

# Hydrodynamic instability of one-dimensional electron flow in semiconductors

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The hydrodynamic instability of one-dimensional flow of electrons in an ungated semiconductor driven by a voltage difference is studied. The governing transport and electrostatic equations are linearized about the steady flow, and the eigenspectrum of perturbations is calculated. The carrier flow is found to be unstable under certain circumstances through oscillations that manifest themselves as planar waves. Higher voltages lead to greater rates of growths of the instability. The frequencies of oscillation are found to be of the order of terahertz, making the phenomenon suitable for consideration as a solid-state radiative source at this frequency range. © 2007 American Institute of Physics. [DOI: 10.1063/1.2753692]

## I. INTRODUCTION

A number of solid-state devices are being currently explored for filling the so-called “terahertz gap”—both narrow and broadband electromagnetic wave emitters spanning the frequency range from 300 GHz to 3 THz (corresponding to wavelengths from 100  $\mu\text{m}$  to 1 mm). Though non-solid-state sources such as free electron lasers<sup>1</sup> and relativistic electron accelerators<sup>2</sup> exist for this purpose, compact solid-state sources offer the unique advantage of portability and the possibility of integration with integrated circuits (ICs). Optical devices such as quantum-cascade lasers (QCLs) are approaching this gap from the high-frequency regime.<sup>3</sup> However, the operation of QCLs is limited to cryogenic temperatures (since  $\hbar\omega \sim 1 \text{ meV} \ll k_B T$  at room temperature). On the other hand, high-speed electronic transistors are rapidly advancing to fill the terahertz gap from the low-frequency end.<sup>4</sup> The switching speeds of ultrascaled transistors, however, are limited by transit time effects and parasitics. Switching speed in such transistors is approaching the 1 THz mark, indicating that a room-temperature all-electronic terahertz emitter is possible in the near future. Compact, tunable, and physically small terahertz sources are needed for the detection of chemical reactions, the characterization of chemical and biological systems, imaging, and a host of other yet-unrealized applications.

In 1993, Dyakonov and Shur proposed that the current flow in a two-dimensional electron gas (2DEG) channel in traditional high-electron mobility transistor (HEMT) can become inherently unstable under certain bias conditions.<sup>5</sup> This instability stems from the fluidlike correlated flow of electrons in the 2DEG, which can result in the formation of plasma waves, much alike the formation of waves on the surface of shallow water. Using Shockley’s gradual channel

approximation for the relationship between the gate potential and the local channel charge, they showed that plasma waves could be generated. A close analogy of this hydrodynamic model with shallow-water equations was obtained.<sup>6,7</sup> Subsequently, they have also shown that current instabilities and plasma waves can be generated in an *ungated* two-dimensional electron layer.<sup>8</sup> Transit time effects in plasma instability related to the electron drift across the high field region in HEMTs have also been studied.<sup>9</sup> Plasma oscillations in both gated two-dimensional layers and strips in HEMTs have been recently analyzed.<sup>10,11</sup>

The proposal for the all-electronic plasma-wave terahertz emitter has inspired a number of experimental efforts. Subterahertz and terahertz radiation in silicon field-effect transistors (FETs) and deep submicron gate length HEMTs due to plasma waves have recently been claimed to be observed.<sup>12,13</sup> Theoretical and experimental studies have showed the nonresonant and resonant detection of terahertz radiation in both Si metal-oxide-semiconductor FETs (MOSFETs) and gated two-dimensional structures such as GaAs HEMT.<sup>14,15</sup> Under special conditions, a nanoscale FET made of InGaAs/InAlAs can also produce terahertz emission.<sup>16,17</sup> Techniques and algorithms to determine the radiation spectrum of terahertz sources have been analyzed.<sup>18</sup>

Thus, a large body of theoretical work has been done, primarily based on the solutions of the hydrodynamic transport equations of field-effect transistors. The problems that have received most attention are those for which the electrostatic part (governed by Poisson’s equation) relating the charge to the electrostatic potential yields an analytically tractable solution. For example, in Ref. 5, the capacitor approximation greatly simplifies the system of transport equations by relating the charge in the channel linearly to the gate voltage. However, the complete solution of the hydrodynamic transport equations has not been analyzed analytically for a general charge distribution. The purpose of this work is to present an analytical formalism for the treatment of insta-

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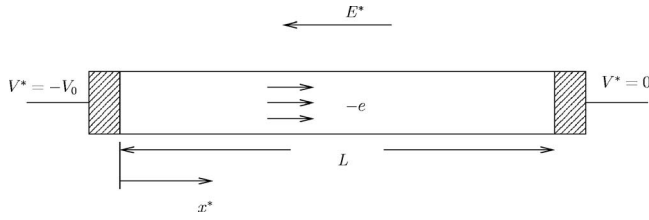


FIG. 1. Schematic of semiconductor material.

bilities in one-dimensional current flow in semiconductors. The formalism makes use of a perturbative treatment of the hydrodynamic transport equations, and yields a broadband spectrum of collective plasma excitations, whose frequencies are given analytically. Various boundary conditions can be easily incorporated, and the formalism should prove useful for the analysis of instability in collisionless hydrodynamic transport in semiconductors.

