Name: Athith Krishna
Net-id: ak857

Assignment 6

Problem 6.1 (Semiconductor Band Parameters)

Verdeyen Problem #11.5.

11.5. An intrinsic GaAs semiconductor is irradiated by a wave with $h\nu - E_g = 0.05$ eV. Assume k conservation and 0 K and ignore light holes ($E_g = 1.43$ eV).

(a) Identify the energy levels in the conduction and valence band that can participate in absorption or gain (i.e., find $E_2 - E_c$ and $E_v - E_1$). (Ans.: $\Delta E_c = 0.044$ eV; $\Delta E_v = 0.0054$ eV.)

(b) What is the minimum number of electron-hole pairs necessary to achieve optical gain at the wavelength? (Ans.: $N > 7.4 \times 10^{17}$ cm^{-3}.)

Solution:

Assumptions - Momentum is conserved, light holes are ignored.

Diagram:

a) Using Eq. 11.4.5a Verdeyen,

$$\Delta \mathrm{E_c} = E_2 - E_c = \frac{m_h}{m_e + m_h}\left(h\nu - E_g\right) = \frac{0.55\, m_o}{0.55\, m_0 + 0.067 m_0}(0.05) = 0.0446\, eV$$

Using Eq. 11.4.5b Verdeyen,

$$\Delta \mathrm{E_v} = E_v - E_1 = \frac{m_e}{m_e + m_h}\left(h\nu - E_g\right) = \frac{0.067 m_0}{0.55\, m_0 + 0.067 m_0}(0.05) = 0.00543\, eV$$

b) Using Eq 11.2.9 Verdeyen,

$$N = \frac{1}{3\pi^2}\left[\frac{2m\Delta E_c}{\hbar^2}\right]^{\frac{3}{2}} = 7.4 \times 10^{23} m^{-3} = 7.4 \times 10^{17} cm^{-3}$$

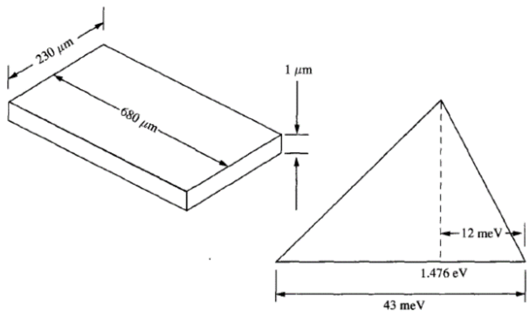
This is the minimum number of electron-hole pairs required to achieve optical gain.

Name: Athith Krishna
Net-id: ak857

Problem 6.3 (GaAs Bulk Laser)

Verdeyen Problem #11.15

11.15. The spontaneous emission from a GaAs semiconductor laser can be approximated by the graph shown below. The length of the wafer is 680 μm, the index of refraction is 3.6, the facet reflectivity is 0.3, the residual absorption coefficient in the crystal is 10 cm^{-1}, and the recombination lifetime is 1 ns.



(a) What is the wavelength of peak gain? (Ans.: 0.84 μm.)

(b) What is the FWHM of the gain coefficient in Hz and cm^{-1}? (Ans.: 5.1×10^{12}Hz and 169 cm^{-1})

(c) What must be the inverted carrier density to bring this wafer to threshold? (Ans.: 6.5×10^{15} cm^{-3})

(d) This carrier density must be sustained by some sort of pumping—carrier injection, photo pumping, or E-beam excitation. Estimate the minimum pumping power to maintain an inversion of 10^{16} cm^{-3} throughout the wafer. (Ans.: 369 mW.)

Solution:

(a)

Using

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{hc}{1.476eV} = 0.84\mu m$$

(b)

From the diagram given in the problem,

$$E' = h\Delta\nu \ \Rightarrow \Delta\nu_{FWHM} = \frac{h}{E'} = \frac{h}{2 \times 12\ meV} = 5.1 \times 10^{12} Hz = 169\ cm^{-1}$$

(c)

