Problem 1. Consider the energy level diagram shown below. There is a pump that moves atoms from the ground state to level 4, from which they non-radiatively relax into level 3 within a picosecond of arriving in level 4. This is a solid state laser, and for each transition the lineshape function, \( g(\nu) \), is the same with value \( g(\nu) = 10^{-13} \). The index of refraction is \( n = 1.50 \).

\[
\begin{align*}
4 \text{ eV} & \quad \text{Level 4} \\
3.5 \text{ eV} & \quad \text{Level 3} \\
2.2 \text{ eV} & \quad \text{Level 2} \\
1 \text{ eV} & \quad \text{Level 1} \\
\text{Ground State} & \\
\end{align*}
\]

1) Pumping occurs optically between the ground state and level 4. What is the wavelength of the pump light? 
2) What is the lifetime of levels 3, 2, and 1? 
3) What is the gain cross-section, \( \sigma_{em} \), for the 3\( \rightarrow \)2 and 3\( \rightarrow \)1 transitions? 
4) If you were going to use this to make a laser, pumped as shown, which transition of all those shown would you expect to see the highest gain? (There are 5 transitions shown, some can be eliminated easily).

Problem 2. Gain in an \text{Cu}^{3+}\text{:YAG} crystal

A 2 cm long Cornellium-doped YAG crystal is pumped using two 976 nm diode lasers. The measured absorption coefficient for the pump light in the crystal is \( \alpha = 3 \text{ cm}^{-1} \). In other words, 
P(z) = P(0) e^{-\alpha z}, \text{ where } z \text{ is measured in centimeters.}

Because of the strong absorption, the crystal is pumped from both sides. The dashed lines in the sketch below show the inversion as a function of position in the crystal.

The Cornellium system forms a 4 level laser, with the upper state lifetime \( \tau = 250 \times 10^{-6} \text{ sec} \), a transition wavelength of 1.239 µm, and a gain cross section of \( \sigma = 3 \times 10^{-21} \text{ cm}^2 \). The radius of the pump beams is \( \omega = 60 \mu \text{m} \).

1) If each pump delivers 1 W of power, what is the expected small signal gain for a single pass through this crystal? You should assume in this case the effective area of the pump volume is \( \pi \omega_0^2 \) (laser diodes have a large \( M^2 \) value, and are not particularly Gaussian) 
2) The crystal is placed in a 2 mirror cavity with a perfect High Reflector on one end (\( R = 100\% \)) and an output coupler of \( R = 92\% \) on the output side. 
   a. What is the threshold inversion for this laser? 
   b. What is \( I_{sat} \) for this laser? 
   c. Assuming your answer to part 1) is correct, what is the expected output power from this laser? In the output case, you should assume the laser beam effective area is \( \pi \omega_0^2/2 \)