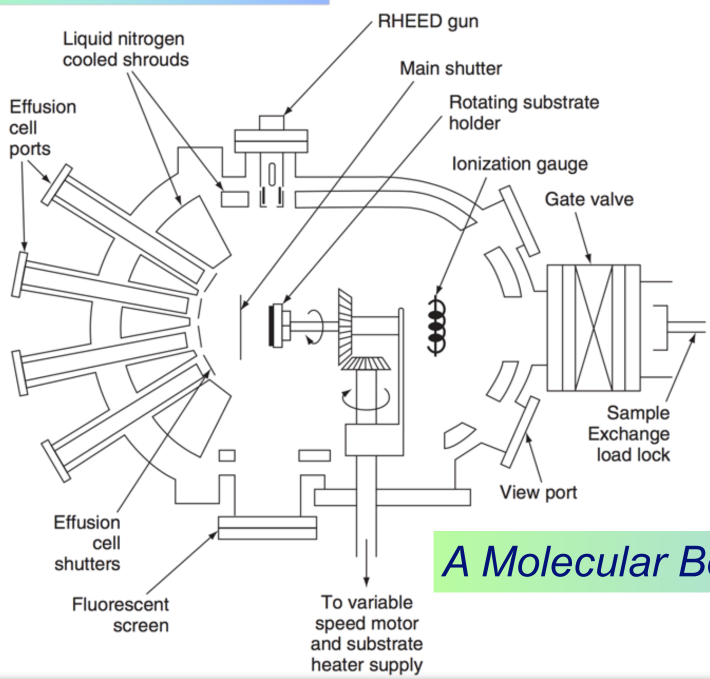
Nanofabrication

MBE growth with atomic level design and control



Photonic and electronic devices





A Molecular Beam Epitaxy System

Quantum Wells to Wires to Dots for Gain

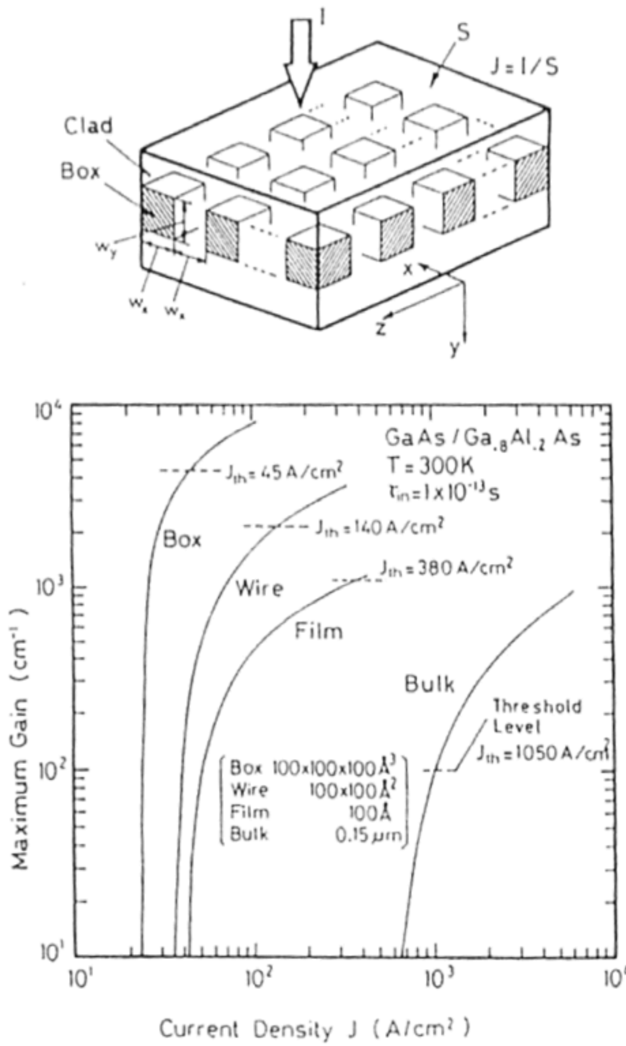
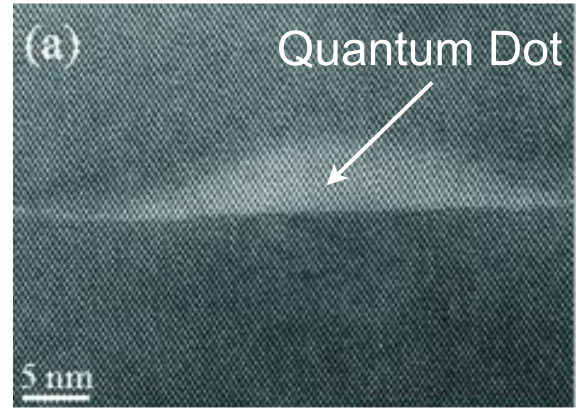
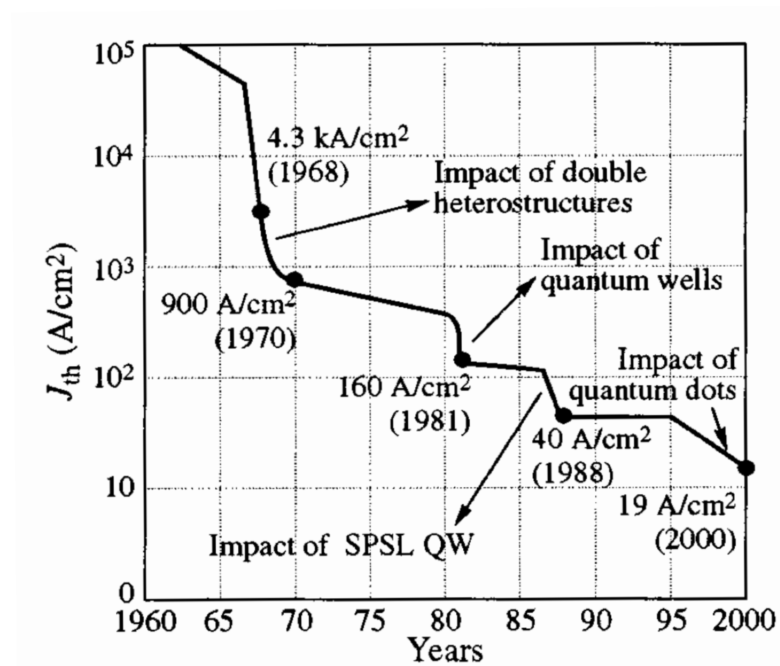


FIG. 114. Schematics of quantum box structure and gain curves 3D, 2D, 1D, 0D lasers, with optimized optical confinement in each case. (Adapted from M. Asada, Y. Miyamoto, and Y. Suematsu, IEEE *J. Quantum Electron.* QE-22, 1915, © 1986 IEEE.)





Alferov: 2000 Physics Nobel Lecture

Vertical Cavity Surface-Emitting Laser (VCSEL)

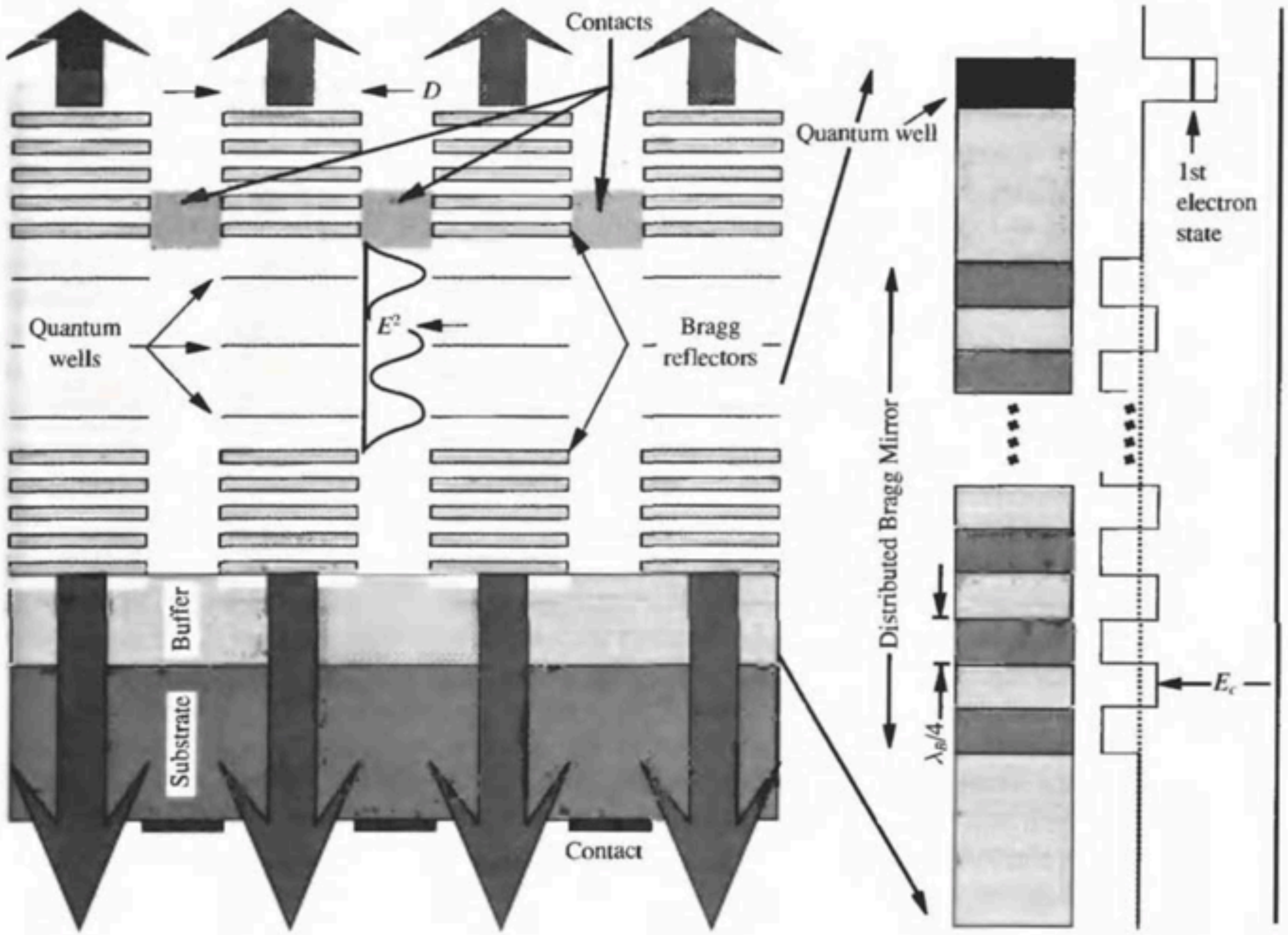
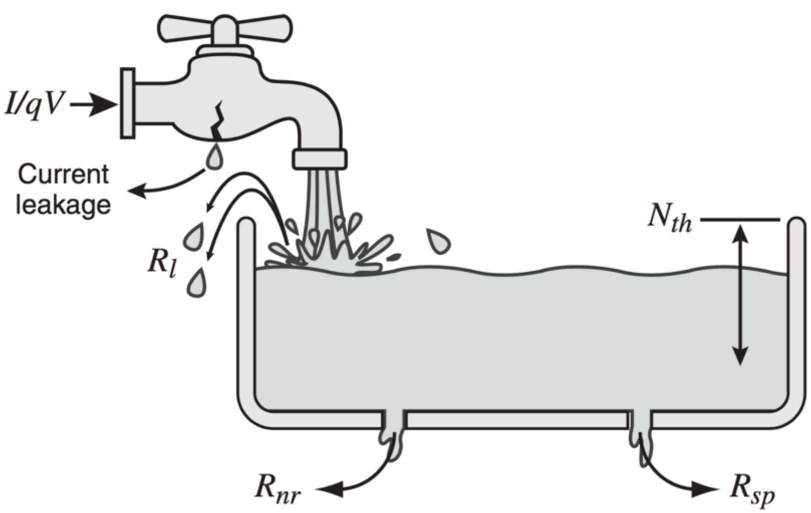
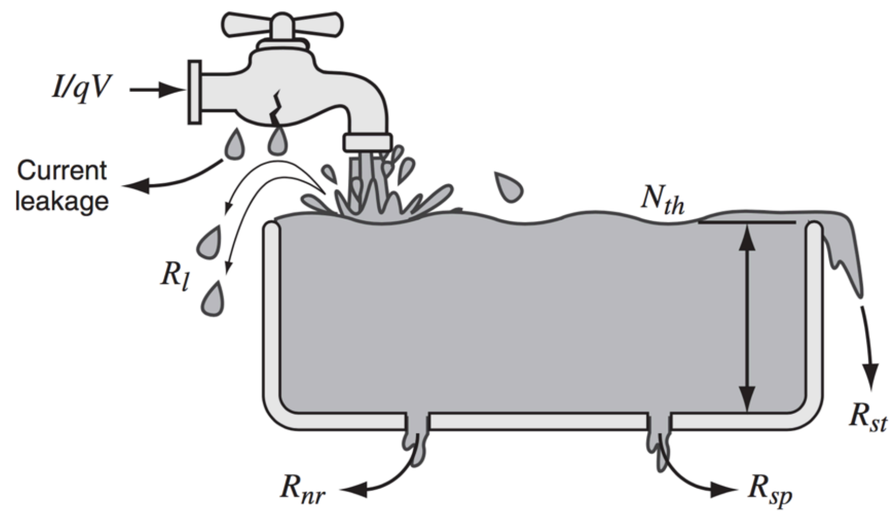
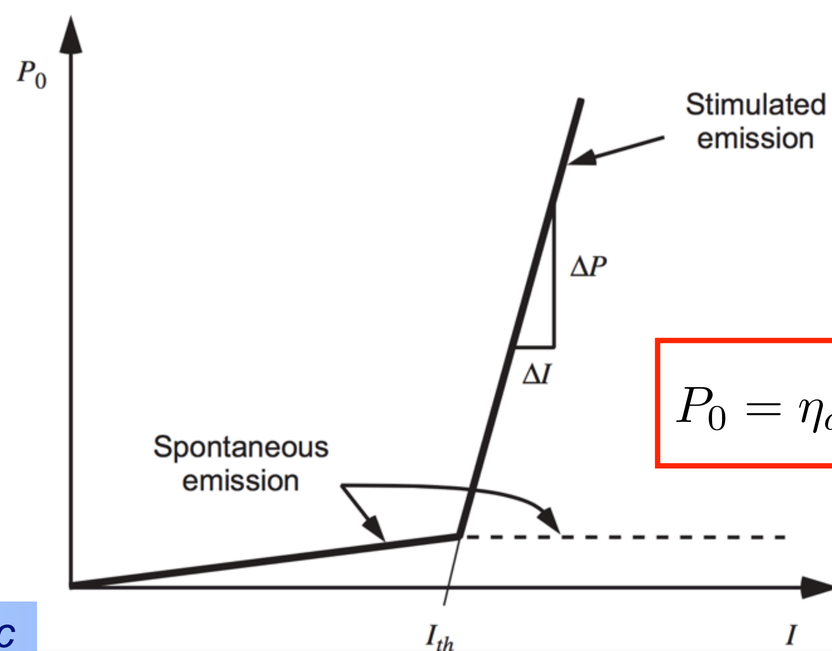