We know that $\int_0^\infty g(\nu)\, d\nu = 1$, From part (a), $c = \lambda\nu \Rightarrow \nu_0 = 3.57 \times 10^{14} Hz$

$$\Delta\nu_{Base} = 10.2 \times 10^{12} Hz$$

$$\gamma_0 = \sigma N g(\nu_0)$$

$$g(\nu_0) = \frac{2}{\Delta\nu_{Base}} = 2 \times 10^{-13} s$$

Condition for threshold:

$$R_1 R_2 \exp[(\gamma_0 - \alpha)2l_g] = 1 \Rightarrow \gamma_0 = \alpha + \frac{1}{2l_g}\ln\frac{1}{R_1R_2} = \frac{3.6}{680\mu m} + \frac{1}{2 \times 680\mu m}\ln\frac{1}{(0.3)(0.3)}$$
$$= 27.7\ cm^{-1}$$

$$\gamma_0 = \frac{\lambda_0^2}{2\pi n^2}\left(\frac{n_e}{\tau}\right) g(\nu_0) = \frac{(0.84\mu m)^2}{2\pi(3.6)^2}\left(\frac{n_e}{1\ ns}\right) 2 \times 10^{-13} = 27.7\ cm^{-1}$$

Inverted carrier density,

$$n_e = \frac{27.7\ cm^{-1}}{4.26 \times 10^{-15} cm^2} = 6.5 \times 10^{15} cm^{-3}$$

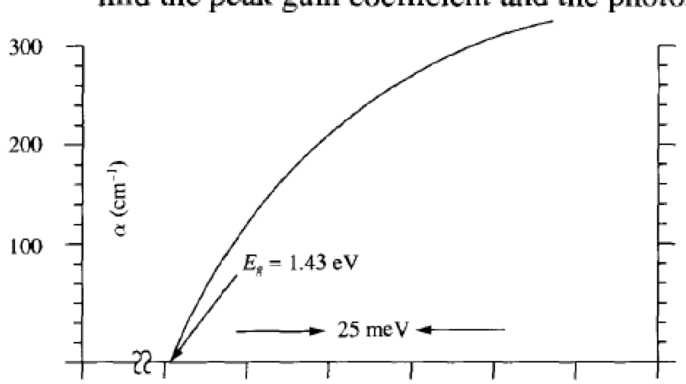
(d)

$$P = \left[\left(\beta n_e^2 + \frac{n}{\tau}\right)(h\nu_0)\right] \cdot Volume$$
$$= \left[\left(\beta(10^{16})^2 + \frac{10^{16}}{1ns}\right)(h(0.84\ \mu m))\right] \cdot (680 \times 230 \times 1)\mu m^3 = 369\ mW$$

Problem 6.4 (Optically Pumped Semiconductor)

Verdeyen Problem #11.16

11.16. The graph below is the absorption coefficient of a semiconductor at $T = 0$ K. If that sample is photopumped such that $F_n - E_c = 0.050$ eV and $E_v - F_p = 2$ meV, find the peak gain coefficient and the photon energy at which it occurs.



Solution:

Assumption: the given diagram is drawn to scale.

Total energy gap,

$$E = E_g + (F_n - E_c) + (E_v - F_p) = 1.43\ eV + (0.052)eV$$

Using the plot given above, we can read the loss coefficient off it which corresponds to an energy of $0.052\ eV$ from E_g.

$$\alpha \approx 235\ cm^{-1}$$

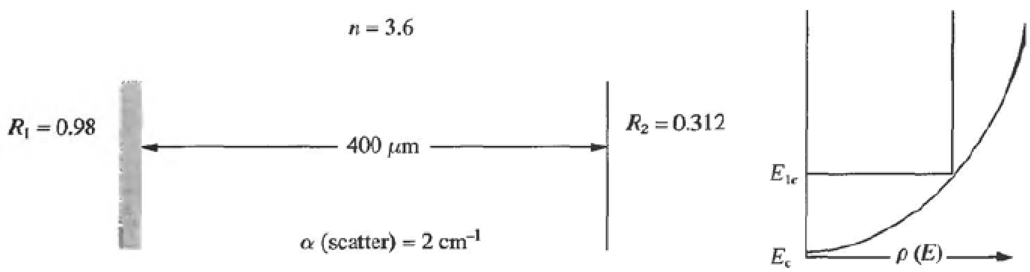
Photon energy,

$$E = h\nu = 1.43 + 0.052 = 1.482\ eV$$

Problem 6.5 (A Quantum Well Laser)

Verdeyen Problem #11.19

11.19. Consider the semiconductor quantum well laser shown in the diagram below along with the density of states diagram for the conduction band. Oscillation takes place in the wavelength region around 8500 Å.