## II. MATHEMATICAL MODEL

We consider a doped semiconductor of one-dimensional geometry as shown in Fig. 1. Current is driven through the device by a voltage difference between the two contacts at  $x=0, L$ . The coordinate  $x$  is in the direction of the electron flow, and  $t$  is time. In this work, we assume the simplest case of instabilities, where all the transverse fluctuations in the  $(y, z)$  plane are neglected, similar to the plane-wave approximation in the propagation of electromagnetic waves. In bulk semiconductors, this forms a subspace of collective excitations, whereas for charge flow in quantum wires, fluctuations in the transverse direction are not allowed. As in previous analyses, we will also neglect the generation of heat due to interaction between electrons and phonons. This can be experimentally realized if the transport in the semiconductor is ballistic, and the voltage drops occur only at the contacts.

The one-dimensional hydrodynamic equations for electron flow are<sup>19</sup>

$$\frac{\partial V^*}{\partial x^*} + E^* = 0, \quad (1a)$$

$$\frac{\partial E^*}{\partial x^*} + \frac{e}{\epsilon_s}(n^* - N_D) = 0, \quad (1b)$$

$$\frac{\partial n^*}{\partial t^*} + \frac{\partial(v^* n^*)}{\partial x^*} = 0, \quad (1c)$$

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial x^*} + \frac{e}{m} E^* + \frac{v^*}{\tau} = 0, \quad (1d)$$

where  $V^*(x^*, t^*)$  is the voltage,  $E^*(x^*, t^*)$  is the electric field,  $n^*(x^*, t^*)$  is the charge density, and  $v^*(x^*, t^*)$  is the average electron velocity. The system parameters are the doping concentration  $N_D$ , the permittivity  $\epsilon_s$ , the charge of an electron  $e$ , its effective mass  $m$ , and the momentum relaxation time  $\tau$ . Here, the Poisson equation [Eq. (1b)], the continuity equation [Eq. (1c)], and the momentum conservation equation [Eq. (1d)] are effectively decoupled from the energy conservation equations by choosing the length of the semiconduc-

tor layer to be less than a mean free path of electrons (i.e., weak elastic scattering in the ballistic regime).

The voltage at both ends and the charge density at  $x^* = 0$  are prescribed boundary conditions. The physical realization is an Ohmic contact at  $x=0$ , which does not allow any perturbations in the majority carrier density due to an infinitely high surface recombination velocity. The carrier density at the right contact allowed perturbations, but across the whole semiconductor layer, charge neutrality is enforced at all times. This form of a contact can be achieved, for example, by an inductive boundary condition, as in Ref. 5. Charge neutrality of the semiconductor layer also implies the absence of space charge injection effects. Then, we must have  $\int_0^L n^*(x^*, t^*) dx^* = N_D L$ , where  $L$  is the length of the device, which can be put in terms of the electric field  $E$  using Eq. (1b).

Thus the boundary conditions are

$$V^*(0, t^*) = -V_0, \quad V(L, t^*) = 0,$$

$$n^*(0, t^*) = n_0, \quad E^*(0, t^*) = E^*(L, t^*).$$

For convenience, the hydrodynamic transport equations are nondimensionalized. Writing  $V = V^*/V_0$ ,  $n = n^*/N_D$ ,  $x = x^*/L$ ,  $v = v^* \sqrt{m/eV_0}$ , and  $t = t^* \sqrt{eV_0/mL^2}$ , the dimensionless versions of Eq. (1) are

$$\frac{\partial V}{\partial x} + E = 0, \quad (2a)$$

$$\frac{\partial E}{\partial x} + \alpha(n - 1) = 0, \quad (2b)$$

$$\frac{\partial n}{\partial t} + \frac{\partial(vn)}{\partial x} = 0, \quad (2c)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + E + \frac{\sqrt{\alpha}}{\gamma} v = 0, \quad (2d)$$

where the dimensionless parametric groups are  $\alpha = eN_D L^2 / V_0 \epsilon_s$  and  $\gamma = \sqrt{\tau^2 e^2 N_D / \epsilon_s m} = \omega_p \tau$ , with  $\omega_p = \sqrt{e^2 N_D / \epsilon_s m}$  being identified as the fundamental plasma frequency of the free electrons in the semiconductor. The dimensionless parameter  $\gamma$  can be expected to play a major role in any phenomena related to collective excitations of the electron gas, as will be seen later. The only electrically tunable parameter is  $\alpha$ , which is inversely proportional to the applied bias  $V_0$ . The dimensionless boundary conditions are

$$V(0, t) = -1, \quad V(1, t) = 0, \quad n(0, t) = \beta, \quad E(0, t) = E(1, t),$$

where  $\beta = n_0 / N_D$ ,  $n_0$  being the free electron density at the left contact. By choosing an Ohmic contact, we clamp the value to  $n_0 = N_D$ , and  $\beta = 1$  at all times.