FIGURE 11.22. A possible arrangement of vertical cavity surface emitting lasers (VCSELs). A typical dimension of D might be 5 to 50 μm. The number of wells might be anywhere from 1 (see [33]) to 20 (see [24]).

Light output characteristics of a Laser



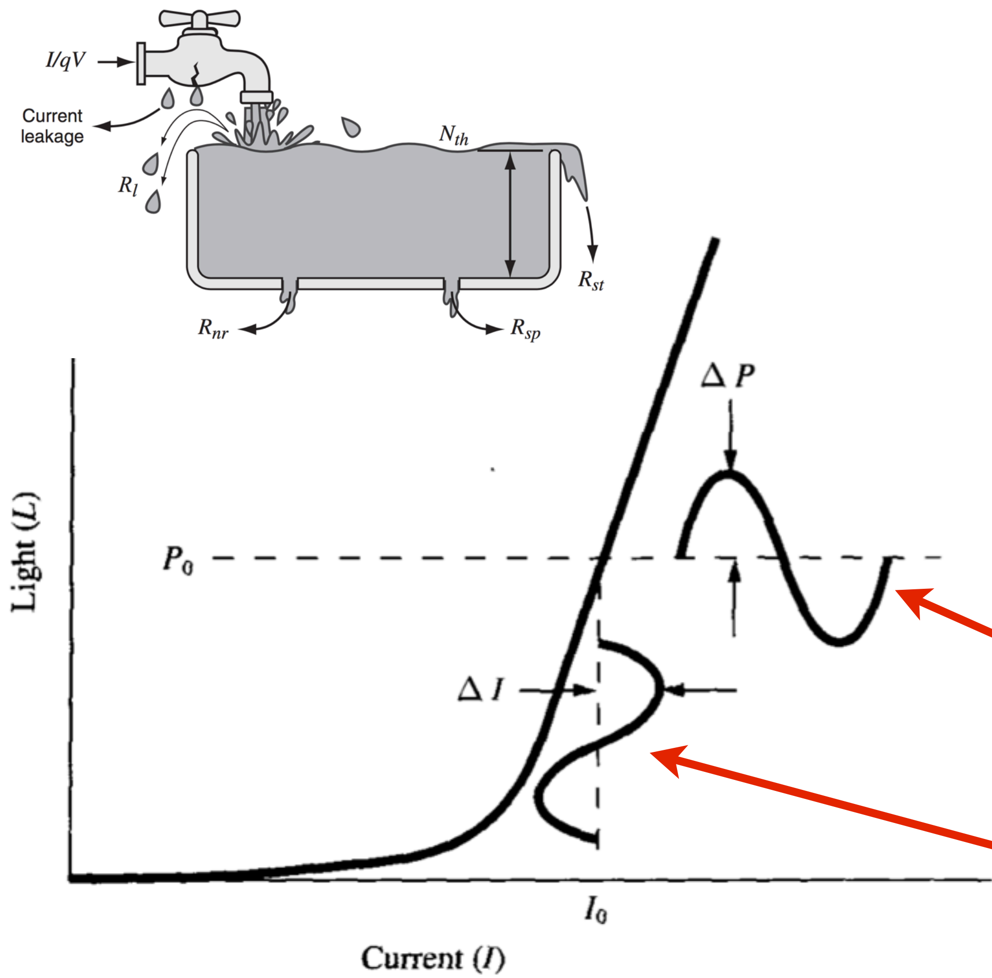




$$P_0 = \eta_d \frac{h\nu}{e}(I - I_{th}) + P_{sp}$$

From: Coldren/Corzine/Masanovic

Modulating the Laser Output: Dynamics



Rate equation for electrons (=holes)

$$\frac{dN}{dt} = \frac{J}{ed} - \frac{N}{\tau_s} - A(N - N_{tr})P$$

Rate equation for photons

$$\frac{dP}{dt} = +A(N - N_{tr})P + \beta \frac{N}{\tau_s} - \frac{P}{\tau_p}$$

Net laser P_{out} vs Injected Current Density

$$P_0 = K(J - J_{th}) + P_{spont.}$$

$$N = N_0 + \Delta N(t)$$

$$J = J_0 + \Delta J(t)$$

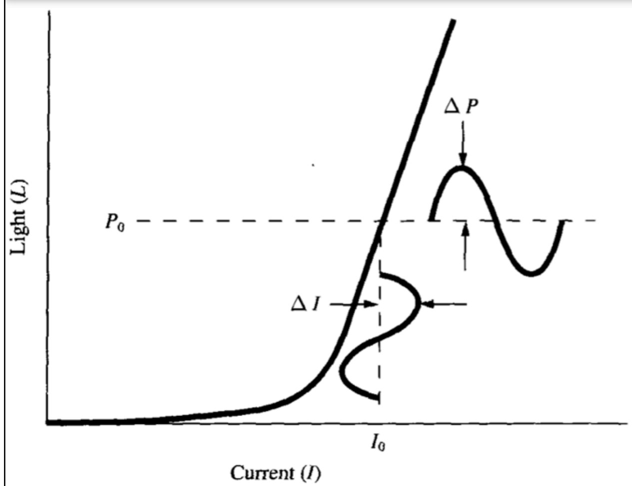
oscillatory injection current

$$P = P_0 + \Delta P(t)$$

oscillatory P_{out}

$$\frac{d\Delta N(t)}{dt} = \frac{J_0}{ed} + \frac{\Delta J(t)}{ed} - \frac{N_0 + \Delta N(t)}{\tau_s} - A[N_0 + \Delta N(t) - N_{tr}] \cdot [P_0 + \Delta P(t)]$$

Modulating the Laser Output: Dynamics



$$\frac{\Delta P_m}{\Delta J_m} = \frac{(1/ed)A\left(P_0 + (\beta/\tau_s)\right)}{\left[(1/\tau_p)\left(AP_0 + (\beta/\tau_s)\right) - \omega^2\right] + j\omega \cdot \left[(1/\tau_s) + AP_0\right]}$$

$$\boxed{\frac{P_{out}(\omega)}{I(\omega)} = \frac{\eta_d \frac{h\nu}{q}}{1 - (\frac{\omega}{\omega_R})^2 + j(\frac{\omega}{\omega_R})[\omega_R \tau_p + \frac{1}{\omega_R \tau_s}]}}$$

$$\omega_R^2 = \frac{1}{\tau_p} \cdot \left[AP_0 + \frac{\beta}{\tau_s}\right] \approx \frac{AP_0}{\tau_p}$$

"How fast can I modulate a semiconductor laser?"

The frequency response is limited by the photon lifetime and the spontaneous emission lifetime.

Today: 10s of GHz

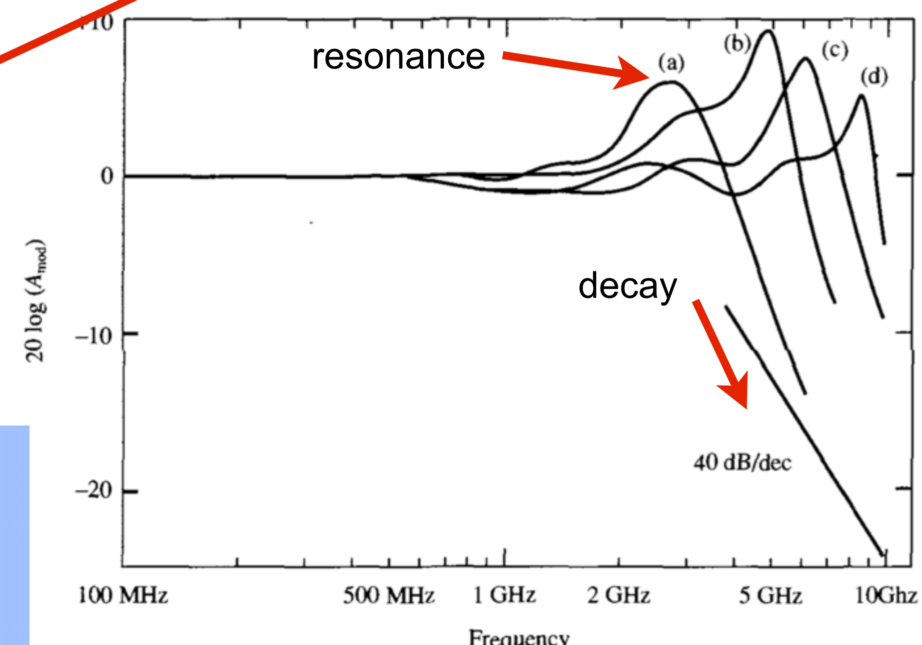
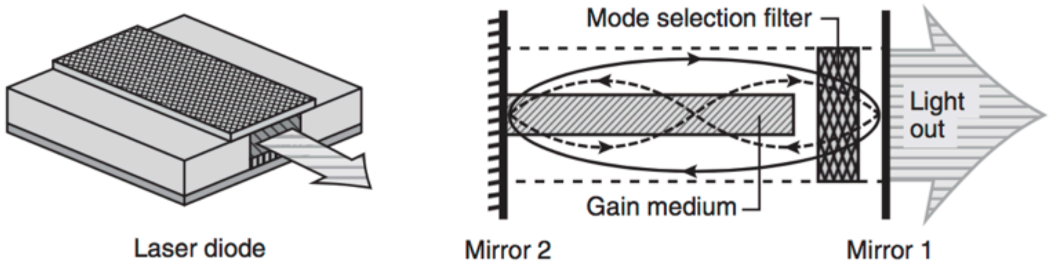
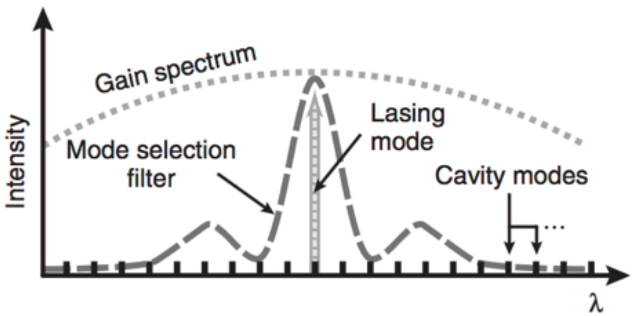


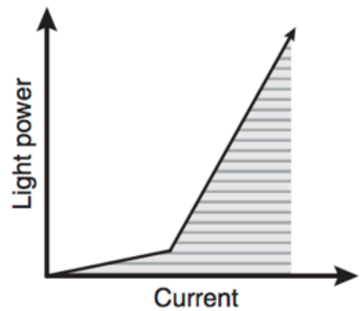
FIGURE 11.24. Modulation characteristics of a short-cavity (120 μm), buried-heterostructure laser as a function of bias levels: (a) 1 mW, (b) 2 mW, (c) 2.7 mW, and (d) 5 mW. (Data from Lau and Yariv [4].)

