
ECE 4070/MSE 6050
Physics of Semiconductors and Nanostructures
Final Exam, May 22, 2017

Debdeep Jena (`djena@cornell.edu`), Depts. of ECE and MSE, Cornell University

---

Instructions:

- There are **FOUR** problems in this exam
- Every problem must be done in the booklet provided
- Always solve **analytically first** before finding numerical values
- Only work done on exam booklets will be graded. Do not attach your own sheets to the exam booklets under any circumstances
- To get partial credit you must show all the relevant work
- Correct answers with wrong reasoning will not get points
- All questions do not carry equal points
- All questions do not have the same level of difficulty, use your time judiciously
- Physical Constants: [Planck's constant: $h = 6.63 \times 10^{-34}$ J·s and $\hbar = h/(2\pi)$], [Electron charge: $q = 1.6 \times 10^{-19}$ Coulomb], [Free electron mass: $m_e = 9.1 \times 10^{-31}$ kg], [Speed of light in vacuum: $c = 3 \times 10^8$ m/s], [Permittivity of vacuum: $\epsilon_0 = 8.85 \times 10^{-12}$ F/m], [Impedance of vacuum: $\sqrt{\mu_0/\epsilon_0} \approx 377\ \Omega$], [Boltzmann constant: $k_b = 1.38 \times 10^{-23}$ J/K], [Room temperature $k_b T \sim 1/40$ eV ~ 26 meV].

---

1 Miscellaneous [25 points]

Give *very* short answers to the following questions. All symbols have their usual meanings.

a) If the electron bandstructure near the conduction band minimum and the valence band maximum of a semiconductor of bulk bandgap E_g is parabolic with effective masses $m_c^\star$ & $m_v^\star$, sketch the band edge density of states for 3D, 2D, 1D, and 0D structures made of this material side by side. Cover an energy range that shows both conduction and valence bands.

b) For the same crystal structure, why do semiconductors with larger lattice constants (e.g. InAs, InN) and larger inter-atomic hopping energies have smaller effective masses and smaller bandgaps (e.g. compared to GaAs, GaN)?

c) The classical Newton's law of motion $F = \frac{dp}{dt}$ changes to $F = \frac{d(\hbar k)}{dt}$ for electrons in a crystal. Here F is the net *external* force on the electron. How does then the effect of the periodic potential of atoms enter the dynamics of the electron, and does $\hbar k$ have the same meaning as $p = mv$ of classical mechanics?

d) By using the tools of semiconductor DOS and Fermi levels, explain the reason why for any field-effect transistor, the off-state drain current has a gate voltage dependence $I_d^{off} \sim e^{\frac{qV_{gs}}{k_b T}}$, whereas in the on-state the dependence switches to $I_d^{on} \sim (V_{gs} - V_T)^\alpha$, where α is a constant.

e) Estimate the *number* of acoustic phonons of energy $\hbar\omega \sim 5$ meV seen by an electron in a hot Silicon nanowire at $T = 400$ K. Estimate the *ratio* of the emission to absorption rates of these acoustic phonons by the electron at this temperature at equilibrium. Which is higher, and why?

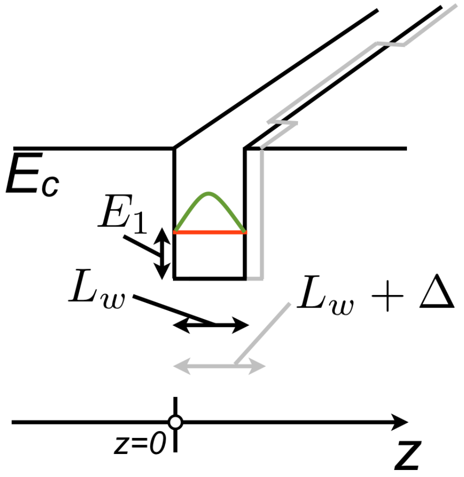

Figure 1: A quantum well with rough interfaces.

2 Electron Mobility in Ultrathin Quantum Wells [25 points]

Consider a 2D electron gas located in the ground state of the conduction band of a heterostructure quantum well of area A as sketched in Figure 1. Assume for this problem that the band offset is infinite. The electron is free to move in the $x-y$ plane, and its motion is quantized in the $z-$direction.