## III. ONE-DIMENSIONAL ELECTRON FLOW

The steady state solutions of Eq. (2) that satisfies the boundary conditions are

$$\bar{E}(x) = -1, \quad \bar{V}(x) = x - 1, \quad \bar{n}(x) = 1, \quad \bar{v}(x) = \frac{\gamma}{\sqrt{\alpha}}.$$

where  $(\bar{\quad})$  indicates time independence. The steady state solution captures the drift flow, with the dimensional form of the electron concentration and velocity given by  $n^*(x) = N_D$ , and  $v^*(x) = (e\tau/m)(V_0/L)$  for all  $x$ . Any instabilities that exist are due to time-dependent fluctuations to this steady state.

Applying small perturbations of the form

$$E = \bar{E} + E'(x, t), \quad V = \bar{V} + V'(x, t), \quad (3)$$

$$n = \bar{n} + n'(x, t), \quad v = \bar{v} + v'(x, t),$$

substituting in Eq. (2) and linearizing, we get

$$\frac{\partial V'}{\partial x} + E' = 0, \quad (4a)$$

$$\frac{\partial E'}{\partial x} + \alpha n' = 0, \quad (4b)$$

$$\frac{\partial n'}{\partial t} + \frac{\gamma}{\sqrt{\alpha}} \frac{\partial n'}{\partial x} + \frac{\partial v'}{\partial x} = 0, \quad (4c)$$

$$\frac{\partial v'}{\partial t} + \frac{\gamma}{\sqrt{\alpha}} \frac{\partial v'}{\partial x} + E' + \frac{\sqrt{\alpha}}{\gamma} v' = 0. \quad (4d)$$

The boundary conditions on the perturbations are

$$V'(0, t) = 0, \quad V'(1, t) = 0, \quad n'(0, t) = 0, \quad E'(0, t) = E'(1, t). \quad (5)$$

Using normal modes of the form

$$\begin{Bmatrix} V' \\ E' \\ n' \\ v' \end{Bmatrix} = \begin{Bmatrix} \hat{V} \\ \hat{E} \\ \hat{n} \\ \hat{v} \end{Bmatrix} e^{kx + \omega t},$$

where the wavenumber  $k$ , the frequency  $\omega$ , and the amplitudes, denoted by  $\hat{\quad}$ , are all complex, Eq. (4) becomes

$$\begin{bmatrix} k & 1 & 0 & 0 \\ 0 & k & \alpha & 0 \\ 0 & 0 & \omega + k\gamma/\sqrt{\alpha} & k \\ 0 & 1 & 0 & \omega + k\gamma/\sqrt{\alpha} + \sqrt{\alpha}/\gamma \end{bmatrix} \begin{Bmatrix} \hat{V} \\ \hat{E} \\ \hat{n} \\ \hat{v} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$

For a nontrivial solution, the determinant should vanish, so that the characteristic polynomial is given by

$$-k \left[ k \left( \omega + \frac{\gamma}{\sqrt{\alpha}} k \right) \left( \omega + \frac{\gamma}{\sqrt{\alpha}} k + \frac{\sqrt{\alpha}}{\gamma} \right) + \alpha k \right] = 0. \quad (6)$$

A critical value of the electron mobility defines two distinct phases of the collective excitations possible in the electron system. This critical mobility is defined by

$$\mu_{cr} = \frac{1}{2} \sqrt{\frac{\epsilon_s}{mN_D}}. \quad (7)$$

It is equivalent to satisfy the critical condition  $\tau = 1/2\omega_p$ . If the mobility of electrons is higher than the critical value, the condition  $4\gamma^2 > 1$  is obtained, and vice versa. For most semiconductors, this critical mobility is in the range of  $1-10 \text{ cm}^2/\text{V s}$ , and therefore the condition  $4\gamma^2 > 1$  is achievable in reasonably clean semiconductors doped to a nominal level  $N_D \sim 10^{17} \text{ cm}^{-3}$ .

### A. For $4\gamma^2 > 1$

The four roots of Eq. (6) are  $k_{1,2} = 0$  and  $k_{3,4} = a \pm ib$ , where

$$a = -\frac{\alpha}{2\gamma^2} \left( \frac{2\gamma\omega}{\sqrt{\alpha}} + 1 \right), \quad (8a)$$

$$b = \frac{\alpha}{2\gamma^2} \sqrt{4\gamma^2 - 1}. \quad (8b)$$

Then we can write the amplitudes of the perturbations as

$$V' = [A + Bx + Ce^{(a+ib)x} + De^{(a-ib)x}] e^{\omega t}, \quad (9a)$$

$$E' = [-B - C(a+ib)e^{(a+ib)x} - D(a-ib)e^{(a-ib)x}] e^{\omega t}, \quad (9b)$$

$$n' = \left[ \frac{1}{\alpha} (C(a+ib)^2 e^{(a+ib)x} + D(a-ib)^2 e^{(a-ib)x}) \right] e^{\omega t}, \quad (9c)$$

$$v' = \frac{\gamma}{\omega\gamma + \sqrt{\alpha}} \left[ (a+ib)Ce^{(a+ib)x} \left\{ \frac{\gamma\omega(a+ib)}{\alpha^{3/2}} + \frac{\gamma^2(a+ib)^2}{\alpha^2} + 1 \right\} + (a-ib)De^{(a-ib)x} \left\{ \frac{\gamma\omega(a-ib)}{\alpha^{3/2}} + \frac{\gamma^2(a-ib)^2}{\alpha^2} + 1 \right\} + B \right] e^{\omega t}. \quad (9d)$$