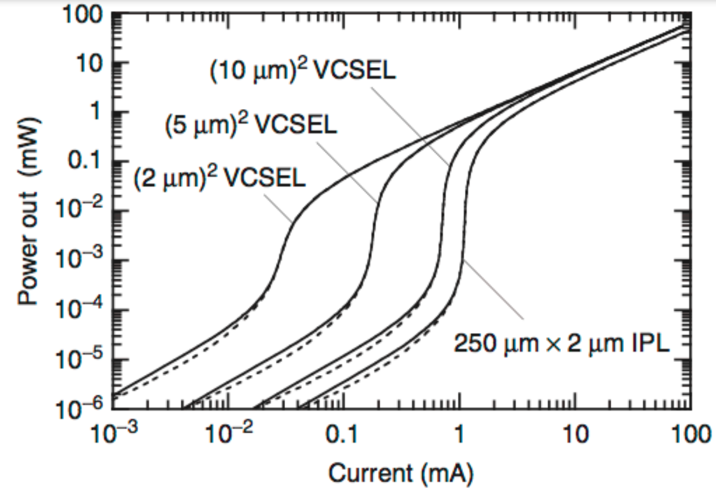
Design of Waveguides, Mirrors, and Laser Arrays

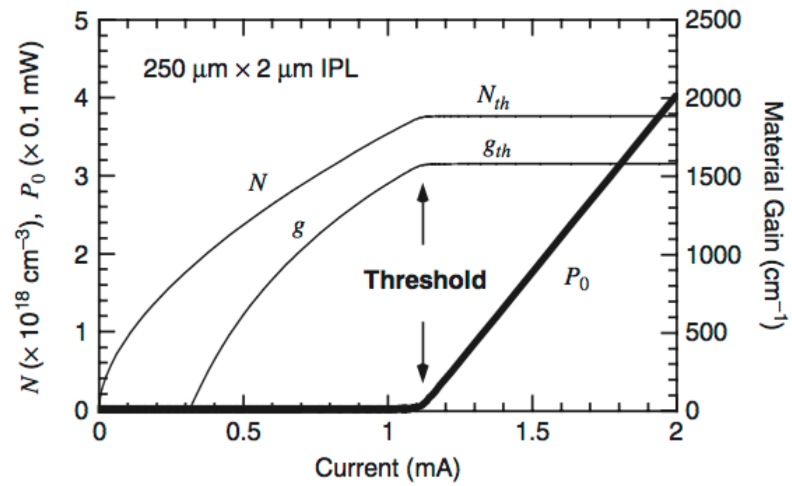


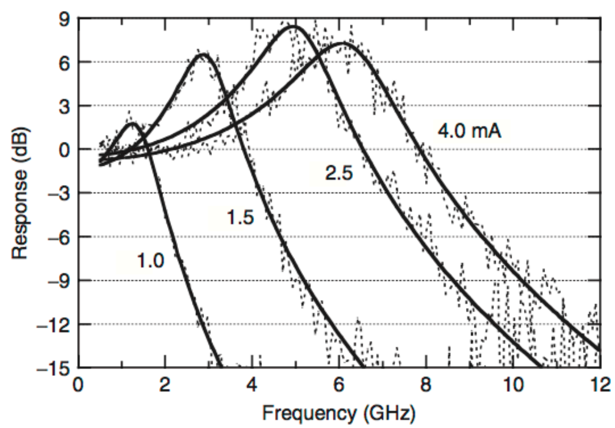
(a) (b)





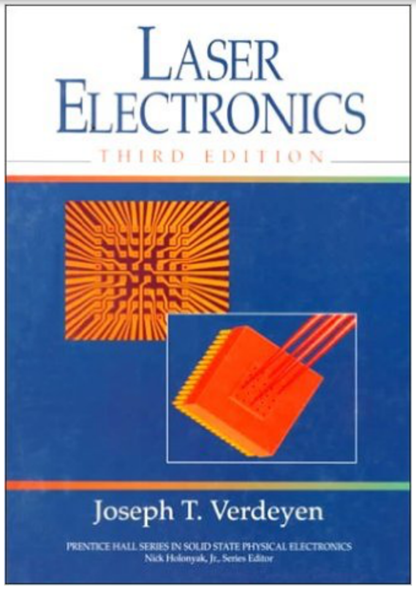






From: Coldren/Corzine/Masanovic

Course Outline



Design of Waveguides, Mirrors, and Laser Arrays

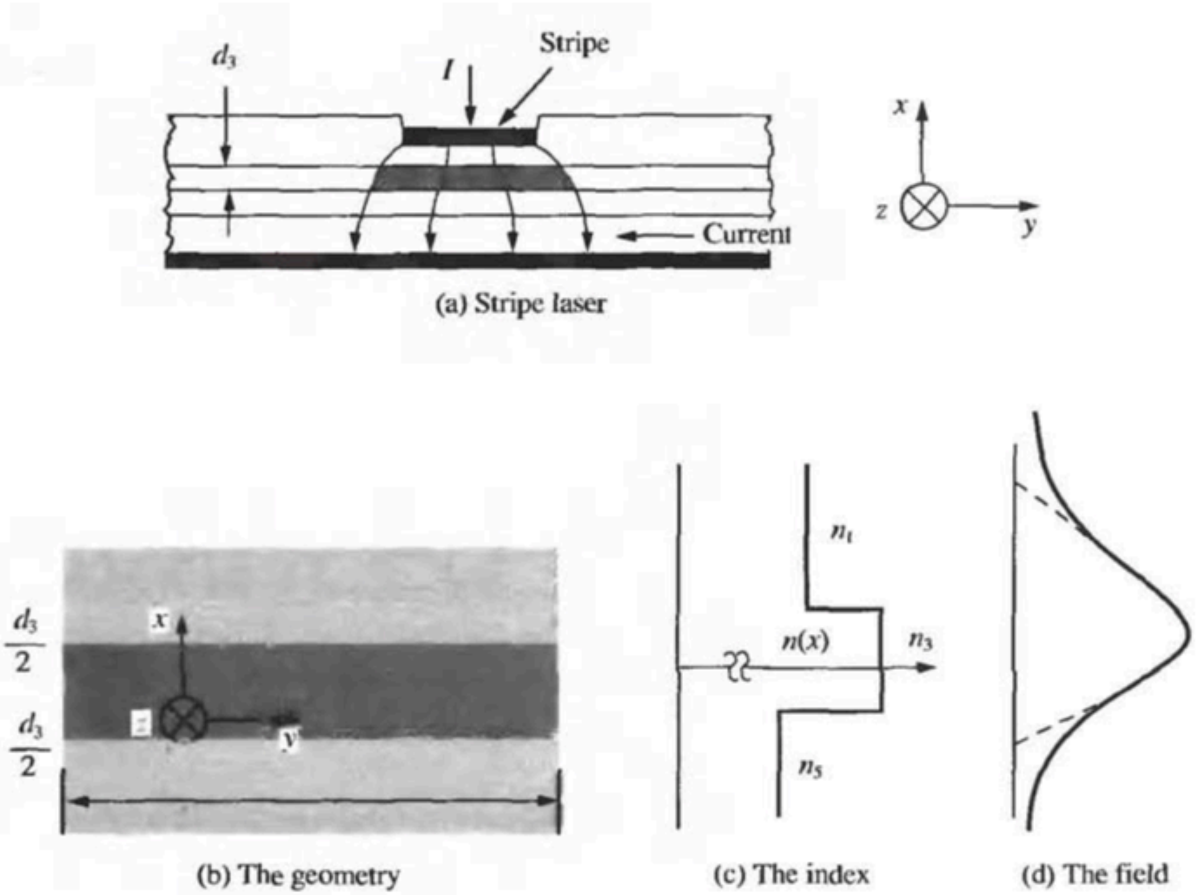


FIGURE 12.1. (a) The geometry of a stripe laser. (b) The asymmetric slab waveguide representative of many semiconductor lasers. (c) A sketch of the mode in the slab.

$$\frac{\partial^2 \Psi}{\partial x^2} + \left[\frac{\omega^2}{c^2} n^2(x, y) - \beta^2 \right] \Psi = 0$$

$$\boxed{n_{1,5} < \frac{\beta}{k_0} < n_3}$$ for guided modes

Design of Waveguides, Mirrors, and Laser Arrays

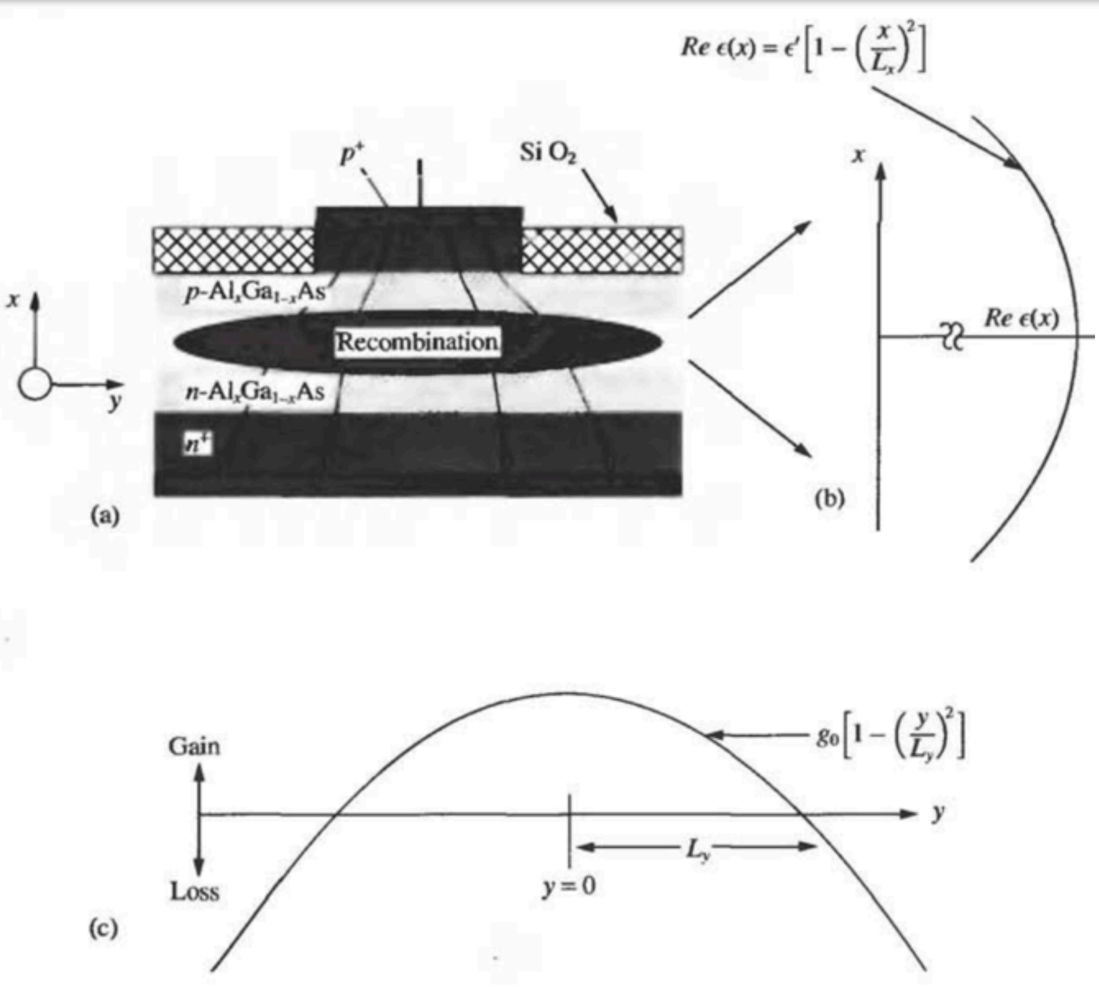


FIGURE 12.5. The assumed variation of the dielectric constant and gain in a heterostructure laser. The dielectric constant is assumed to be uniform with y. The cross-hatch region is to indicate an etched region back filled with SiO_2 to obtain a real index guided laser (see Sec. 12.4).

