# ECE 4070/MSE 6050
# Physics of Semiconductors and Nanostructures
# Exam 1, February 28, 2017

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## Instructions:

- There are **THREE** problems in this exam
- Every problem must be done in the booklet provided
- Always solve **analytically first** before finding numerical values
- Only work done on the blue exam booklets will be graded. Do not attach your own sheets to the exam booklets under any circumstances
- To get partial credit you must show all the relevant work
- Correct answers with wrong reasoning will not get points
- All questions do not carry equal points
- All questions do not have the same level of difficulty, use your time judiciously
- Common Physical Constants: [Planck's constant: $h = 6.63 \times 10^{-34}$ J·s and $\hbar = h/(2\pi)$], [Electron charge: $q = 1.6 \times 10^{-19}$ Coulomb], [Free electron mass: $m_e = 9.1 \times 10^{-31}$ kg], [Speed of light in vacuum: $c = 3 \times 10^8$ m/s], [Permittivity of vacuum: $\epsilon_0 = 8.85 \times 10^{-12}$ F/m], [Boltzmann constant: $k_b = 1.38 \times 10^{-23}$ J/K].

## DO NOT WRITE IN THIS SPACE

# 1 Miscellaneous [20 points]

Give *very* short answers to the following questions:

**a)** What is the wavelength of a free electron moving at a velocity of $v = 10^6$ m/s?

**b)** What is the reason for the enormous velocities and momenta of free electrons in a metal even at 0K?

**c)** When there is no applied electric field, why is there no *net current* in spite of the enormous velocities of electrons in metals?

**d)** Why can't we put 2 electrons in the same quantum state?

# 2 Fermi Energy, Average Energy, and Conductivity of Graphene [40 points]

In assignment 1, you used the energy-momentum dispersion relationship for electrons in 2-dimensional graphene $E(k_x, k_y) = \hbar v_F \sqrt{k_x^2 + k_y^2}$, where $v_F = 10^6$ m/s is a parameter with dimensions of velocity. Each allowed $k-$state holds $g_s = 2$ spins, and there are $g_v = 2$ valleys. Using this energy dispersion, you found that the density of states for graphene is $g(E) = \frac{g_s g_v}{2\pi(\hbar v_F)^2}|E|$.

(a) If the sheet of graphene has $n_{2d} = 10^{13}/\text{cm}^2$ electrons, find the Fermi wavevector $k_F$ at $T = 0$ K.

(b) Find the Fermi energy $E_F$ at $T = 0$ K. Express its numerical value in eV units.

(c) Find the average energy of the electron distribution at $T = 0$ K.

(d) Find an expression for the group velocity of electrons occupying the state $(k_x, k_y)$. Remember that the group velocity is a vector in 2D.

(e) Make a sketch of the direction and magnitude of the group velocity of a few $(k_x, k_y)$ points using part (d). How is this case different from a free-electron?

(f) When I apply an electric field, a current will flow in graphene. Sketch how the $\mathbf{k}-$space picture changes from the equilibrium value to support the flow of current.

# 3 Classical and Quantum Current Flow [40 points]

Answer the following questions regarding electron transport:

(a) Use the classical Drude model for this part. A current density of $J = 1000$ A/cm$^2$ is flowing through a metal wire in response to an electric field $E = 10$ V/cm. I have managed to find that the average time between electron collisions is $\tau = 10^{-14}$ s. Find the electron density $n_{3d}$ in the metal.

(b) Find the current carried by an electron in the superposition state $\psi(x) = A_+ e^{ikx} + A_- e^{-ikx}$.

(c) Consider now a current $I$ current flowing in a 1D ring of circumference $L$ because of a *single* electron. Find the wavelength $\lambda$ of that electron. Show that not all currents are allowed, the current is *quantized.*