ECE 4070, Spring 2017
Physics of Semiconductors and Nanostructures
Handout 4070

Contents

Problems

Present your solutions *neatly*. Do not turn in rough unreadable worksheets - learn to **take pride in your presentation**. Show the relevant steps, so that partial points can be awarded. $\boxed{\text{BOX}}$ your final answers where applicable. Draw figures wherever necessary. Always provide the analytical answer before finding numerical values. **Please print out the question sheet(s) and staple to the top of your homework.** Write your name, email address, and date/time the assignment is turned in on the cover. Assignments must be turned in before class on the due date. The time the assignment is turned in should be written. There will be a 10% penalty each day of delay, and assignments will not be accepted beyond 3 days after the due date. There will be no exceptions to this rule. You are allowed to work with other students in the class on your homeworks. The name(s) of the student(s) you worked with must be included in your homework. But what you turn in must be in your *own* writing, and have your *own* plots and figures. Turning in plots/figures/text that are exact replicas of others *is considered cheating*.

4070.1 HW 1

Posted: 02/04/2017, Due: 02/13/2017

Problem 1: Semiconductor History

Write a short <1-page critique of the paper "The History of Semiconductors" handed out in class.

Problem 2: Mirror Mirror on the Wall

Believe it or not, coating glass with metal to make make a mirror was a *big* technological breakthrough back in the time [1]. In this problem, we answer why metals are shiny - why they reflect most of the visible light incident on them. Not surprisingly, this has to do with the conduction electrons in the metal. Here is our mode of attack: when the light wave experiences a mismatch in the refractive index, part of it is transmitted, and part reflected. So we will ask Maxwell's equations to give us the reflection coefficient when a light beam is incident from air on a metal. If we find that this reflection coefficient is very high for visible wavelengths, we have succeeded in explaining why metals are shiny.

The reflection coefficient Γ_r will depend on the refractive index $\sqrt{\epsilon(\omega)}$ of the metal, which in turn will depend on how the conduction electrons respond to the oscillating electric field of the light beam. This is where Drude's free electron model of the metal - the same model that explained electrical and thermal conductivity - will help us through.

Figure 4070.1 shows the measured reflectance of three common metals as a function of the wavelength of light incident on it. Your job in this problem is to calculate and make

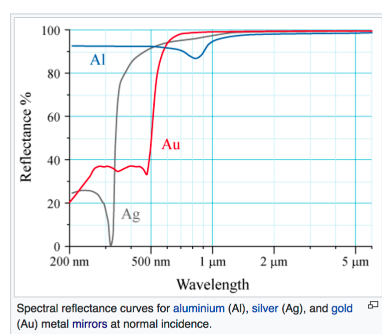

Fig. 4070.1: Reflectance spectra of three metals from Wikipedia

[1] Quite possibly marking the birth of fashion?

your own plot by standing on Maxwell, Drude, and Newton's shoulders. If things go right, you may even explain the dips and wiggles in figure 4070.1. I will roughly outline the method and trust you can finish the story:

The electric field of the light beam according to Maxwell oscillates in time as $E(t) = E_0 e^{i\omega t}$, where E_0 is the amplitude, and $\omega = 2\pi f$ is the radial frequency, $f = c\lambda$ with c the speed of light, and the wavelength λ of light is the $x-$axis in the plot. The reflection coefficient for light is $\boxed{\Gamma_r = \frac{E_r}{E_i} = \frac{\sqrt{\epsilon_0} - \sqrt{\epsilon(\omega)}}{\sqrt{\epsilon_0} + \sqrt{\epsilon(\omega)}}}$, where $\sqrt{\epsilon}$ is the refractive index of the media. The reflectance is $R = |\Gamma_r|^2$, which can be found for various wavelengths; this is the $y-$axis of the plot. Note that all symbols have their usual meanings.

(a) From Maxwell's equation $\nabla \times \mathbf{H} = \mathbf{J} + i\omega\epsilon_0\mathbf{E}$ in material media, show that the dielectric constant of the metal is $\boxed{\epsilon(\omega) = \epsilon_0[1 + i\frac{\sigma(\omega)}{\omega\epsilon_0}]}$.

(b) Now if you have the frequency-dependent conductivity $\sigma(\omega)$, you can make your plot by looking up the properties of the metal! But we have only covered the DC Drude model for conductivity in class, where we obtained $\sigma(0) = \frac{nq^2\tau}{m_e}$. Here you need to use Newton's laws again and solve to show the following:

$$qE_0 e^{i\omega t} = m_e \frac{dv}{dt} - \frac{m_e v}{\tau} \implies \boxed{\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} = \underbrace{\frac{\sigma_0}{1 + (\omega\tau)^2}}_{\mathrm{Re}(\sigma(\omega))} + i \underbrace{\frac{\omega\tau\sigma_0}{1 + (\omega\tau)^2}}_{\mathrm{Im}(\sigma(\omega))}}. \tag{4070.1}$$

(c) Now you are close to the finish line. Use the above modified Drude ac conductivity, look up the required properties of the three metals, and plot the reflectances of all the three metals. Compare with figure 4070.1.