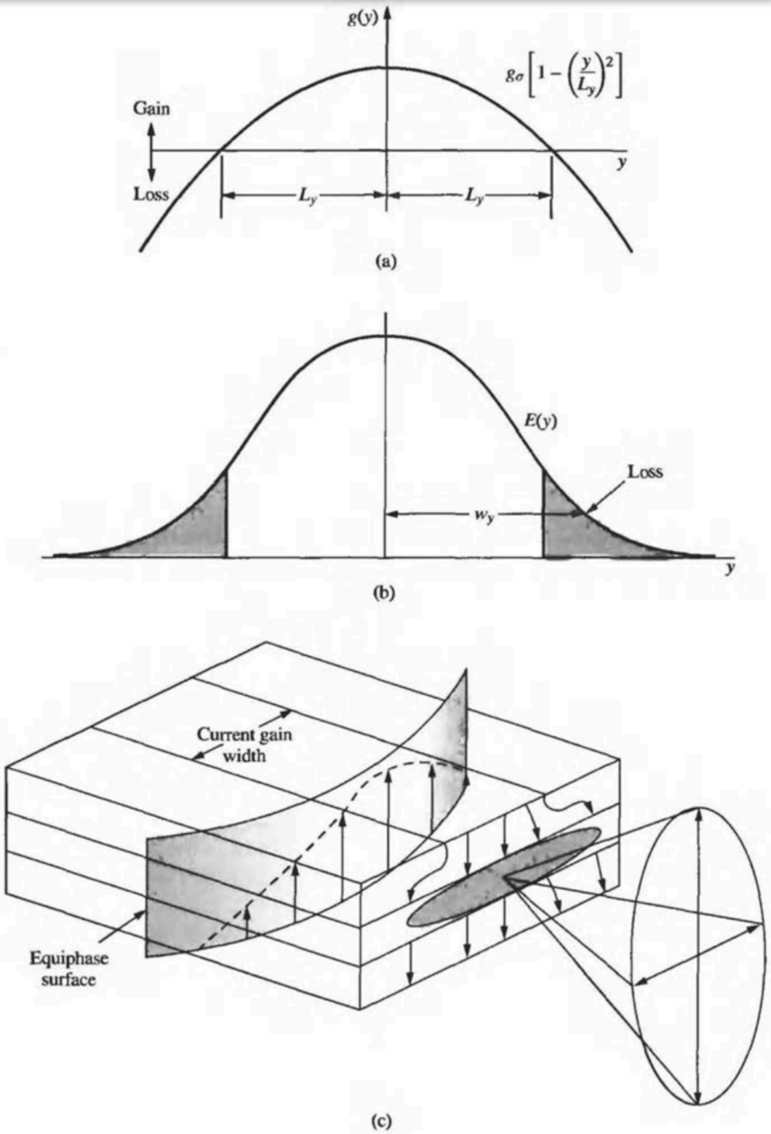
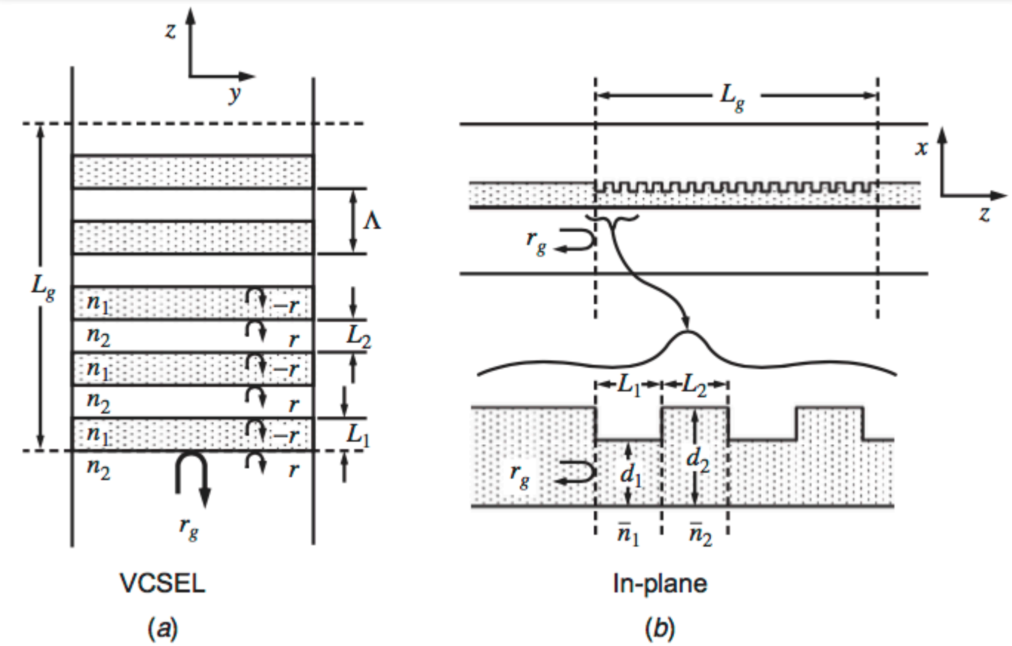
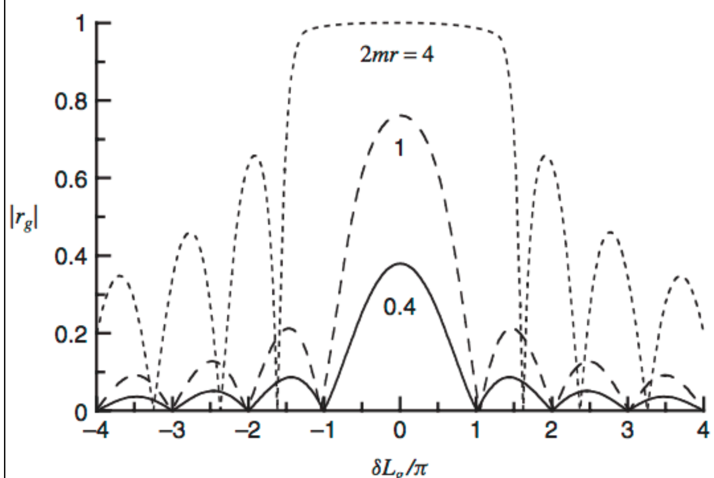
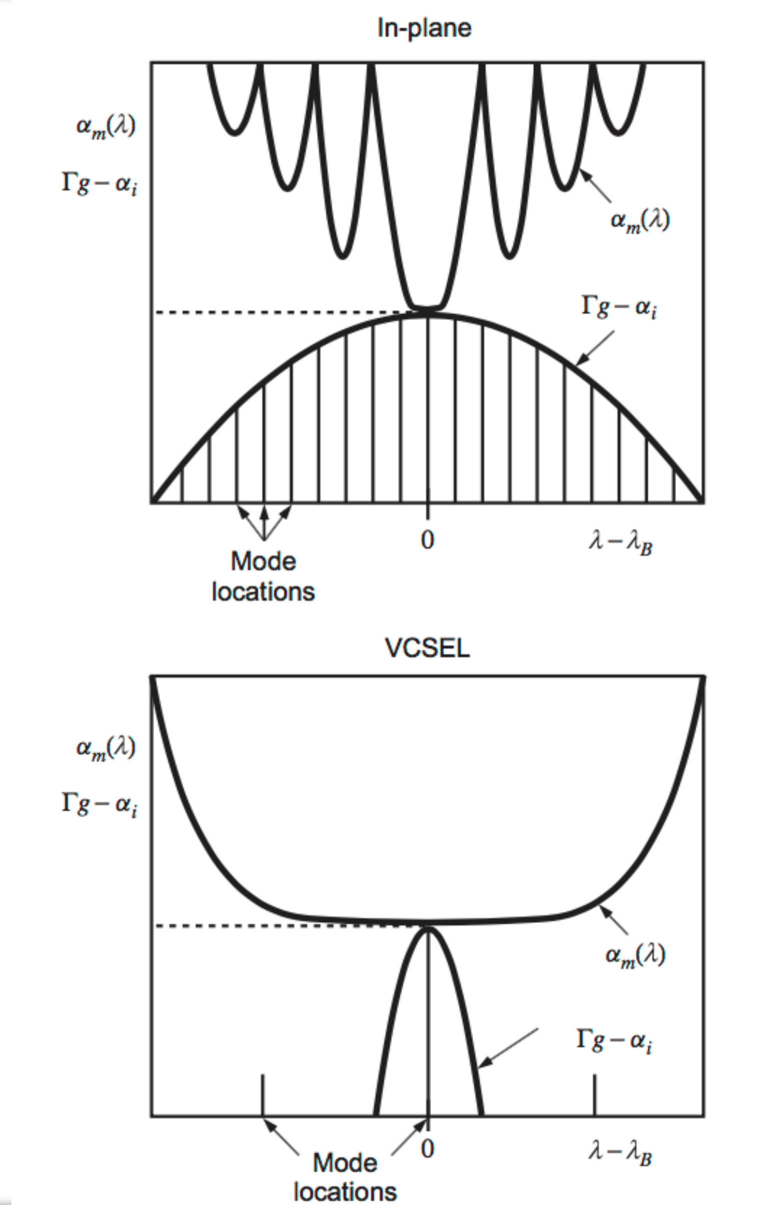


FIGURE 12.6. (a) The variation of the gain along the plane of the junction. (b) The resulting electric field of the mode. (c) The equiphase surface for a gain-guided laser.