The boundary conditions [Eq. (5)] can be written as

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & e^{a+ib} & e^{a-ib} \\ 0 & 0 & (a+ib)^2 & (a-ib)^2 \\ 0 & 0 & (a+ib)(1-e^{a+ib}) & (a-ib)(1-e^{a-ib}) \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$

For a nontrivial solution the determinant must vanish, which gives

$$(a+ib)(1-e^{a-ib}) - (a-ib)(1-e^{a+ib}) = 0,$$

from which the condition

$$e^{a-b \cot b} (a - b \cot b) = -\frac{be^{-b \cot b}}{\sin b}$$

is obtained. This can be written as

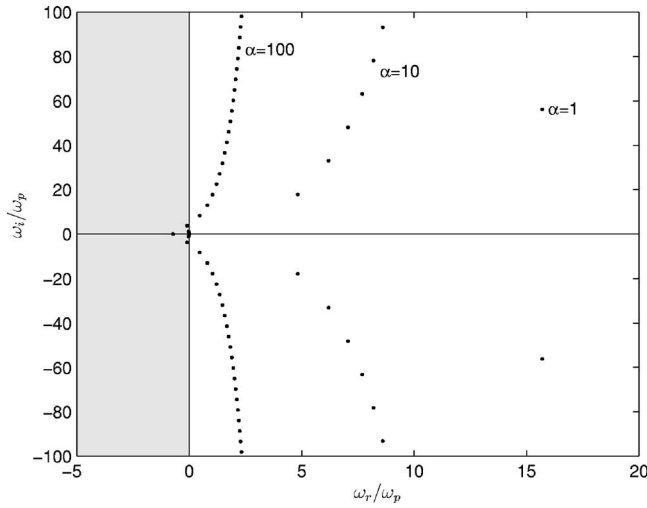


FIG. 2.  $\omega$  spectra for  $\beta=1$  and  $\gamma=20$ .

$$W\left(-\frac{be^{-b \cot b}}{\sin b}\right) = a - b \cot b.$$

where the Lambert  $W$  function is defined as the solution of  $W(z)e^{W(z)}=z$ .<sup>20,21</sup> Finally, substituting  $a$  from Eq. (8a), we get

$$\omega = -\frac{\gamma}{\sqrt{\alpha}} \left[ \frac{\alpha}{2\gamma^2} + b \cot b + W\left(-\frac{be^{-b \cot b}}{\sin b}\right) \right]. \quad (10)$$

$W(z)$  has an infinite number of complex values even for real  $z$  (as it is here), so that  $\omega$  also has an infinite number of

values corresponding to the branches of the function. We can split the complex frequency into its real and imaginary parts as  $\omega = \omega_r + i\omega_i$ , and then normalize them using  $\omega_p = \sqrt{\alpha}$  (a physical meaning for  $\omega_p$  is given in Sec. IV B). Positive  $\omega_r$  represents unstable, growing exponentials in time in Eq. (9), whereas  $\omega_i$  is the frequency of oscillation. A portion of the  $\omega$  spectra for three different values of  $\alpha$  is shown in Fig. 2. The complex frequencies shift to the left and the oscillations become more stable as  $\alpha$  is increased; this corresponds to a decrease in the applied voltage. The distribution of electron density perturbation  $n'(x, t)$  for  $\alpha=10$  and for an unstable eigenvalue at different instants in time within the first period of oscillation is shown in Fig. 3. The behavior is the same to that of a wave that is growing in time. The corresponding unstable behavior of the small signal voltage  $V'(x, t)$ , electric field  $E'(x, t)$ , electron density  $n'(x, t)$ , and velocity  $v'(x, t)$  at  $x=0.5$  is shown in Fig. 4; again the variables are seen to be growing in time.

Thus the hydrodynamic instability of electron flow in an ungated doped bulk semiconductor exhibits a broadband spectrum, which can be tuned from the damped stable oscillations (for  $\omega_r < 0$ ) to growing, unstable ones (for  $\omega_r > 0$ ) by increasing the applied bias. These collective oscillations of the electron density will lead to broadband radiation at those frequencies. With appropriate cavity design, the energy in selected modes can be enhanced at the expense of others, similar to what is done in Ref. 5.