(a) Write the total wavefunction in the effective mass approximation of the electron in the lowest (or ground-state) subband as a function of the lateral wavevector $\mathbf{k} = (k_x, k_y)$, the lateral real-space coordinate $\mathbf{r} = (x, y)$ and area A, the quantum well width L_w and the $z-$coordinate, and the quantum well lattice-periodic function $u_c(r)$. Identify the envelope function and retain it for the rest of the sections of this problem.

(b) Write the effective mass equation for the quantum well of width L_w and show that the energy eigenvalues are given by $E = \frac{\hbar^2}{2m_c^\star}(k_x^2 + k_y^2) + E_1(L_w)$ where $E_1(L_w)$ is the ground state energy of the quantum well.

(c) Show that the ground state energy of the quantum well depends on the quantum well width as $E_1(L_w) = \frac{h^2}{8m_c^\star L_w^2}$.

(d) Due to fluctuations in the epitaxial growth of the quantum well, the well width was not exactly L_w everywhere in the (x, y) plane, but fluctuated by a thickness $\Delta(x, y)$ such that the fluctuation is much smaller than the well width $\Delta(x, y) << L_w$ and the net fluctuation is zero $\langle \Delta(x, y) \rangle = 0$ as shown in gray lines in Figure 1. Show then that the ground state energy fluctuates by $\Delta E_1(x, y) = -\frac{h^2}{4m_c^\star L_w^3}\Delta(x, y)$.

(e) Show using the effective mass equation that this fluctuation in the ground state energy due to well-width variations can be treated as a perturbation $\Delta E_1(x, y) = W(x, y) = W(\mathbf{r})$ in the motion of the electron in the $x-y$ plane.

(f) The scattering rate for the 2DEG electrons $\frac{1}{\tau(\mathbf{k} \to \mathbf{k}')} = \frac{2\pi}{\hbar}|\langle \mathbf{k}'|W(\mathbf{r})|\mathbf{k}\rangle|^2 \delta(E_{\mathbf{k}} - E_{\mathbf{k}'})$ due to these fluctuations from Fermi's Golden rule depends on the square of the matrix element $\langle \mathbf{k}'|W(\mathbf{r})|\mathbf{k}\rangle$. Show that $|\langle \mathbf{k}'|W(\mathbf{r})|\mathbf{k}\rangle|^2 = \frac{h^4}{16(m_c^\star)^2 L_w^6} \cdot |\int \frac{d^2r}{A} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}\Delta(x, y)|^2$. There is no need to evaluate the integral.

(g) Because the electron mobility $\mu = \frac{q\langle\tau\rangle}{m_c^\star}$, show that the mobility limited by this quantum well interface roughness (IR) scattering decreases as the *sixth power* of the quantum well width according to $\mu_{IR} \propto \frac{qm_c^\star L_w^6}{h^3}$. Show this proportionality, there is no need to evaluate integrals. This is a severe scattering mechanism that is a hurdle to achieving high electron mobilities for 2DEGs in very thin quantum wells, because $\mu_{IR} \downarrow asL_w \downarrow$ as the sixth power.

Contd...

3 A 2DEG as a parallel array of 1D conductors [25 points]

Electrons of sheet carrier density n_s sit in the conduction band of a 2D electron system of energy bandstructure $E(k_x, k_y) = \frac{\hbar^2}{2m_c^\star}(k_x^2 + k_y^2)$ with the $k-$space occupation of carriers shown in Figure 2. Assume a spin degeneracy of $g_s = 2$ and a valley degeneracy of $g_v = 1$. The width of the 2D system is W, the length L, and ohmic source and drain contacts are made to connect to the electrons to flow a current in the $x-$direction. Solve this problem entirely at $T = 0$ K. The allowed discrete points in the $k-$space $(k_x, k_y) = (\frac{2\pi}{L}n_x, \frac{2\pi}{W}n_y)$ where (n_x, n_y) are integers are considered individual *modes* of the 2DEG as indicated in Figure 2. The collection of modes with the same n_y is considered a 1D mode of the 2DEG.