Design of Waveguides, Mirrors, and Laser Arrays

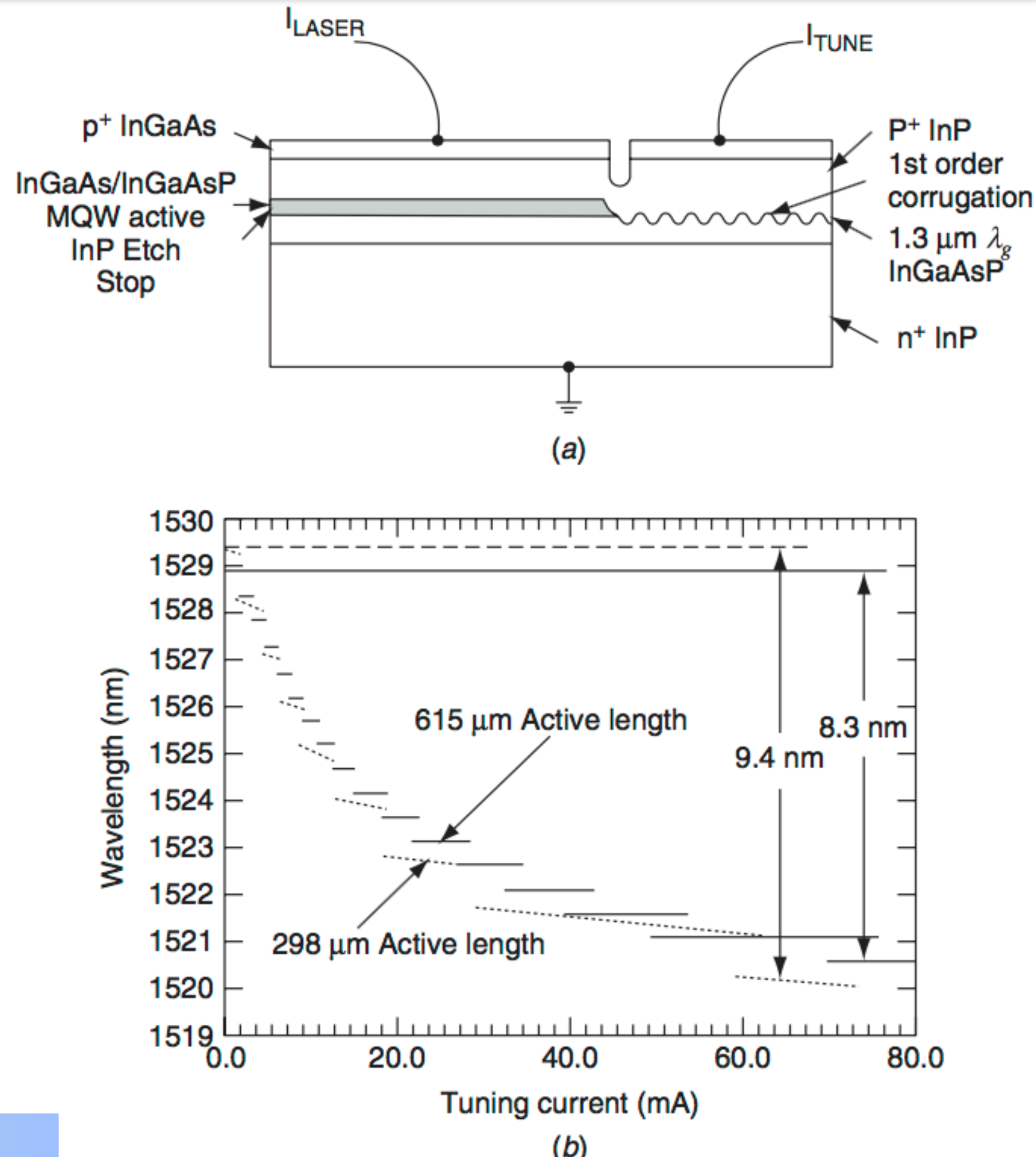






From: Coldren/Corzine/Masanovic

Design of Waveguides, Mirrors, and Laser Arrays



Tunable lasers by actively changing the mirror transmit wavelength window

From: Coldren/Corzine/Masanovic

Design of Waveguides, Mirrors, and Laser Arrays

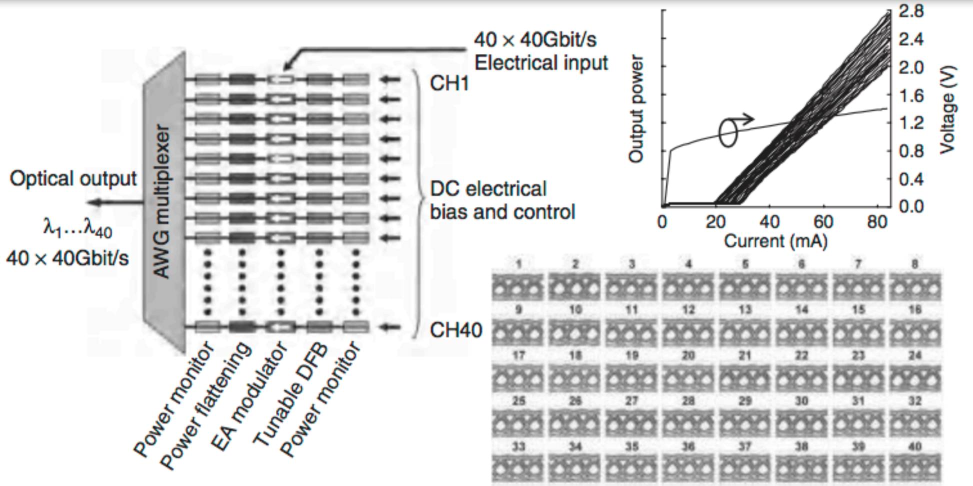


FIGURE 8.21: A 40-channel transmitter PIC, consisting of an array of 40 Gbps DFB-EAM EMLs, which are combined using an AWG multiplexer. The LIV curves and eye diagrams at 40 Gbps for each channel are shown. (© IEE 2006, [21].)

Photonic Integrated Circuits (PICs):
Multi-channel Transmitters and Receivers

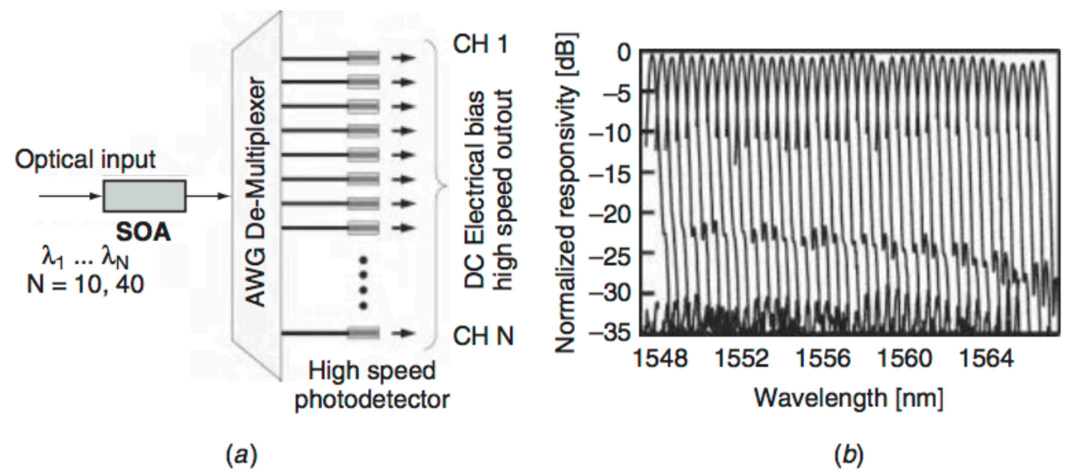


FIGURE 8.22: (a) A 40-channel receiver PIC, consisting of a preamplifier SOA, an arrayed waveguide grating demultiplexer and high-speed photodiodes operating at 40 Gbps. (b) Normalized spectral response for all 40 channels. (*Reprinted by permission from OSA, [22].*)

From: Coldren/Corzine/Masanovic

Design of Waveguides, Mirrors, and Laser Arrays

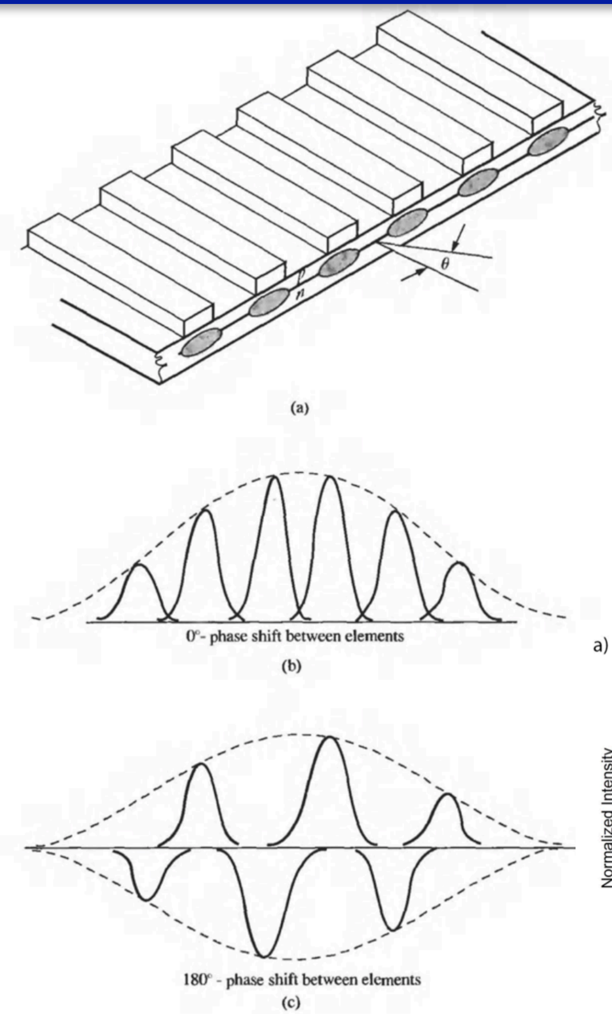
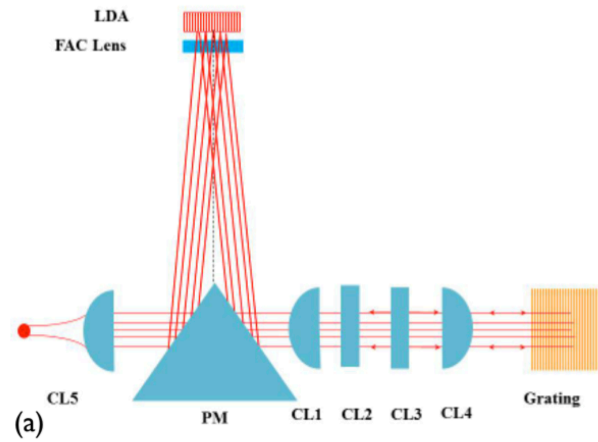
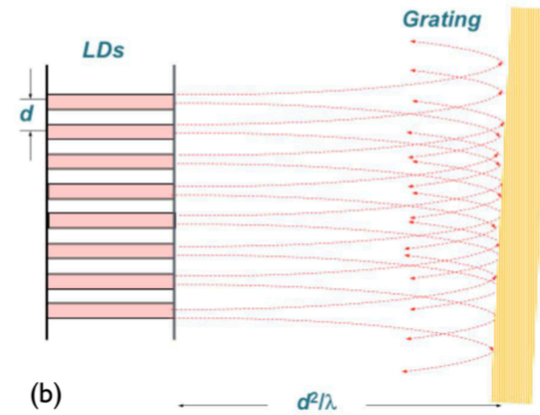
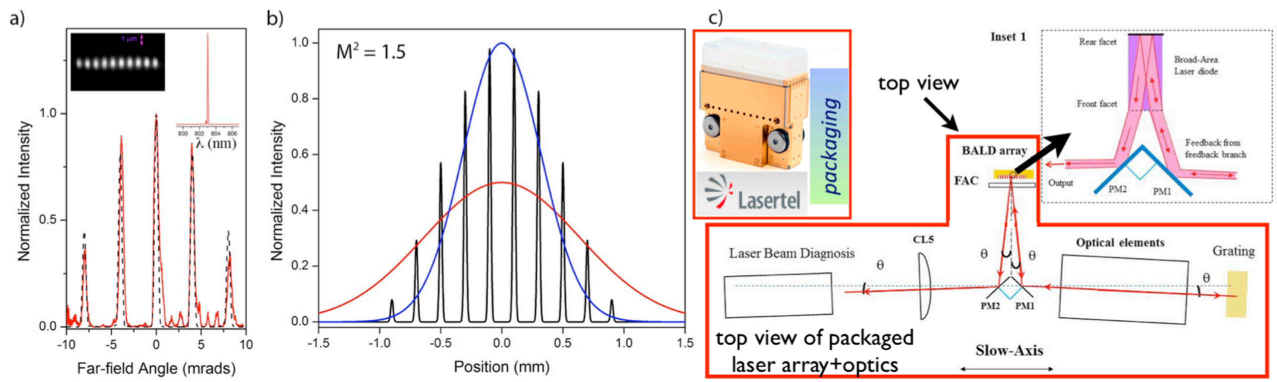


FIGURE 12.35. (a) A typical six-stripe semiconductor array. (b) An in-phase field distribution. (c) The same field elements as (b) but with a 180° phase shift between adjacent stripes.

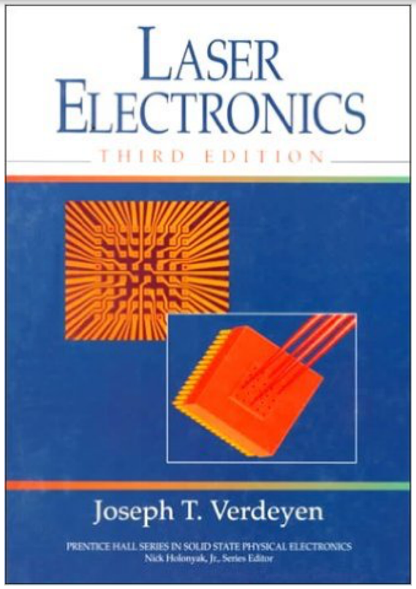






Laser Arrays for More Output Power

Course Outline



Classical theory of the Einstein A-Coefficient

$$\Delta \mathbf{x}_a(t) = \Delta \mathbf{x}_0 \sin \omega_{21} t$$

$$\frac{\partial P_a}{\partial t} = -e \cdot \Delta \dot{\mathbf{x}}_a(t) = -e \cdot \omega_{21} \cdot \Delta \mathbf{x}_0 \cos \omega_{21} t = I_0 \Delta \mathbf{x}_0 \cos \omega_{21} t$$

$$P_{\rm rad} = \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \cdot \frac{\pi}{3} \cdot \frac{(I_0 \Delta \mathbf{x}_0)^2}{\lambda_{21}^2} = \frac{1}{3}\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\omega_{21}^4}{c^3}(e\Delta \mathbf{x}_0)^2$$

$$e\Delta \mathbf{x}_0 = \mu_{21}$$

Classical Antenna Model

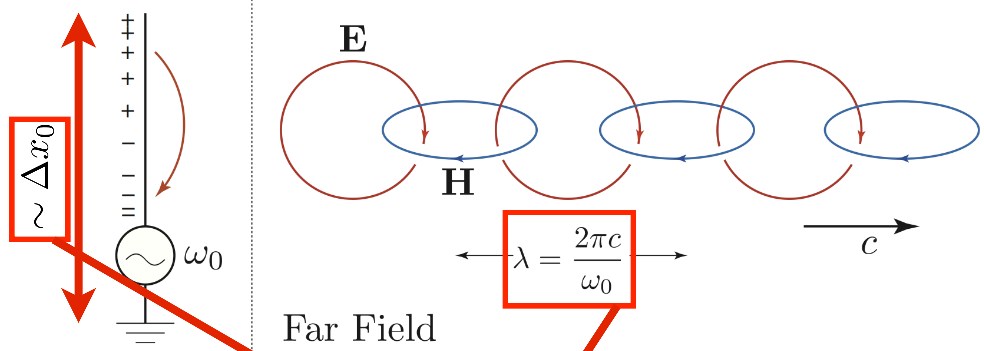


FIGURE 20.2: Antenna producing an electromagnetic wave.