(a) What is the photon lifetime for this cavity?

(b) What is the separation (in Å) between the longitudinal modes?

(c) What is the threshold gain coefficient for a mode in this cavity?

(d) Indicate on the density of states diagram where the quasi-Fermi level F_n must be in order to obtain gain.

(e) Assume $T = 0$ K and an electron density of 10^{18} cm^{-3}, $m_e^* = 0.067m_0$, $L_z = 100$ Å, and $x_{1e} = 1.13$. What is the separation $F_n - E_1$ in meV?

(f) Assume that the equal electron and hole densities of (e) were created by photopumping at 5145 Å and are lost by recombination (at a rate βnp, $\beta = 2 \times 10^{-10}$ cm^3/ sec) and by diffusion (n/τ_D, $\tau_D = 10$ ns). What must be the absorbed power per unit of volume to maintain this electron-hole population?

Solution:

(a)

For the given cavity,

$$\tau_p = \frac{\left(\frac{2nd}{c}\right)}{1-\exp(-2\alpha d R_1 R_2)} = \frac{\left(\frac{2\times 3.6\ \times 400\mu m}{c}\right)}{1-\exp(-2\times 2cm^{-1}\times 400\mu m\times (0.98\times 0.312))}$$
$$= 1.35\times 10^{-11} s$$

(b)

FSR for the given cavity,

$$FSR = \frac{c}{2nd} = 104.2\times 10^{12} GHz$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\nu}{\nu} \rightarrow \Delta\lambda = \frac{\Delta\nu}{\nu}\cdot\lambda = 104.2\times 10^{12} GHz\times 850nm = 2.51\ A^0$$

(c)

Condition for threshold:

$$R_1 R_2 \exp[(\gamma_{th}-\alpha_s)\cdot 2d] = 1$$

$$\gamma_{th} = \alpha_s + \frac{1}{2d}\ln\left(\frac{1}{R_1R_2}\right) = 2 + \frac{1}{2\cdot(400\mu m)}\ln\left(\frac{1}{0.98\times 0.312}\right) = 16.54\ cm^{-1}$$

(d), (e)

Using Eq. 11.4.15c Verdeyen, For gain, we need,

$$E_g < [h\nu = E_2 - E_1] < F_n - F_p$$

For quantum well lasers, the density of states in the energy interval dE is (Eq. 11.6.6)

$$\rho(E)dE = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)\left(\frac{2x_{1e}}{l_z}\right)dE$$

When T=0,

$$n = \int_{E_1}^{F_n}\rho(E)dE = \int_{E_1}^{F_n}\frac{1}{2\pi^2}\left(\frac{2(0.067m_0)}{\hbar^2}\right)\left(\frac{2(1.13)}{100A^0}\right)dE$$
$$= \frac{1}{2\pi^2}\left(\frac{2(0.067m_0)}{\hbar^2}\right)\left(\frac{2(1.13)}{100A^0}\right)[F_n - E_1] = 10^{18}cm^{-3}$$

$$F_n - E_1 = \frac{10^{18}cm^{-3}}{\frac{1}{2\pi^2}\left(\frac{2(0.067m_0)}{\hbar^2}\right)\left(\frac{2(1.13)}{100A^0}\right)} = 0.050\ eV = 50meV$$

(f)

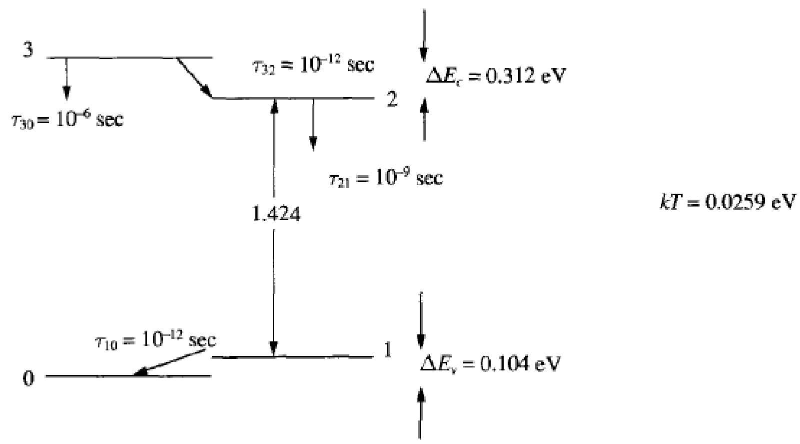
Recombination dominates when we have $n = p = 2\times 10^{18}$