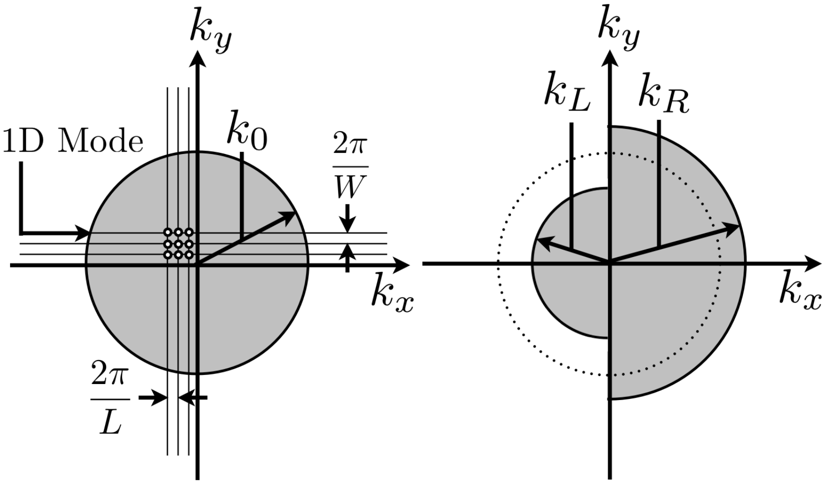

Figure 2: Lateral Modes of a 2D Electron System.

(a) When the applied voltage across the source/drain contacts is $V_{ds} = 0$, find the Fermi wavevector k_0 as shown in the left of Figure 2.

(b) Show that the number of 1D modes with current flow in the x-direction because of the finite width of the 2D conductor is $M_0 = \frac{k_0 W}{\pi}$. Use part (a) to write this in terms of the 2DEG density.

(c) Now a voltage V_{ds} is applied across the drain and the source such that the net sheet carrier density of the 2DEG does not change. Assume ballistic transport and show that in Figure 2, $k_R = \sqrt{k_0^2 + \frac{m_c^\star}{\hbar^2}(qV_{ds})}$ and $k_L = \sqrt{k_0^2 - \frac{m_c^\star}{\hbar^2}(qV_{ds})}$.

(d) Show that the voltage V_{ds} reduces the total number of left going modes M_L and increases the total number of right going modes M_R. Find expressions for M_L and M_R.

(e) Find the voltage V_{ds} at which carriers in all modes move to the right and no carriers move to the left.

(f) Find how many right-going 1D modes are present in the above situation when all carriers move to the right.

(g) Because each 1D mode in the ballistic limit can provide the maximum conductance of a quantum of conductance $G = \frac{g_s g_v q^2}{h}$, find the 'saturation' current I_d when the critical V_{ds} of part (e) is reached.

Contd...

4 Photonic Processes in Semiconductor Solar Cells [25 points]

For red photons of energy $\hbar\omega \sim 1.8$ eV, the equilibrium optical absorption coefficient of Si with $E_g = 1.1$ eV is $\alpha_0(\hbar\omega) \sim 3 \times 10^3$/cm whereas that of GaAs with $E_g = 1.4$ eV is $\alpha_0(\hbar\omega) \sim 2 \times 10^4$/cm.

(a) Explain why it is so much higher for GaAs with representative sketches.

(b) How thick should a Si or GaAs layer be to absorb $(1 - e^{-3}) \sim 1 - \frac{1}{20} = 85\%$ of the photons that enter the semiconductor?

(c) The absorption of a photon creates an electron in the conduction band, and a hole in the valence band. Sketch this process using the bandstructure $E(k)$ first for GaAs and then for Si.

(d) In GaAs and Si, the extra electron in the conduction band and hole in the valence band must lose their excess energies and relax to the edges of the respective bands. What process enables this energy loss?

(e) Once they have reached the band edges, what process will get rid of the extra electron and hole? How is this process fundamentally different between GaAs and Si, and which is much faster? Why?

(f) In a solar cell, a p-n junction is used to separate the electrons and holes spatially and generate an open-circuit voltage well before they meet their undesired fates of part (e). Argue why the voltage generated will be close to the bandgap of the semiconductor.

End.