$$P_{rad} = \frac{\pi}{3}\eta_0 I_0^2 \left(\frac{\Delta x_0}{\lambda}\right)^2$$

Power radiated by a classical Hertzian Dipole Antenna

$$P_{\rm rad} = \frac{16}{3}\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\pi^4 \nu_{21}^4}{c^3}|\mu_{21}|^2 N_2 \cdot (\text{volume})$$

equate

$$P_{\rm rad} = h\nu_{21} A_{21} N_2 \cdot (\text{volume})$$

Einstein Rate Equation

$$A_{21} = \frac{16}{3}\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\pi^4 \nu_{21}^3}{c^3}\frac{|\mu_{21}|^2}{h}$$

Classical Einstein A-Coefficient

compare

$$A_{21} = \frac{64\pi^4}{3\epsilon_r}\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\nu^3}{(c/n)^3}\frac{\mu_{21}^2}{h}$$

Quantum Einstein A-Coefficient = 4X(Classical Value)

Classical theory of the Einstein A-Coefficient

$$\boxed{\nabla^2 \mathbf{E} - \frac{n^2}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_a}{\partial t^2}}$$

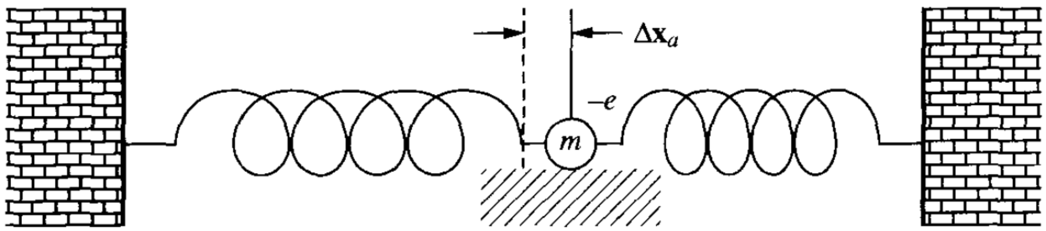


FIGURE 13.1. The mechanical model of an atom.

$$\mathbf{P}_a = \langle N_a \boldsymbol{\mu} \rangle = -\langle N_a e \Delta \mathbf{x}_a \rangle$$

$$\frac{d^2 \Delta \mathbf{x}_a}{dt^2} + \frac{1}{\tau}\frac{d\Delta \mathbf{x}_a}{dt} + \omega_{21}^2 \Delta \mathbf{x}_a = -\frac{e}{m}\mathbf{E}$$

$$\Delta \mathbf{x}_a(t) = \Delta \mathbf{x}_0 \exp(j\omega t) \qquad \mathbf{E} = \mathbf{E}_0 \exp(j\omega t)$$

$$\boxed{\Delta \mathbf{x}_0 = -\frac{e}{m}\frac{\mathbf{E}_0}{(\omega_{21}^2 - \omega^2) + j\omega/\tau}}$$

Dipole length in response to an oscillating electric field from a light beam

Atomic polarization in response to an oscillating electric field from a light beam

$$\boxed{\mathbf{P}_a(t) = \epsilon_0 \left\{ \frac{N_a e^2}{m\epsilon_0}\left[\frac{\omega_{21}^2 - \omega^2}{(\omega_{21}^2 - \omega^2)^2 + (\omega/\tau)^2} - j\frac{(\omega/\tau)}{(\omega_{21}^2 - \omega^2)^2 + (\omega/\tau)^2} \right] \right\} \mathbf{E}_0 \exp[j\omega t]}$$

Classical theory of the Einstein A-Coefficient

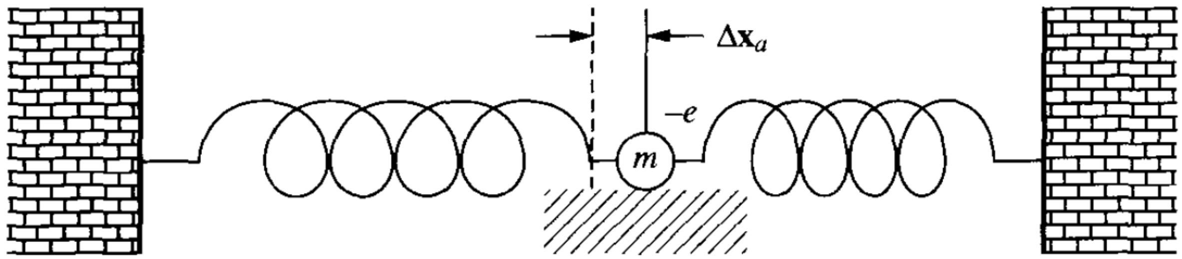


FIGURE 13.1. The mechanical model of an atom.

$$\mathbf{P}_a(t) = \epsilon_0 \left\{ \frac{N_a e^2}{m\epsilon_0} \left[\frac{\omega_{21}^2 - \omega^2}{(\omega_{21}^2 - \omega^2)^2 + (\omega/\tau)^2} - j\, \frac{(\omega/\tau)}{(\omega_{21}^2 - \omega^2)^2 + (\omega/\tau)^2} \right] \right\} \mathbf{E}_0 \exp[j\omega t]$$

$$\boxed{\mathbf{P}_a(t) \triangleq \epsilon_0(\chi_a' - j\chi_a'') \cdot \mathbf{E}_0 \exp[j\omega t]}$$

$$\mathbf{E}(z,t) = \mathbf{E}_0 \exp[(\gamma/2 - jk)z] \exp[+j\omega t]$$

$$\boxed{\nabla^2 \mathbf{E} - \frac{n^2}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_a}{\partial t^2}}$$

$$k^2 - (\gamma/2)^2 = \frac{\omega^2}{c^2}(n^2 + \chi_a')$$

$$-k\gamma = \frac{\omega^2}{c^2}\chi_a''$$

Real part of the wavevector

$$\boxed{k = \frac{\omega n}{c}\left(1 + \frac{\boxed{\chi_a'}}{2n^2}\right)}$$

$$\boxed{\gamma = -\frac{\omega n}{c}\left(\frac{\boxed{\chi_a''}}{n^2}\right)}$$

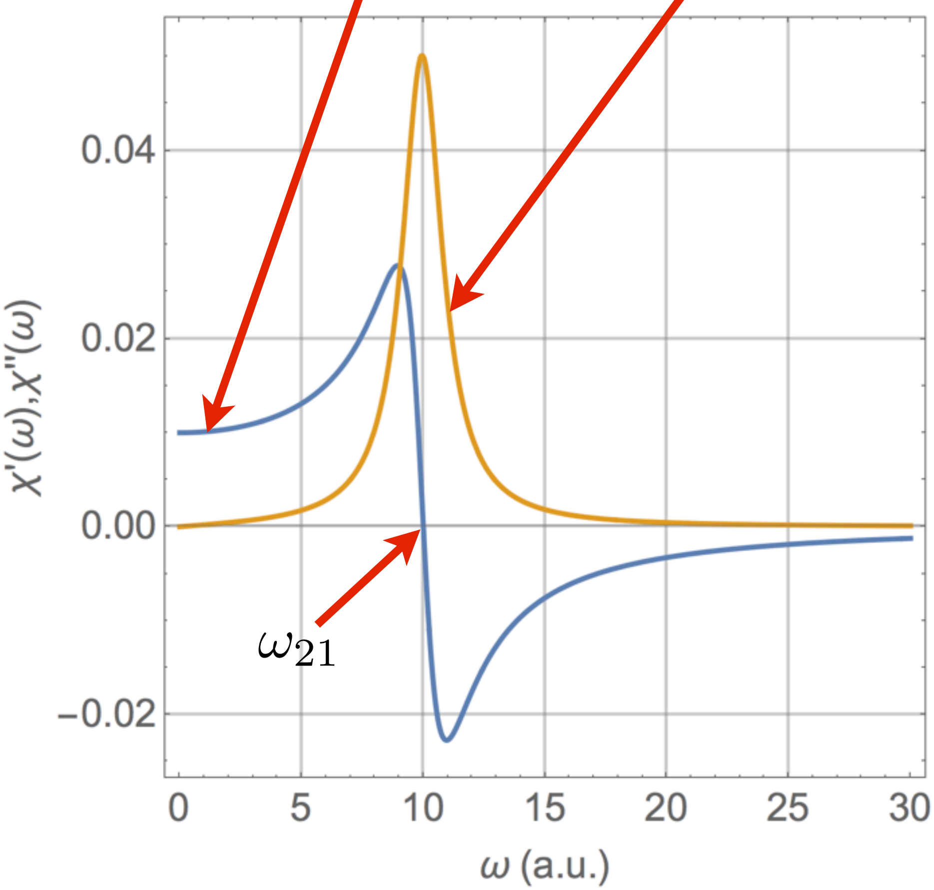
Imaginary part of the wavevector

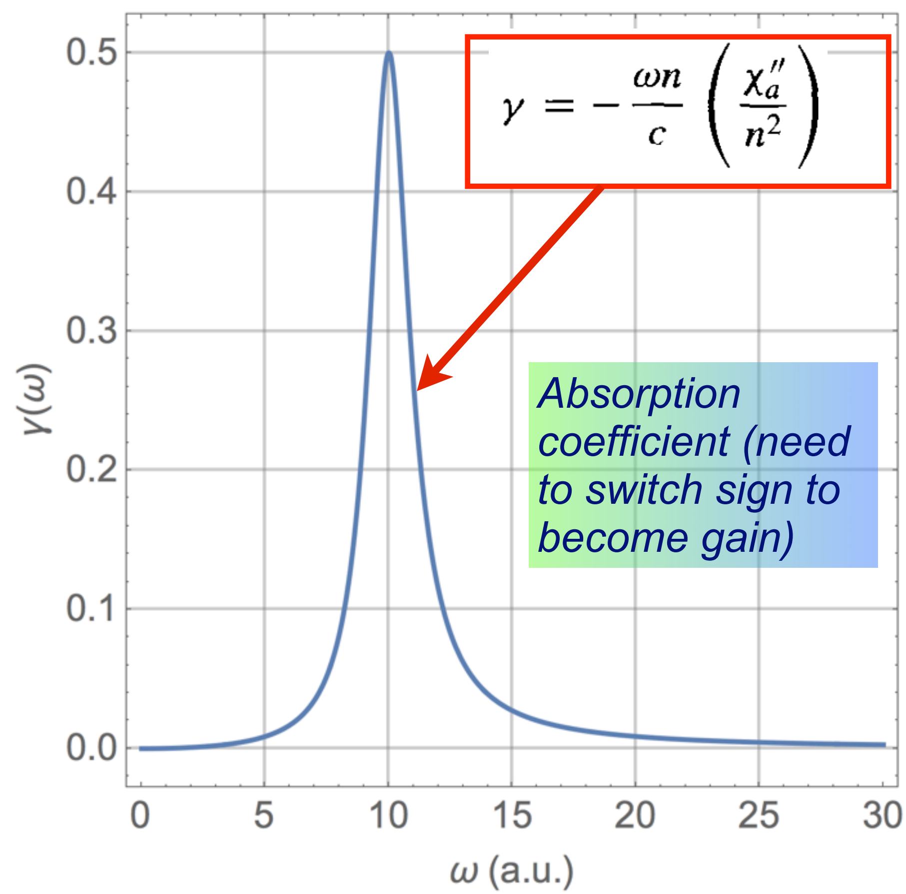
Classical theory of the Einstein A-Coefficient

$$\mathbf{P}_a(t) = \epsilon_0 \left\{ \frac{N_a e^2}{m\epsilon_0} \left[\boxed{\frac{\omega_{21}^2 - \omega^2}{(\omega_{21}^2 - \omega^2)^2 + (\omega/\tau)^2}} - j \boxed{\frac{(\omega/\tau)}{(\omega_{21}^2 - \omega^2)^2 + (\omega/\tau)^2}} \right] \right\} \mathbf{E}_0 \exp[j\omega t]$$