Given, $\lambda = 514.5\ nm$ and $\beta = 2 \times 10^{-10} cm^3/s$

$$P = \left[\left(\beta n_e^2 + \frac{n}{\tau}\right)(h\nu_0)\right] \rightarrow p = \frac{P}{h\nu_0} = \left[\left(\beta n_e^2 + \frac{n}{\tau}\right)\right] = 385 \times \frac{10^6 W}{cm^2}$$

Problem 6.6 (Quantum Well Laser as a 4-Level System)
Verdeyen Problem #11.20

11.20. Consider the following four-level system. Any possible resemblance to a quantum well laser is intentional, but semiconductor theory is at most incidental. The system can be pumped via the $0 \rightarrow 3$ route. State 3 can relax back to 0 at a rate of $10^6\ \text{sec}^{-1}$ or to state 2 at a rate of $10^{12}\ \text{sec}^{-1}$ and can receive population back from 2 at a rate of $5.87 \times 10^6\ \text{sec}^{-1}$. State 2 decays to 1 at a rate of $10^9\ \text{sec}^{-1}$ and 1 relaxes to 0 at a rate of $10^{12}\ \text{sec}^{-1}$. Thermal processes keep the populations in (3, 2) and in (1, 0) related by the Boltzmann factor involving the appropriate energies. All states have a degeneracy equal to 2. In the absence of pumping, the absorption coefficient on the $2 \rightarrow 1$ transition is $20\ \text{cm}^{-1}$, and the density of active atoms is $10^{20}\ \text{cm}^{-3}$.



(a) In the absence of pumping, what is the population in state 1?
(b) What is the absorption cross section on the $2 \rightarrow 1$ transition?
(c) What is the ratio N_3/N_2?
(d) What must be the density of atom in state 2 to reach optical transparency?
(e) How much pump power must be expended on the $0 \rightarrow 3$ route to maintain a population in state 2 of $2 \cdot \times 10^{+18}\ \text{cm}^{-3}$?

Solution:

(a)

When there is no pumping, as it is given that the populations in (1,0) are related by the Boltzmann factor, we have,

$$N_1 = \left(\frac{\exp - \frac{\Delta E_v}{kT}}{1 + \exp - \frac{\Delta E_v}{kT}} \right) \times 10^{20} cm^{-3} = 1.77 \times 10^{18} cm^{-3}$$

(b) Given that absorption coeff. in the absence of pumping is $20\ cm^{-1}$.

$$20 = N_1 \sigma_{abs} \rightarrow \sigma_{abs} = \frac{20\ cm^{-1}}{1.77 \times 10^{18} cm^{-3}} = 1.17 \times 10^{-17} cm^{-2}$$

(c)

We are given that thermal processes keep populations in (3,2) related by Boltzmann factor, therefore,

$$\frac{N_3}{N_2} = \exp -\Delta E_c/kT = \left(\exp - \frac{0.312}{kT} \right) = 5.87 \times 10^{-6}$$

(d)

Using principle of conservation of Atoms,

$$N = N_0 + N_1 + N_2 + N_3$$

But, we know that $N_3 \ll N_1$ & N_2 and can be ignored.

And we also know that $N_1 + N_2 < N_0$ & N.

Thus, $N_1 \approx N_2 = 1.77 \times 10^{18} cm^{-3}$ when we have optical transparency.