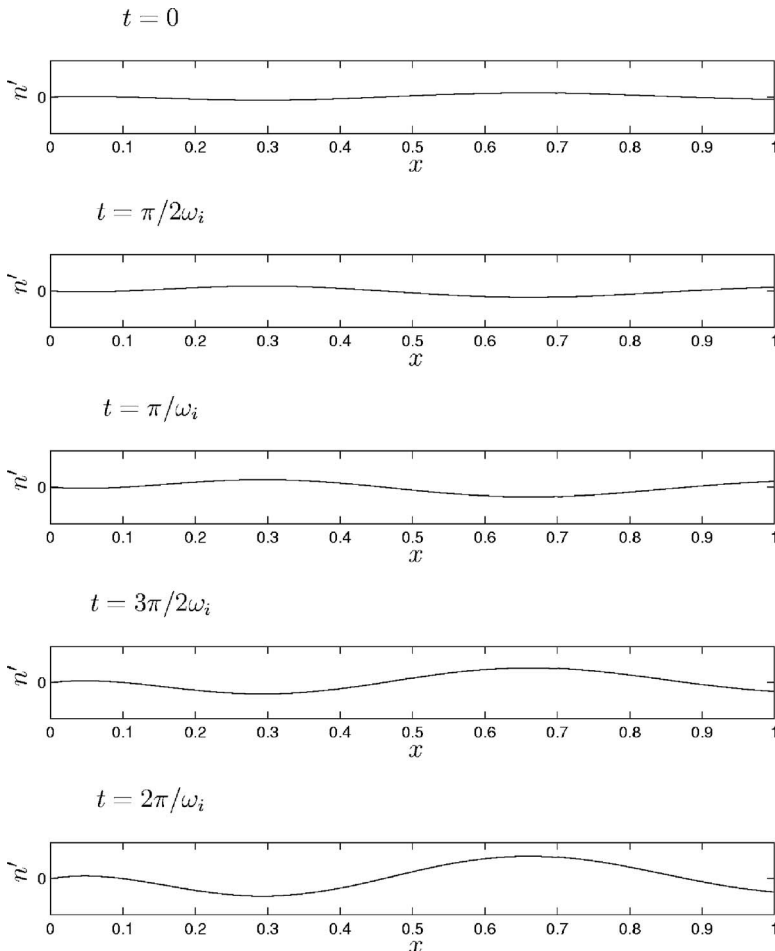


FIG. 3. Spatial distribution of electron density  $n'(x, t)$  in arbitrary scale for  $\alpha=10$ ,  $\beta=1$ , and  $\gamma=20$  at  $\omega_i \sim 20\omega_p$ .

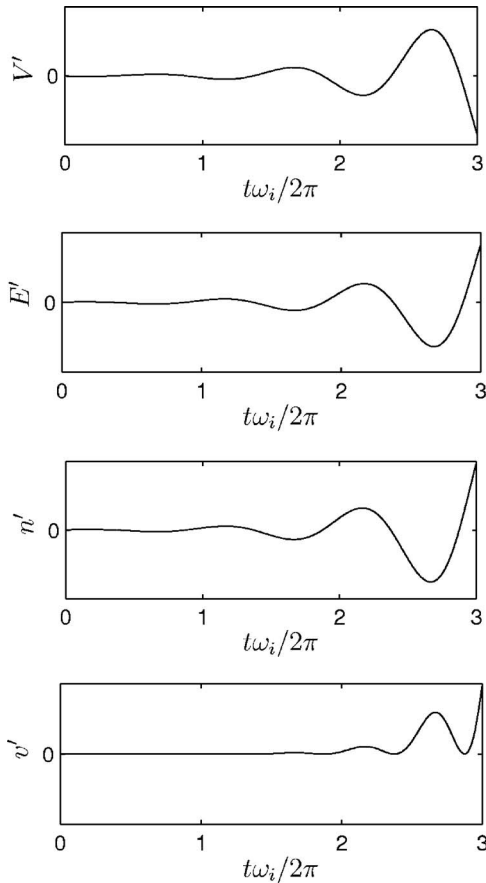


FIG. 4. Time variation of voltage  $V'(x,t)$ , electric field  $E'(x,t)$ , electron density  $n'(x,t)$ , and electron velocity  $v'(x,t)$  in arbitrary scale at  $x=0.5$  for  $\alpha=10$ ,  $\beta=1$ , and  $\gamma=20$  with  $\omega_i \sim 20\omega_p$ .

The low-mobility cases exhibit similar behavior, the major difference being that higher voltages are necessary to achieve undamped plasma oscillations.

**B. For  $4\gamma^2=1$**

Equation (6) now has two pairs of repeated roots. Instead of Eq. (10), we have

$$\omega = -\frac{\gamma}{\sqrt{\alpha}} \left[ \frac{\alpha}{2\gamma^2} + 1 + W(-e^{-1}) \right].$$

The  $\omega$  spectra for three different values of  $\alpha$  are shown in Fig. 5. Again, the tendency is to move toward greater stability as  $\alpha$  is increased. For  $\alpha=1$ , the spatial distribution of the electron density  $n'(x,t)$  at an unstable eigenvalue and its evolution in time within the first period of oscillation is shown in Fig. 6. It describes a wave that is growing in time. However, as is clear from a comparison of Figs. 5 and 2, the hydrodynamic instability is damped strongly, and larger voltages are now necessary to achieve growing unstable oscillations with  $\omega_r > 0$ .

