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**ECE 5390/MSE 5472, Fall Semester 2017**  
**Quantum Transport in Electron Devices & Novel Materials**  
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**Assignment 1**

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**Policy on assignments:** Please turn them in by 5pm of the due date. The due date for this assignment is **Thursday, Sept 7th, 2017**.

**General notes:** Present your solutions *neatly*. Do not turn in rough unreadable worksheets - learn to **take pride in your presentation**. Show the relevant steps, so that partial points can be awarded. BOX your final answers. Draw figures wherever necessary. Please print out this question sheet and staple to the top of your homework. Write your name and email address on the cover. Some problems may lead to publishable results - be on the lookout!

### **Problem 1.1) The electron that escaped**

From the surface of a round wire of radius  $a$  carrying a dc current  $I$  an electron escapes with a velocity  $v_0$  perpendicular to the surface. Find the maximum distance the electron travels from the axis of the wire before it turns back towards the wire.

### **Problem 1.2) Electron Transport in Crossed E and B Fields**

An electron of charge  $q$  and mass  $m$  at  $t = 0$  is at rest at the origin of a region that has a  $y$ -directed electric field  $\mathbf{E} = E\hat{y}$  and a  $z$ -directed magnetic field  $\mathbf{B} = B\hat{z}$ . Neglect relativistic effects to find:

- (a) the trajectory  $x(t)$  and  $y(t)$  the electron traverses due to the fields and sketch/plot it,
- (b) the lengths the electron travels between successive moments of rest,
- (c) the mean particle velocity projected along the  $x$ -axis, which is also called the *drift* velocity. How does the drift velocity depend on the charge and mass of the electron?

### **Problem 1.3) Exactly solved problems of quantum mechanics**

Quantum mechanical states of *definite energy* for electrons are of central importance in this class. Make a table where your columns are a) the electric potential  $V(r)$ , b) the definite energy wavefunctions in real space  $\psi_E(x)$ , c) The corresponding energy eigenvalues  $E_n$ , and d) a sketch of the potential, eigenfunctions, and eigenvalues for

- 1) The completely free electron in 1D  $V(x) = 0$  in  $-\infty \leq x \leq +\infty$ ,
- 2) The 'quasi'-free electron in 1D  $V(x) = 0$  in a 'circle' of length  $L$ ,
- 3) The electron in a box with  $V(x) = 0$  for  $0 \leq x \leq L$  and  $V(x) = \infty$  for  $x < 0$  and  $x > L$ ,
- 4) The harmonic oscillator  $V(x) = \frac{1}{2}kx^2$ , and
- 5) The hydrogen atom with  $V(r) = -\frac{e^2}{4\pi\epsilon_0 r}$ .

We will refer to this table frequently in the course.

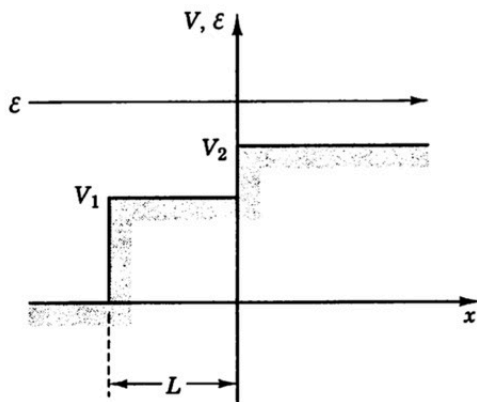
### **Problem 1.4) A Quantum Mechanical anti-reflection Coating**

Solve this interesting problem shown in Fig 1 from Kroemer's QM text. One can implement the scheme in semiconductor heterostructure devices for electrons incident at energy  $\mathcal{E}$  from the left as shown in the figure. Discuss how the design will work only for specific electron energies, and

what happens if the electron energy is changed for a particular design. Provide sketches wherever appropriate.

### #5-3-1: Zero-Reflection Conditions at a Double-Step Barrier

Devise a quantum-mechanical “antireflection coating”; that is, determine the proper width  $L$  and height  $V_1$  of an intermediate potential step to suppress the reflection of a wave of a specific incident energy at a barrier of height  $V_2 < \epsilon$  (Fig. 5-3-3).



**Figure 5-3-3.** Zero-reflection barrier.

Figure 1: Designing an anti-reflection coating for an electron incident on a barrier.

### Problem 1.5) Electron ‘dripping’ through a cone

An electron “sits” in the ground state in a cone-shaped “bag” under the influence of gravity. The lower end of the plastic bag is cut with scissors. Find the time it takes for the electron to fall out of the bag.