(e)

$$P = V \left(\frac{n_2}{\tau_{21}} \right) (h\nu_{3,0}) \rightarrow \frac{P}{V} = \frac{(2 \times 10^{18})}{10^{-9}} \times (h\nu_{3,0}) = 471 \times 10^{6} W/cm^2$$

Problem 6.2

Solution by Kevin Lee

(a)

Spontaneous emission rate can be written as following.

$$R_{sp} = A \times \rho_{jnt}(v) f_C(E_2)[1 - f_V(E_1)]$$

$$R_{sp} = A \frac{1}{2\pi^2} (\frac{2m_r^*}{\hbar^2})^{3/2} \sqrt{hv - E_g} \frac{1}{1 + e^{\frac{E_2 - F_n}{kT}}} \frac{e^{\frac{E_1 - F_p}{kT}}}{1 + e^{\frac{E_1 - F_p}{kT}}}$$

We want to express everything in terms of photon energy. From (11.4.5ab), we have

$$E_2 - E_C = \frac{m_h^*}{m_e^* + m_h^*}(hv - E_g) = \frac{m_r^*}{m_e^*}(hv - E_g)$$

$$E_V - E_1 = \frac{m_e^*}{m_e^* + m_h^*}(hv - E_g) = \frac{m_r^*}{m_h^*}(hv - E_g)$$

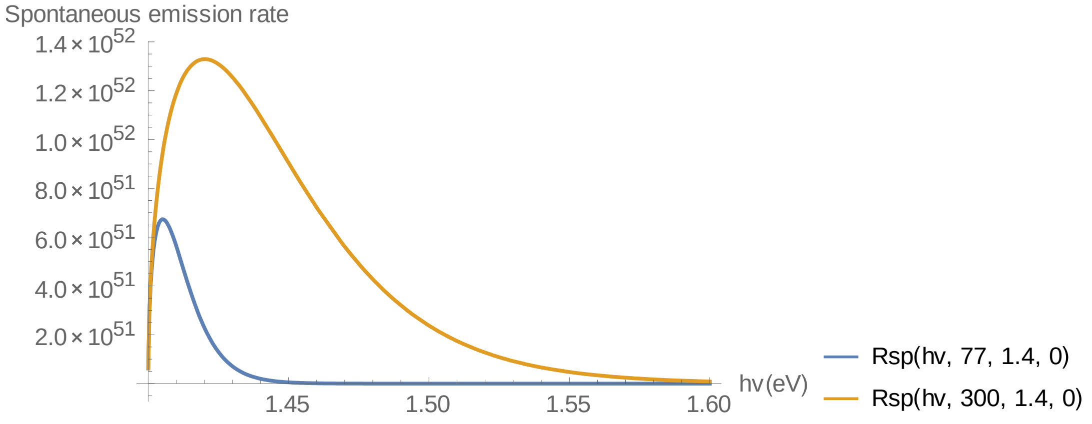
$$m_r^* = \frac{m_e^* m_h^*}{m_e^* + m_h^*}$$

Let's assume the energy reference point is the valence band maximum point, meaning $E_V = 0, E_C = E_g$.

$$E_2 = \frac{m_r^*}{m_e^*}(hv - E_g) + E_g$$

$$-E_1 = \frac{m_r^*}{m_h^*}(hv - E_g)$$

Substitute these two into the first equation, then we can plot it.



Blue curve corresponds to T=77K. Orange corresponds to T=300K.

- First, high temperature has higher peak intensity in the plot. This can be understood easily. Because the spontaneous emission results from the carrier recombination. For higher temperature, it means that the carrier distribution tail will go to higher energy in conduction, while hole will go lower into valence band. So we have more carriers to recombine and higher intensity.
- Higher temperature's peak intensity is at higher energy. This is similar behavior. Because higher temperature moves electron distribution upward in band

diagram, while hole distribution moves downward. Therefore, the maximum emission has higher energy.

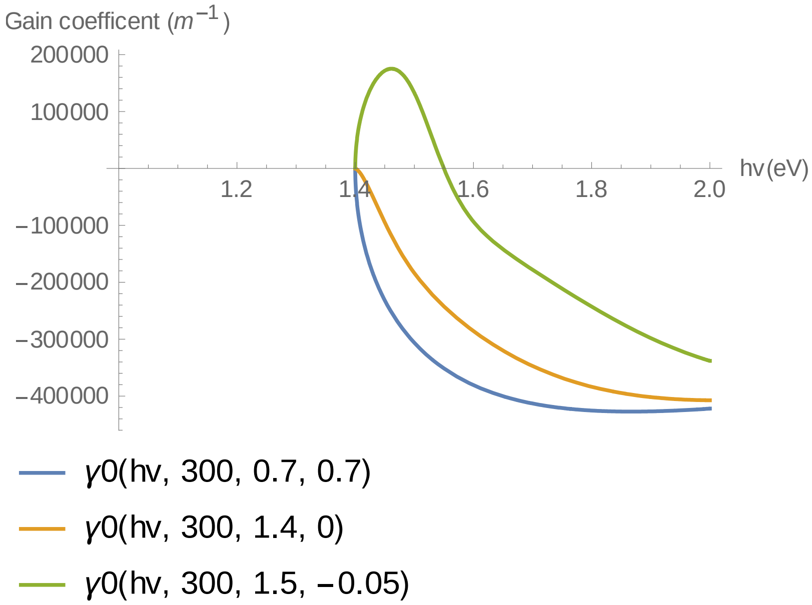
(b)