**C. For  $4\gamma^2 < 1$**

The roots of Eq. (6) are now all real. The analysis of Sec. III A still holds as long as it is remembered that  $b$  is imaginary. On writing  $b=ib'$ , where  $b'=\alpha\sqrt{1-4\gamma^2}/2\gamma^2$  is now real, Eq. (10) becomes

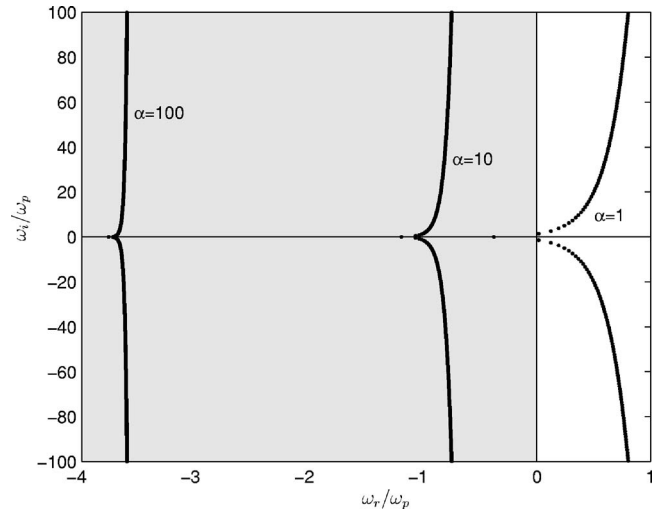


FIG. 5.  $\omega$  spectra for  $\beta=1$  and  $\gamma=1/2$ .

$$\omega = -\frac{\gamma}{\sqrt{\alpha}} \left[ \frac{\alpha}{2\gamma^2} + b' \coth b' + W\left(-\frac{b' e^{-b' \coth b'}}{\sinh b'}\right) \right].$$

The complex values of  $\omega$  for three different values of  $\alpha$  are shown in Fig. 7. As before, stability is increased on increasing  $\alpha$ . For  $\alpha=1$ , the spatial distribution of the electron density  $n'(x,t)$  at an unstable eigenvalue, and its evolution in time within the first period of oscillation, is shown in Fig. 8. A growing wavelike behavior is also observed here.

**D. Phase and group velocities**

The constant  $b$  in Eq. (8b) is real, but  $a$  in (8a) is complex, so that

$$a = -\frac{\alpha}{2\gamma^2} \left[ \left( \frac{2\gamma\omega_r}{\sqrt{\alpha}} + 1 \right) + i \frac{2\gamma\omega_i}{\sqrt{\alpha}} \right],$$

where  $\omega_i$  is the frequency of oscillation of the wave. The wave number  $k=a+ib$  can also be separated into its real and imaginary parts as

$$k = -\frac{\alpha}{2\gamma^2} \left[ \left( \frac{2\gamma\omega_r}{\sqrt{\alpha}} + 1 \right) + i \left( \frac{2\gamma\omega_i}{\sqrt{\alpha}} + \sqrt{4\gamma^2 - 1} \right) \right],$$

where the imaginary part is the wave number of the spatial oscillations. From this dispersion relation, the phase and group velocities can be deduced to be

$$v_p = \frac{\gamma}{\sqrt{\alpha}} \frac{2\gamma\omega_i}{2\gamma\omega_i + \sqrt{\alpha(4\gamma^2 - 1)}},$$

$$v_g = \frac{\gamma}{\sqrt{\alpha}},$$

respectively. Waves running in both  $+x$  and  $-x$  directions are possible. Since the magnitude of the group velocity of the plasma excitation is found to be exactly the same as the steady state electron velocity, it indicates that the fluctuation rides on the steady state velocity, and in a stationary rest frame, the effective plasma velocity is  $v_g \pm \bar{v}=0$  and  $2\gamma/\sqrt{\alpha}$ ,