Real part of the susceptibility

Imaginary part of the susceptibility





$$\boxed{\gamma = -\frac{\omega n}{c}\left(\frac{\chi_a''}{n^2}\right)}$$

Absorption coefficient (need to switch sign to become gain)

Classical theory of the Einstein A-Coefficient

$$\text{integrated spectral absorption} = \int_0^\infty [-\gamma(\nu)]\, d\nu$$

$$\gamma(\omega) = -\left(\frac{\omega n}{c}\right) \cdot \left(\frac{\chi_a''(\omega)}{n^2}\right)$$

$$\frac{\chi_a''(\omega)}{n^2} = \frac{1}{n^2} \cdot \frac{Ne^2}{m\epsilon_0} \frac{(\omega/\tau)}{(\omega_{21}^2 - \omega^2)^2 + (\omega/\tau)^2}$$

$$-\gamma(\nu) = \frac{\pi}{n} \cdot \frac{Ne^2}{mc} \cdot \left(\frac{1}{4\pi\epsilon_0}\right) \cdot \left\{\frac{1/2\pi\tau}{2\pi[(\nu_{21} - \nu)^2 + (1/4\pi\tau)^2]}\right\}$$

$$= \frac{\pi}{n} \cdot \frac{Ne^2}{mc} \cdot \left(\frac{1}{4\pi\epsilon_0}\right) \cdot \left\{\frac{\Delta\nu}{2\pi[(\nu_{21} - \nu)^2 + (\Delta\nu/2)^2]}\right\}$$

$$\text{integrated spectral absorption} = \int_0^\infty -\gamma(\nu)\, d\nu = \frac{\pi}{n} \frac{Ne^2}{mc} \cdot \left(\frac{1}{4\pi\epsilon_0}\right)$$

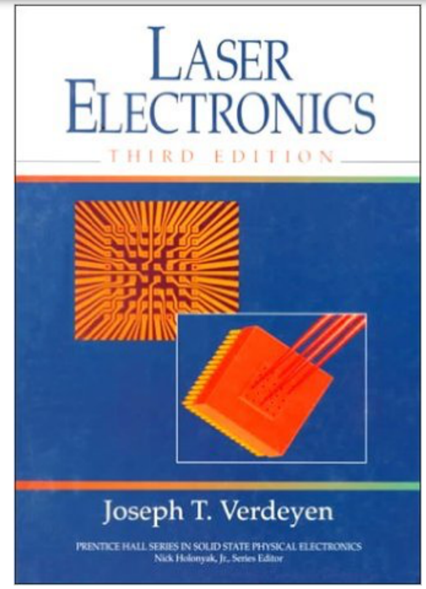
$$\text{integrated spectral absorption} = \frac{\pi}{n} \frac{(Nf_{12})e^2}{mc} \cdot \left(\frac{1}{4\pi\epsilon_0}\right) = A_{21} \frac{\lambda_0^2}{8\pi n^2} \frac{g_2}{g_1} N_1$$

$$N \to N_1 f_{12} \to f_{21}\left[\frac{g_2}{g_1} N_1 - N_2\right]$$

$$\boxed{A_{21} = 8\pi^2 n \cdot \frac{\nu_{21}^2}{c^3} \cdot \left(\frac{e^2}{m}\right) \cdot \frac{g_1}{g_2} \cdot f_{12} \left(\frac{1}{4\pi\epsilon_0}\right)}$$

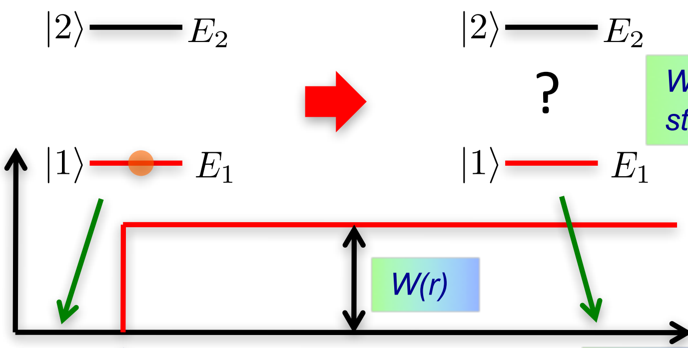
Another approach to obtain the classical Einstein A-coefficient without invoking Antenna theory

Course Outline

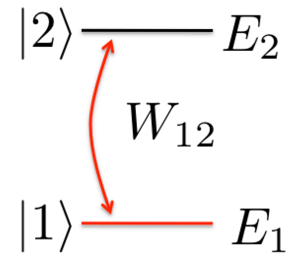


Quantum theory of the Einstein A & B Coefficients

Simplest case: A 2-level system with step perturbation



What is the occupation of states at time t?



Perturbation: W(r,t)

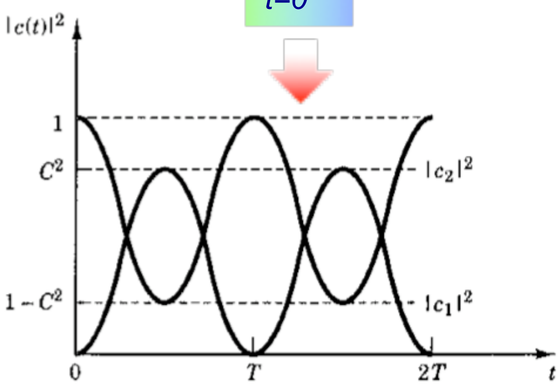
time

t=0

$$\hbar\delta\omega = \hbar\omega_{21} = (E_2 - E_1) + (W_{22} - W_{11})$$

$$i\hbar\frac{dc_2(t)}{dt} = c_1(t)e^{i\delta\omega t}W_{21}$$

$$i\hbar\frac{dc_1(t)}{dt} = c_2(t)e^{-i\delta\omega t}W_{12}$$



setting $c_1(t) = b_1 e^{i(\omega - \frac{1}{2}\delta\omega)t}$ and $c_2(t) = b_2 e^{i(\omega + \frac{1}{2}\delta\omega)t}$ to get

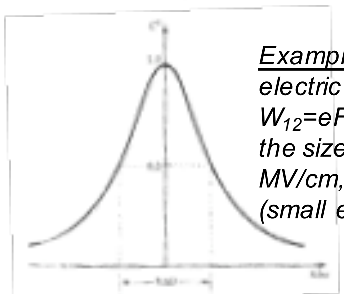
$$b_1\hbar(\omega - \frac{1}{2}\delta\omega) + b_2 W_{21} = 0$$

$$b_1 W_{12} + b_2\hbar(\omega + \frac{1}{2}\delta\omega) = 0$$

$$|c_1(t)|^2 = 1 - C^2\sin^2\Omega t$$

$$|c_2(t)|^2 = C^2\sin^2\Omega t$$

$$C^2 = \frac{|W_{12}|^2}{(\frac{1}{2}\hbar\delta\omega)^2 + |W_{12}|^2}$$



Example: Electrons in an atom with electric field perturbation. W=eFx, W_{12}=eF<1|x|2>=eFx_{12}~eFr, r is ~ the size of the atom. For F~1 MV/cm, r~0.1 nm, W_{12}~10 meV (small energy, sharp resonance).

Quantum theory of the Einstein A & B Coefficients

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V + v\right)\Psi = j\hbar\frac{\partial\Psi}{\partial t}$$

$$v = e\phi \qquad \text{and} \qquad \phi = -\int_0^{\mathbf{x}} \mathbf{E}\cdot d\mathbf{l}$$

$$\dot{c}_2(t) = j\Omega\, c_1(t)\exp[+j(\omega_{21} - \omega)t$$

$$\dot{c}_1(t) = j\Omega\, c_2(t)\exp[-j(\omega_{21} - \omega)t]$$

$$\boxed{\Omega \triangleq \frac{\mu_{21x}E_{0x}}{2\hbar} = \text{Rabi "flopping frequency"}}$$

$$\frac{\hbar\Omega}{e} = \frac{\mu_{21x}E_{0x}}{2e} = \frac{\langle x\rangle\, E_{0x}}{2}\ \text{(volts)}$$

A 2-level atomic system interacting with an oscillating electric field

Quantum theory of the Einstein A & B Coefficients

$$|c_2(t)|^2 = |c_1(0)|^2\Omega^2 \left\{\frac{\sin \Delta\omega t/2}{\Delta\omega t/2}\right\}^2 t^2$$

$$\frac{dN_2}{dt} \sim B_{12} \cdot N_1 \cdot g(\nu) \cdot [\rho_\nu \sim E_{0x}^2]$$

$$N_2 \sim N_1 \cdot B_{12} \cdot g(\nu) \cdot [\rho_\nu \sim E_{0x}^2] \cdot t$$

$$\int_0^\infty g(\nu'_{21})\, d\nu'_{21} = 1 \qquad N_1 = |c_1(0)|^2$$

$$\left\langle |c_2(t)|^2 \right\rangle = |c_1(0)|^2\Omega^2 \int_0^\infty \left[\frac{\sin \pi(\nu'_{21}-\nu)t}{\pi(\nu'_{21}-\nu)t}\right]^2 t^2 g(\nu'_{21})\, d\nu'_{21}$$

$$\left\langle |c_2(t)|^2 \right\rangle = |c_1(0)|^2\Omega^2 g(\nu) t \left[\int_0^\infty \frac{1}{\pi}\left(\frac{\sin x}{x}\right)^2 dx\right]$$

$$\boxed{\left\langle |c_2(t)|^2 \right\rangle = |c_1(0)|^2 \cdot \Omega^2 \cdot g(\nu) \cdot t}$$

$$\cancel{|c_1(0)|^2} \cdot B_{12} \cdot \cancel{g(\nu)} \cdot \cancel{t} \cdot \rho_\nu = \frac{g_2}{g_1} \cancel{|c_1(0)|^2} \cdot \Omega^2 \cdot \cancel{g(\nu)} \cdot \cancel{t}$$

$$B_{12} = \frac{g_2}{g_1}\frac{\Omega^2}{\rho_\nu} \qquad \rho_\nu = \frac{1}{2}\epsilon_r \cdot \epsilon_0 E_{0x}^2$$

$$\boxed{B_{12} = \frac{2}{3}\frac{\pi^2}{\epsilon_r \cdot \epsilon_0}\frac{g_2}{g_1}\frac{\mu_{21}^2}{h^2} = \frac{g_2}{g_1}B_{21}}$$

Quantum Einstein B-coefficient

$$\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$$

$$\boxed{A_{21} = \frac{64\pi^4}{3\epsilon_r}\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\nu^3}{(c/n)^3}\frac{\mu_{21}^2}{h}}$$

Quantum Einstein A-coefficient

Field-Quantization: Quantum Electrodynamics

Summary

Quantization of EM Field, Spontaneous & Stimulated Emission

$\mathcal{H}_\omega$: $\boxed{E_\omega = \frac{\Omega}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 \Omega \omega^2 A^2}$ $\qquad$ $\boxed{E_{osc} = \frac{p^2}{2M} + \frac{M}{2}\omega^2 x^2}$

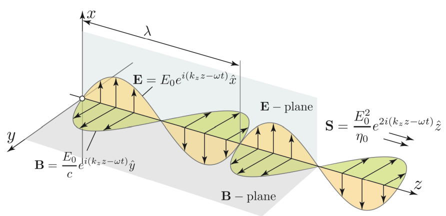
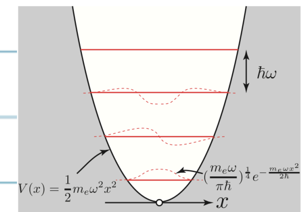


FIGURE 20.3: Electromagnetic wave.

$$\frac{dE}{dt} = \omega^2 A \qquad \frac{dp}{dt} = -M\omega^2 x$$

$$\frac{dA}{dt} = -E \qquad \frac{dx}{dt} = \frac{1}{M} p$$



$$A \equiv x$$
$$-\epsilon_0 \Omega E \equiv p$$
$$\epsilon_0 \Omega \equiv M$$

(Electric field amplitude of one photon!) $\boxed{E_0 = \sqrt{\frac{\hbar\omega}{2\epsilon_0 \Omega}}}$

$$[\hat{A}, -\epsilon_0 \Omega \hat{E}] = i\hbar$$
$$\hat{A} = \sqrt{\frac{\hbar}{2\epsilon_0 \Omega \omega}}(a^\dagger + a)$$
$$\hat{E} = -i\sqrt{\frac{\hbar\omega}{2\epsilon_0 \Omega}}(a^\dagger - a)$$