To plot the gain spectrum, we can use the formula in the class.

$$\gamma_0(hv) = A\frac{\lambda^2}{8\pi n^2}[h \times \rho_{jnt}(hv)][f_c(E_2) - f_v(E_1)]$$

The same strategy can be used here to replace E_1 and E_2.

$$\rho_{jnt}(hv) = \frac{1}{2\pi^2}(\frac{2m_r^*}{\hbar^2})^{3/2}\sqrt{hv - E_g}$$



1) First case is the blue curve. This case means that the both quasi-electron and hole Fermi levels are in the middle of the band gap. Therefore, there is no population inversion, we have negative gain, which means absorption.
2) Second case is the orange curve. Here the quasi-Fermi levels of electron and hole are at the band edge. This is the threshold point that the system is going to have gain.
3) Third case is green curve. Now quasi-Fermi level of electron is above conduction band minimum, which means excess electrons in the conduction band. Quasi-Fermi of hole is also below the valence band maximum, meaning excess hole exists. Therefore, we have population inversion created in the semiconductor. There is gain in the material.

(c)

Now I am going to plot the quantum well laser gain spectrum. The only difference from the previous problem is that the density of states is different for quantum well system.

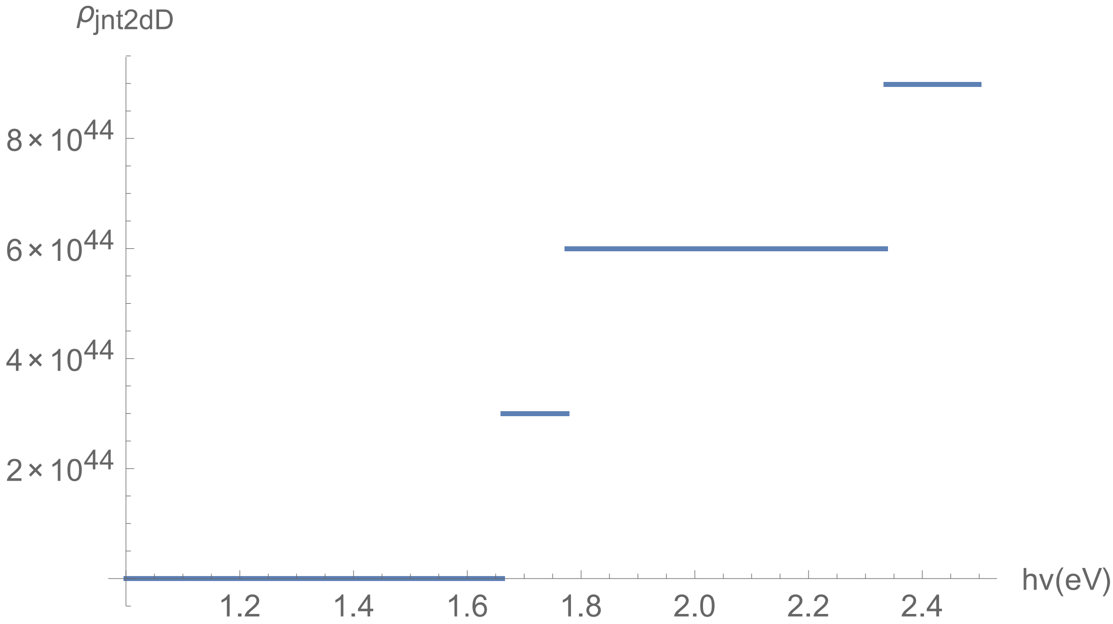
Let's first observe what the density of states.