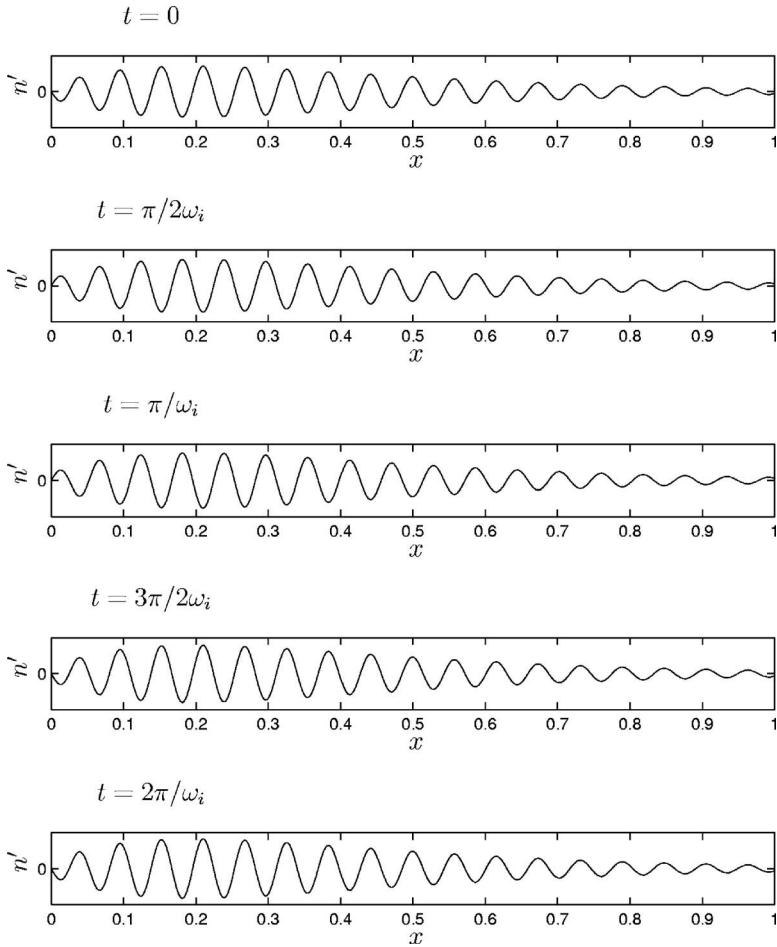


FIG. 6. Spatial distribution of electron density  $n'(x,t)$  in arbitrary scale for  $\alpha=1$ ,  $\beta=1$ , and  $\gamma=1/2$  at  $\omega_i \sim 20\omega_p$ .

meaning the “right going” plasma wave is twice as fast as the steady state flow, and there are no left going waves.

**IV. SPECIAL CASES**

**A. Asymptotic approximation**

The Lambert  $W$  function can be approximated for values of  $\omega$  around the origin such that the ratio between the real and imaginary components is much less than 1. In this case, Eq. (10) becomes

$$\omega = -\frac{\gamma}{\sqrt{\alpha}} \left[ \frac{\alpha}{2\gamma^2} + b \cot b + \log \left( -\frac{be^{-b \cot b}}{y_n \sin b} \right) + iy_n \right],$$

where  $y_n = \pm(\pi/2)(4n \pm 1)$  (keeping the argument of the log function positive) and  $n=1, 2, 3, \dots$

**B. Zero electron flow**

A possible dimensionless steady state solution of Eq. (2) is  $\bar{E}=0$ ,  $\bar{V}=\text{const}$ ,  $\bar{v}=0$ , and  $\bar{n}=1$ . For small perturbations, as in Eq. (3), we have upon linearization

$$\begin{aligned} \frac{\partial V'}{\partial x} + E' &= 0, \\ \frac{\partial E'}{\partial x} + \alpha n' &= 0, \\ \frac{\partial n'}{\partial t} + \frac{\partial v'}{\partial x} &= 0, \\ \frac{\partial v'}{\partial t} + E' + \frac{\sqrt{\alpha}}{\gamma} v' &= 0. \end{aligned}$$

It is possible to reduce the equations to a single fourth-order equation,

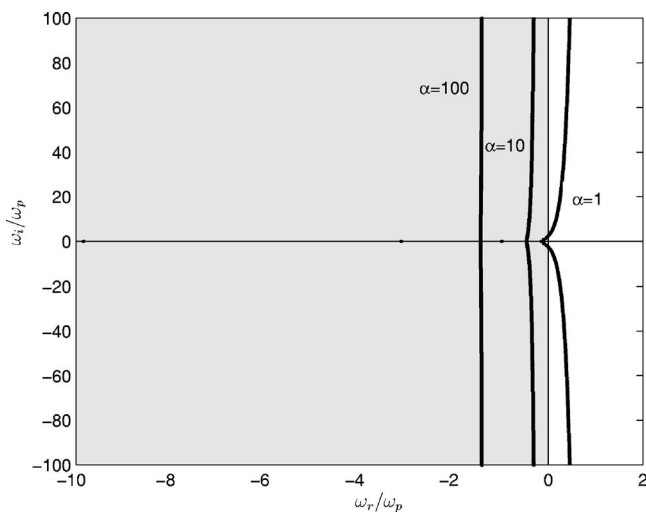


FIG. 7.  $\omega$  spectra for  $\beta=1$  and  $\gamma=1/3$ .