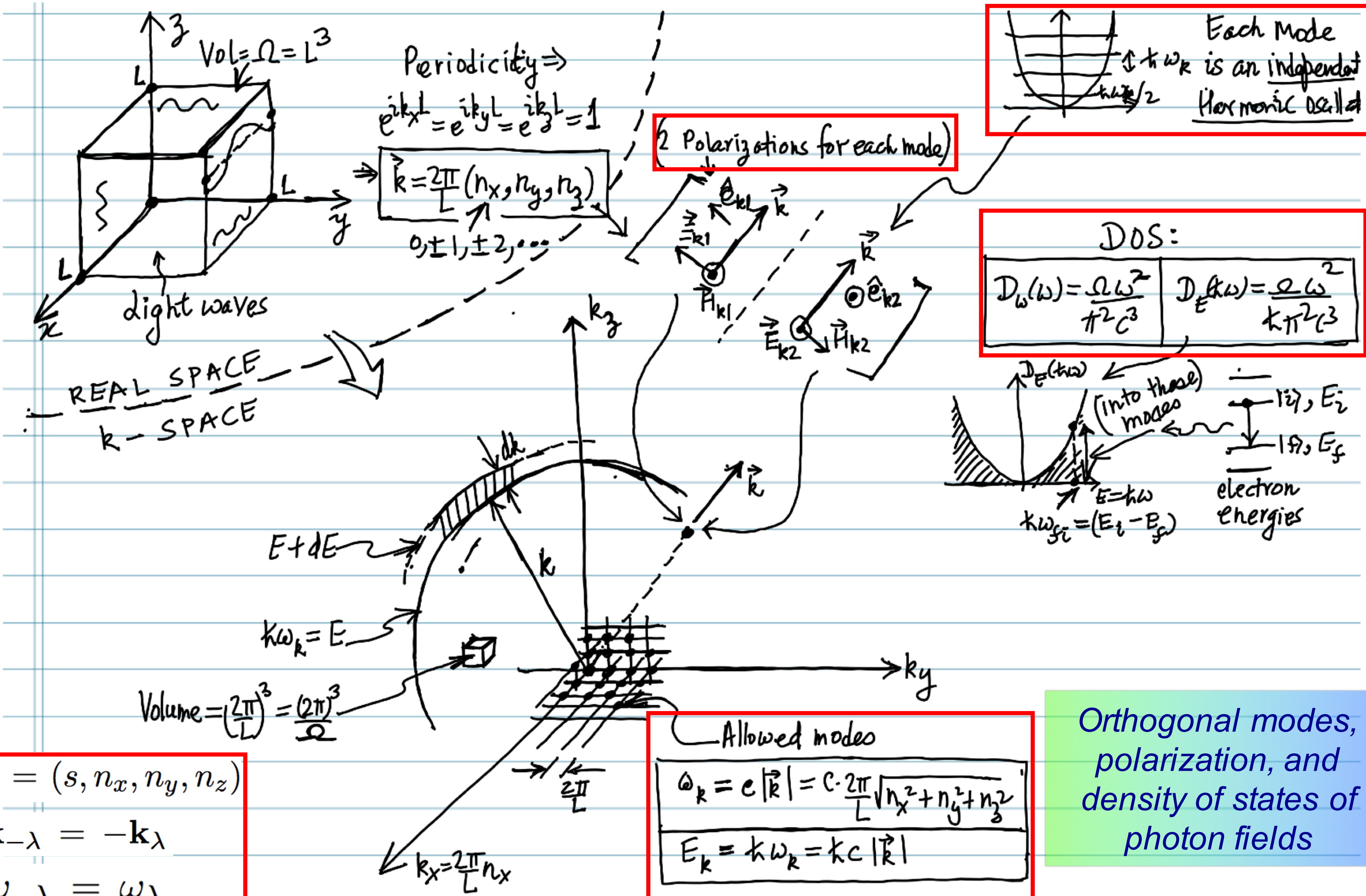
$$\boxed{H(\omega) = (a^\dagger a + \tfrac{1}{2})\hbar\omega}$$ (dipole approx)

$$[\hat{x}, \hat{p}_x] = i\hbar \qquad \hat{x} = \sqrt{\frac{\hbar}{2M\omega}}(a^\dagger + a)$$
$$\hat{p}_x = i\sqrt{\frac{\hbar\omega M}{2}}(a^\dagger - a)$$

$$\boxed{H_{osc} = (a^\dagger a + \tfrac{1}{2})\hbar\omega}$$

The electromagnetic field is quantized in the same way as the harmonic oscillator

Field-Quantization: Quantum Electrodynamics



$$e^{ik_xL} = e^{ik_yL} = e^{ik_zL} = 1$$

$$\vec{k} = \frac{2\pi}{L}(n_x, n_y, n_z), \quad 0, \pm 1, \pm 2, \ldots$$

$$D_\omega(\omega) = \frac{\Omega\omega^2}{\pi^2 c^3} \qquad D_E(\hbar\omega) = \frac{\Omega\omega^2}{\hbar\pi^2 c^3}$$

$$\omega_k = c|\vec{k}| = c\cdot\frac{2\pi}{L}\sqrt{n_x^2+n_y^2+n_z^2}$$

$$E_k = \hbar\omega_k = \hbar c|\vec{k}|$$

$$\lambda = (s, n_x, n_y, n_z)$$

$$\mathbf{k}_{-\lambda} = -\mathbf{k}_\lambda$$

$$\omega_{-\lambda} = \omega_\lambda$$

Orthogonal modes, polarization, and density of states of photon fields

Field-Quantization: Quantum Electrodynamics

$$\boxed{\hat{H}_{\text{tot}} = \frac{(\hat{\mathbf{p}} + e\hat{\mathbf{A}})^2}{2m} + V(\mathbf{r}) + \sum_\lambda \hbar\omega_\lambda(\hat{a}_\lambda^\dagger \hat{a}_\lambda + \frac{1}{2})}$$

Light-Matter combined Hamiltonian. Both matter and the electromagnetic field are now quantized.

$$\hat{H}_{\text{tot}} = \underbrace{[\frac{\hat{\mathbf{p}}^2}{2m} + V(\mathbf{r})]}_{\hat{H}_{\text{matter}}} + \underbrace{[\sum_\lambda \hbar\omega_\lambda(\hat{a}_\lambda^\dagger \hat{a}_\lambda + \frac{1}{2})]}_{\hat{H}_{\text{light}}} + \underbrace{[\frac{e}{2m}(\hat{\mathbf{p}} \cdot \hat{\mathbf{A}} + \hat{\mathbf{A}} \cdot \hat{\mathbf{p}}) + \frac{e^2 \hat{\mathbf{A}} \cdot \hat{\mathbf{A}}}{2m}]}_{\hat{W} = \hat{H}_{\text{light-matter}}}.$$

$$\hat{\mathbf{A}}(\mathbf{r}, t) = \sum_\lambda \hat{\mathbf{A}}_\lambda = \sum_\lambda \sqrt{\frac{\hbar}{2\epsilon_0 \Omega \omega_\lambda}} \hat{\mathbf{e}}_\lambda [\hat{a}_\lambda e^{i(\mathbf{k}_\lambda \cdot \mathbf{r} - \omega_\lambda t)} + \hat{a}_\lambda^\dagger e^{-i(\mathbf{k}_\lambda \cdot \mathbf{r} - \omega_\lambda t)}].$$

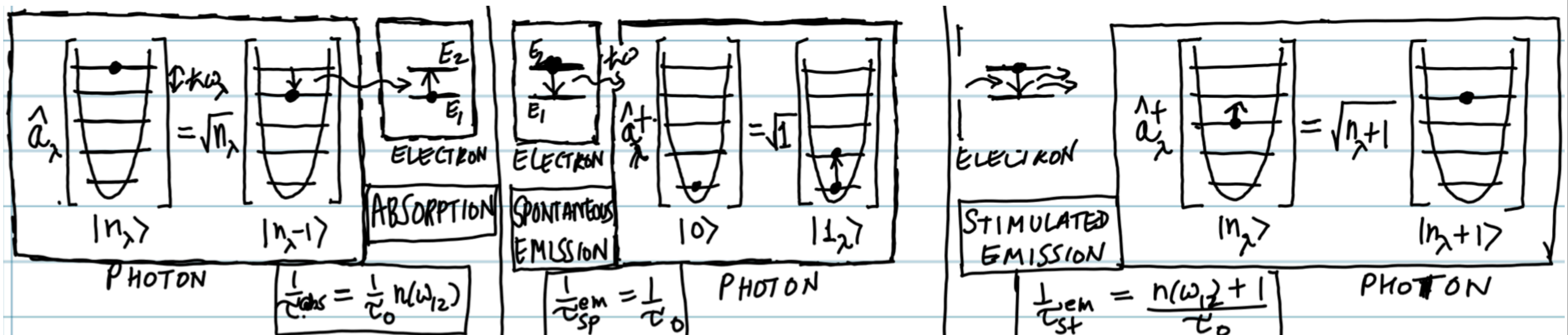
$$\boxed{\hat{W}_\lambda = \frac{e}{m} \hat{\mathbf{A}}_\lambda \cdot \hat{\mathbf{p}} = \frac{e}{m} \sqrt{\frac{\hbar}{2\epsilon_0 \Omega \omega_\lambda}} \Big(\hat{a}_\lambda e^{i\mathbf{k}_\lambda \cdot \mathbf{r}} \underbrace{e^{-i\omega_\lambda t}}_{\text{absorption}} + \hat{a}_\lambda^\dagger e^{-i\mathbf{k}_\lambda \cdot \mathbf{r}} \underbrace{e^{+i\omega_\lambda t}}_{\text{emission}} \Big) \hat{\mathbf{e}}_\lambda \cdot \hat{\mathbf{p}}}$$

If the interaction term is treated as a perturbation, we can identify the absorption and emission terms. Note the annihilation and creation operators make this very clear.

$$\frac{1}{\tau_{abs}} \approx \frac{2\pi}{\hbar} \times |\langle f|\hat{W}_\lambda^{abs}|i\rangle|^2 \delta[E_f - (E_i + \hbar\omega_\lambda)], \quad \frac{1}{\tau_{em}} \approx \frac{2\pi}{\hbar} \times |\langle f|\hat{W}_\lambda^{em}|i\rangle|^2 \delta[E_f - (E_i - \hbar\omega_\lambda)].$$

Transition rates from Fermi's golden rule

Einstein A & B Coefficients from QED



absorption

spontaneous emission

stimulated emission

$$\frac{1}{\tau^{abs}_{i\to f}} = \frac{e^2\omega^3_{fi}}{\pi\epsilon_0\hbar c^3}\overline{(\hat{\mathbf{e}}_\lambda\cdot\mathbf{r}_{if})^2} n(\omega_{fi}) = \frac{1}{\tau_0} n(\omega_{fi}).$$

$$\frac{1}{\tau^{em}_{i\to f}} = \frac{1}{\tau_0}[n(\omega_{fi})+1] = \frac{1}{\tau_0}[\underbrace{n(\omega_{fi})}_{\text{stimulated}} + \underbrace{1}_{\text{spontaneous}}].$$

$$\overline{(\hat{\mathbf{e}}_\lambda\cdot\mathbf{r}_{if})^2} = \frac{\int d\Omega(\hat{\mathbf{e}}_\lambda\cdot\mathbf{r}_{if})^2}{\int d\Omega} = r^2_{if}\frac{\int_{\phi=0}^{2\pi} d\phi\int_{\theta=0}^{\pi} d\theta \sin\theta(\cos\theta)^2}{\int_{\phi=0}^{2\pi} d\phi\int_{\theta=0}^{\pi} d\theta\sin\theta} = \frac{1}{3}r^2_{if}$$

Summing over isotropic 3D photon polarizations for a fixed dipole orientation

$$\frac{1}{\tau_0} = \frac{e^2\omega^3_{fi}r^2_{if}}{3\pi\epsilon_0\hbar c^3} \approx (\frac{900\text{ nm}}{\lambda})^3 \times (\frac{r_{if}}{1\text{ nm}})^2 \times (10^9\text{ sec}),$$

Characteristic time scale of transitions. Depends on the (photon wavelength)^3 and on (dipole length)^2. Typical ~ns.

Extra Slides