Its analytical form is the following.

$$\rho_{jnt}^{2D} = \frac{m_r^*}{\pi\hbar^2 L_z}\Theta[hv - (E_g + E_n + E_p)]$$

$$E_n = \frac{\hbar^2}{2m_e^*}(\frac{\pi n_z}{L_z})^2, n_z = 1,2,3 \dots$$

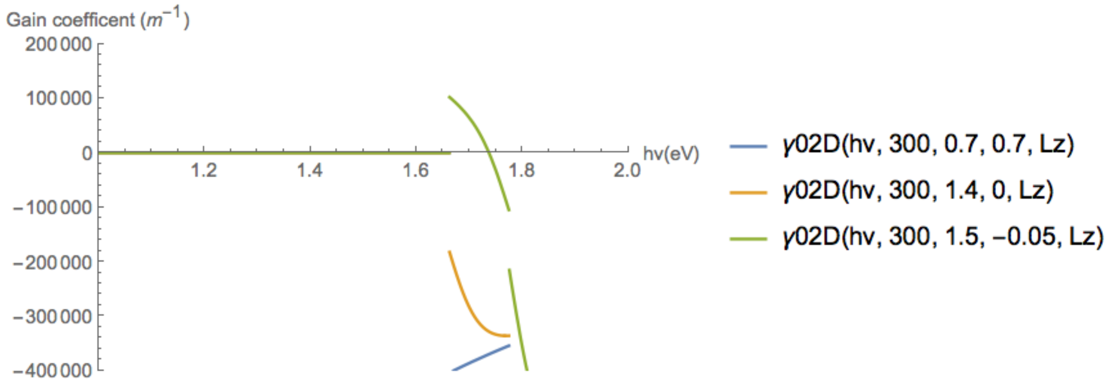
$$E_p = \frac{\hbar^2}{2m_h^*}(\frac{\pi n_z}{L_z})^2, n_z = 1,2,3 \dots$$



$$f_c(hv) = \frac{1}{1 + e^{\frac{\frac{m_r^*}{m_e^*}(hv - E_g) - E_n - F_n}{kT}}}$$

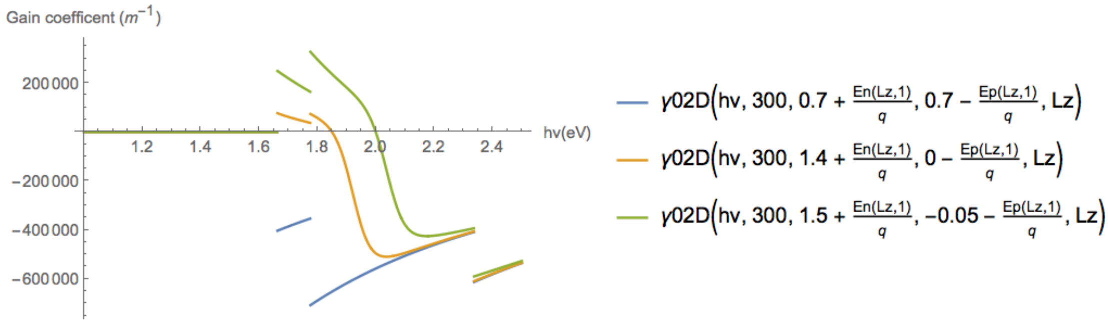
$$f_v(hv) = \frac{1}{1 + e^{\frac{-\frac{m_r^*}{m_e^*}(hv - E_g + F_p) - E_p}{kT}}}$$

There are two cases I can plot this problem for the inversion. Let's say the electron and hole quasi-Fermi levels are both sitting where they were, not shifted.



So you can see that the peak gain is actually decreased. This is expected. Because the quantum confinement shifts the conduction and valence band edges. If the quasi-Fermi levels stay where they were, effectively, we are having less carrier in the bands.

Let's assume that quasi-Fermi levels' differences are away from the new band edges due to the quantum confinement.