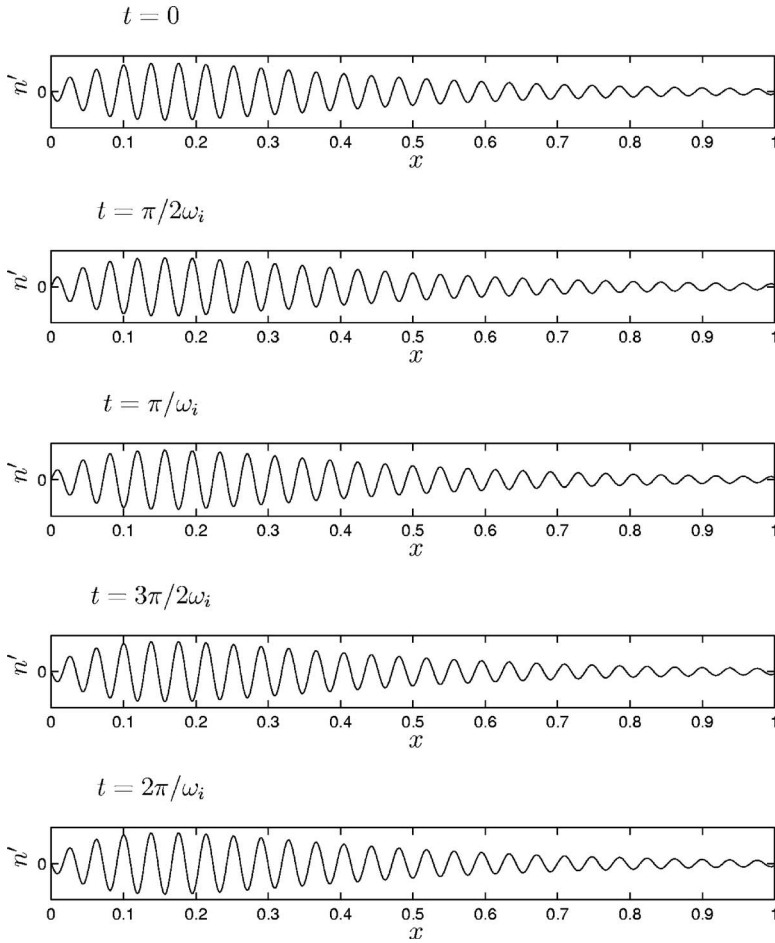


FIG. 8. Spatial distribution of electron density  $n'(x,t)$  in arbitrary scale for  $\alpha=1$ ,  $\beta=1$ , and  $\gamma=1/3$  at  $\omega_i \sim 20\omega_p$ .

$$\frac{\partial^4 V'}{\partial t^2 \partial x^2} + \frac{\sqrt{\alpha}}{\gamma} \frac{\partial^3 V'}{\partial t \partial x^2} + \alpha \frac{\partial^2 V'}{\partial x^2} = 0.$$

This can be solved by writing it first as

$$\frac{\partial^2}{\partial x^2} \left[ \frac{\partial^2 V'}{\partial t^2} + \frac{\sqrt{\alpha}}{\gamma} \frac{\partial V'}{\partial t} + \alpha V' \right] = 0,$$

and then integrating twice with respect to  $x$  to get

$$\frac{\partial^2 V'}{\partial t^2} + \frac{\sqrt{\alpha}}{\gamma} \frac{\partial V'}{\partial t} + \alpha V' = A(t)x + B(t).$$

The homogeneous part of the temporal solution is obtained as usual by setting  $V' \sim e^{rt}$ , where

$$r = \frac{\sqrt{\alpha}}{2\gamma} [-1 \pm \sqrt{1 - 4\gamma^2}].$$

Damped oscillatory solutions are possible if  $4\gamma^2 > 1$ . The solution then takes the form

$$V'(x,t) = e^{-t\sqrt{\alpha/2}\gamma} [C(x)\sin \omega t + D(x)\cos \omega t] + g(x,t),$$

where

$$\omega = \frac{\sqrt{\alpha}}{2\gamma} \sqrt{4\gamma^2 - 1},$$

and  $g$  is the particular solution corresponding to  $A(t)x + B(t)$ . If the momentum relaxation time  $\tau \rightarrow \infty$ , then  $\gamma \rightarrow \infty$  also, so

that damped oscillations at the plasma frequency  $\omega_p = \sqrt{\alpha}$  may occur.

## V. CONCLUSIONS

In summary, a perturbative formalism for studying instabilities in the hydrodynamic model of carrier flow in semiconductors is presented. The treatment allows an analytic evaluation of plasma oscillation modes in a generic semiconductor. The eigenvalue spectrum indicates that there is an unstable region which depends strongly on the applied voltage; the spectrum becomes more stable as the applied voltage decreases. Considering GaAs with an electron effective mass  $m=6.6\%$  of the free electron mass,  $\epsilon_s=12.5\epsilon_0$ ,  $L=100$  nm,  $N_D=5 \times 10^{17}$  cm $^{-3}$ ,  $\tau=0.4$  ps, and  $V_0=1$  V gives  $\alpha=7.072$  and  $\gamma=17.365$ . The corresponding plasma frequency is  $\omega_p = \sqrt{e^2 N_D / \epsilon_s m} = 6.9$  THz. Eigenvalues around the origin with positive real parts thus have frequencies of the order of terahertz, so that the ungated semiconductor could be a possible source of radiation in this frequency range. The major result is a broadband spectrum of eigenvalues tunable by the applied voltage due to the instabilities, and these frequencies can be analytically evaluated for general boundary conditions using the perturbative formalism presented. The formalism presented should prove useful in the analysis and design of plasma-wave instability based terahertz sources in the future.

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