Problem 3: Lord of the Ring

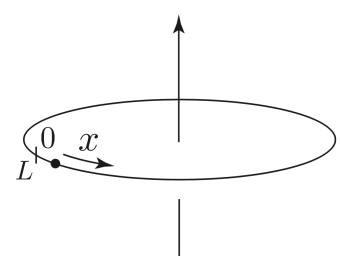

Fig. 4070.2: Electron on a ring

We derived in class that the allowed wavefunctions representing an electron on a circular ring of circumference L is $\psi_n(x) = \frac{1}{\sqrt{L}} e^{ik_n x}$, where $k_n = \frac{2\pi}{L} n$ are *quantized* because $n = 0, \pm 1, \pm 2, ...$ The *angular momentum* of a particle is defined as $\mathcal{L} = \mathbf{r} \times \mathbf{p}$, where $\mathbf{r}$ is the 'radius' of the circle, and $\mathbf{p}$ is the linear momentum.

(a) Show that the angular momentum of an electron in state $\psi_n(x)$ is $\mathcal{L}_n = n\hbar$, where $\hbar = \frac{h}{2\pi}$ is the 'reduced' Planck's constant. This implies that the angular momentum is *quantized* to values $0, \pm\hbar, \pm 2\hbar, ...$ Compare the quantized angular momentum $\mathcal{L}_1$ for $n = +1$ with the classical angular momentum $\mathcal{L}_{cl}$ of a mass $m = 1$ kg being spun by a string of length $R = 1$ m with tangential velocity $v = 1$ m/s to appreciate how 'nano' is the quantum of angular momentum.

(b) By balancing the classical centrifugal force and the electromagnetic Lorentz force, show that for an electron to be in the quantum state $\psi_n(x)$ on the ring, we need a *magnetic* field B_n such that the magnetic flux is $\Phi_n = B_n \cdot A = n \times \frac{h}{2e}$. Here A is the area of the ring, e is the electron charge and $h = 2\pi\hbar$. $\Phi_0 = \frac{h}{2e}$ is known as the *quantum of magnetic flux*, and has been measured experimentally in nanostructured rings.

(c) Consider the quantum state obtained by the superposition $\psi(x) = a[\psi_{n=1}(x) + \psi_{n=-1}(x)]$ from the eigenstates of the electron on the ring. Normalize the state to find the constant a. You may need the result $\int_0^L \cos^2(\frac{2\pi}{L}x)dx = \frac{L}{2}$. Does this superposition state have a definite momentum?

(d) We derived that the quantum expression for current flux is $\mathbf{j} = \frac{1}{2m}(\psi^\star \hat{\mathbf{p}}\psi - \psi\hat{\mathbf{p}}\psi^\star)$, where $\hat{\mathbf{p}} = -i\hbar\nabla$ is the momentum operator, which takes the form $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ for the particle on the ring. Show that even though the states $\psi_{n=1}(x)$ and $\psi_{n=-1}(x)$ carry *net* currents, their superposition state of part (c) does not. Explain.

Problem 4: Born to be free

We discussed in class that because of the Pauli exclusion principle, Fermions must follow the Fermi-Dirac distribution, and they have half-integer spins. Now imagine we have a metal with $n = 10^{23}/\text{cm}^3$ electrons in a cubic box of side L, and we know that electrons are Fermions. Assume the electrons are completely free to move around in the box, meaning there are no atoms in their way. If that that much freedom is not enough for you, how about this: completely neglect the Coulomb interactions due the charge of the electrons![2] Find the following at $T = 0$K:

[2]This may all be very unsettling, but we will explain later why it is actually OK to do so - because with great freedom comes great responsibility! In fact this problem could be formulated for *any* Fermion - for example the uncharged neutron - and the analytical answers will be the same.

(a) The Fermi wavevector k_F.

(b) The Fermi momentum p_F.

(c) The Fermi energy E_F.

(d) The *average* energy of electrons $u = \frac{U}{N}$. What is the origin of this energy?

(e) What is the average energy of the electrons if they did not follow quantum mechanics, but were subject to classical mechanics?

Fig. 4070.3: Andre Geim

Problem 5: Graphene Density of States, Fermi-Dirac distribution

The electrons in the conduction band of graphene are free to move in 2-dimensions, forming a 2-dimensional electron gas (2DEG). The energy-momentum dispersion relationship for the 2DEG electrons in graphene is $E(k_x, k_y) = \hbar v_F\sqrt{k_x^2 + k_y^2}$, where v_F is a parameter with dimensions of velocity. For graphene, it is $v_F = 10^8$cm/s.

Fig. 4070.4: Kotsya Novoselov Geim and Novoselov were awarded the Nobel prize in physics in 2010 for the discovery of graphene, the thinnest 2D crystal with remarkable electron transport properties.

(a) Make a sketch of the energy as a function of the (k_x, k_y) points in the 2D k-space plane, and show that the dispersion results in a conical shape.

(b) Show that the density of states for these electrons is $g(E) = \frac{g_s g_v}{2\pi(\hbar v_F)^2}|E|$, where $g_s = 2$ is the spin degeneracy of each (k_x, k_y) state, and g_v is the number of cones in the energy dispersion. For graphene, $g_v = 2$.

(c) Show that at thermal equilibrium, when the Fermi level is at $E_f = 0$, the number of conduction electrons per unit area in 2D graphene is $n_i = \frac{\pi}{6}(\frac{kT}{\hbar v_F})^2$. Make a plot of this density as a function of temperature for $0K \leq T \leq 500K$. Explain why your plot sets the bar on the lowest possible density of carriers achievable in graphene at those temperatures.