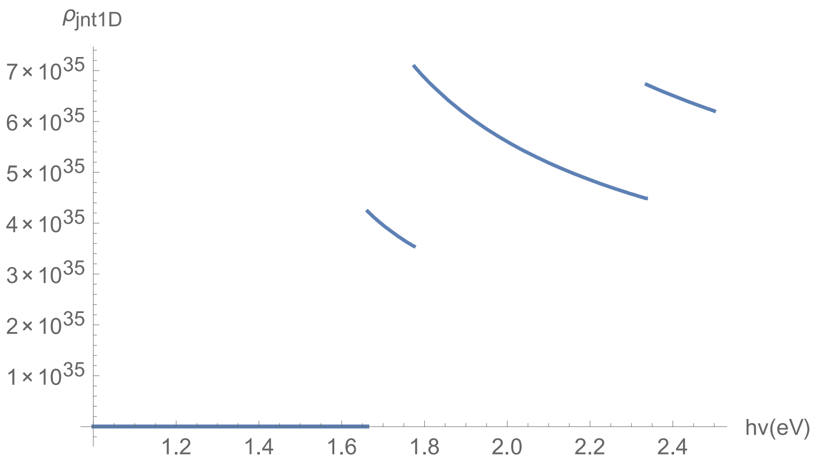
So now I shift all the cases in problem (b) with respect to the quantized energies of first hole and electron states. We can see that the maximum gain is actually higher than the bulk case.

(d)

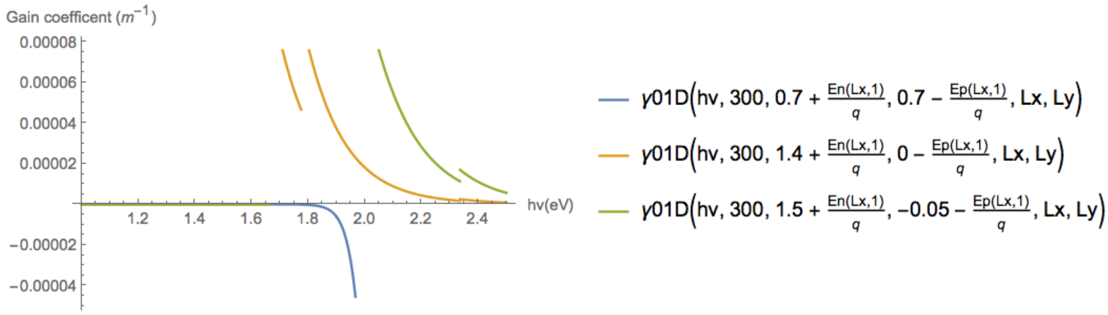
For 1D quantum wire, the density of states should be modified as following.

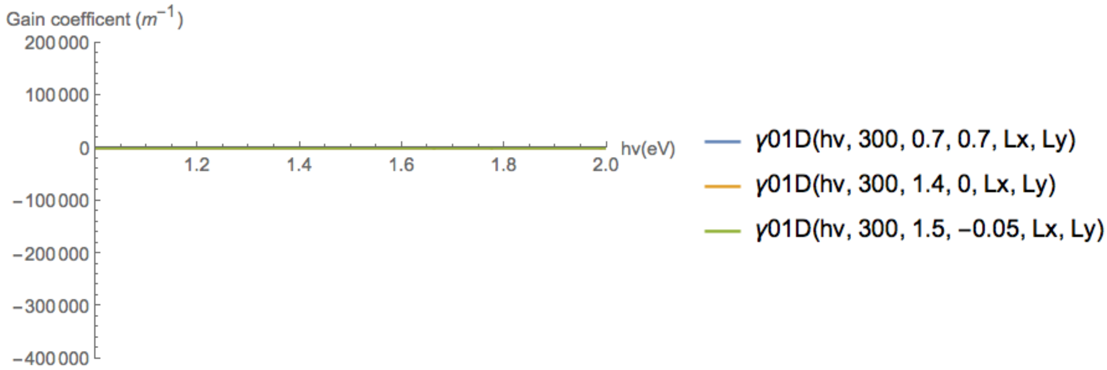
$$\rho_{jnt}^{1D} = \frac{2}{\pi L_x L_y}\sqrt{\frac{2m_r^*}{\hbar^2}}\Theta[hv - (E_g + E_n + E_p)]$$

If I plot it out, it should look something like the following.



So if I redo the two plots I had in the previous problem.





So we don't have any gain for quasi-Fermi levels are the same with respect to the original band edges. This is also expected. Because quantum confinement is too large, that the quasi-Fermi levels are less than the 1 bound states' energies.

For example, the 1^{st} electron bound state energy is 0.624eV and 1^{st} hole bound state energy is 0.1eV. They're both larger than the original quasi-Fermi levels.