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Hydrodynamic instability of confined two-dimensional electron flow in semiconductors

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Hydrodynamic instabilities in two-dimensional electron flow in ungated semiconductors are studied here. The driving force for the electrons is an imposed voltage difference that generates a unidimensional electric field inside the semiconductor and its surroundings. The governing equations are linearized for small perturbations around the steady-flow solution. The eigenvalue spectrum determining the rates of growth and wave numbers of the oscillations is calculated. The electron flow undergoes oscillatory instability and becomes more unstable as the voltage difference is increased. The results show that it is possible to obtain oscillation frequencies of the order of terahertz, indicating the possibility of radiative power at this frequency. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3158551]

I. INTRODUCTION

Terahertz radiation sources are of current interest due to their advantages in leading edge applications such as bioimaging and sensing.[1,2] Features such as compactness and tunability are required in a terahertz source for applications in chemical and biological systems, imaging, and radio astronomy, among others. Semiconductors are among the possible candidates as potential sources. A necessary step for this application is a fundamental understanding and control of charged-particle interactions and dynamics in solid state devices. This motivated both theoretical and experimental studies of electron flow in semiconductors and it has been found that instabilities can produce oscillations even when the imposed electric field is steady.

Electrons in semiconductors scatter as a result of collisions between themselves and with the lattice and impurities. The most common theoretical approach for the analysis of electron flow in semiconductors is to neglect energy transfer between electrons and the lattice, for which hydrodynamic models provide a useful description. This has been an important perspective to explain physical phenomena involved in electron transport in semiconductors and several inroads have been made in this direction. Dyakonov and Shur[3] found analogies with shallow water equations that predict plasma oscillations at terahertz frequencies and radiation emissions in ballistic transport in an AlGaAs/InGaAs field effect transistor (FET). Subsequently, this description was generalized and applied to high electron mobility transistors (HEMT).[4–6] The mechanism of current saturation in a FET due to choking of electron flow and plasma waves was seen to show similarities to shallow water phenomena in fluid dynamics. Under this idea, nonlinear oscillations due to ballistic transport in FETs and effects similar to hydraulic jumps were also described.[7] Nonlinear dynamic response and how the boundary conditions influence the nonlinear effects were studied in two-dimensional electron plasmas in FETs by using the hydrodynamic model.[8] It was found that current and plasma waves in an ungated two-dimensional electron layer may present instability similar to that for a gated electron layer.[9] Transit-time effects in plasma instabilities were related to the electron drift across the high field region in HEMTs.[10] Microscopically bounded plasma due to current-driven plasma instability has been reported in lower-dimensional solid-state systems.[11] Plasma oscillations were analyzed in both gated two-dimensional layers and HEMTs.[12,13] Instabilities in multilayered semiconductor structures have been studied numerically and theoretically.[14] Drift wave instabilities have been found in semiconductor electron-hole plasmas.[15]

Experiments have also been performed to detect and understand the mechanisms of terahertz radiation. Subterahertz and terahertz radiation have been found in silicon FETs and nanometer-scale gate-length HEMTs due to plasma waves.[16,17] A better understanding of the strengths and limitations of experimental techniques, for instance in the two-color diode laser,[18] also helped in the study of terahertz radiation. In addition, new techniques and algorithms to determine the radiation spectrum of terahertz sources have been analyzed recently.[19] Experimental and theoretical studies have shown nonresonant and resonant detection of terahertz radiation in both Si metal oxide semiconductor field effect transistors (MOSFETs) and gated two-dimensional structures such as GaAs HEMTs.[20,21] Under particular conditions, a nanoscale FET made of InGaAs/InAlAs can produce terahertz emission.[22,23]

Calderón-Muñoz *et al.*[24] determined analytically the spatial and time dependent instabilities in one-dimensional electron flow in ungated semiconductors. In this earlier work, on assuming plane-wave propagation of electrons between the contacts, the instabilities that can arise perpendicular to the direction of particle flow were neglected. From analogy to fluid dynamics, such instabilities are likely to be present, and

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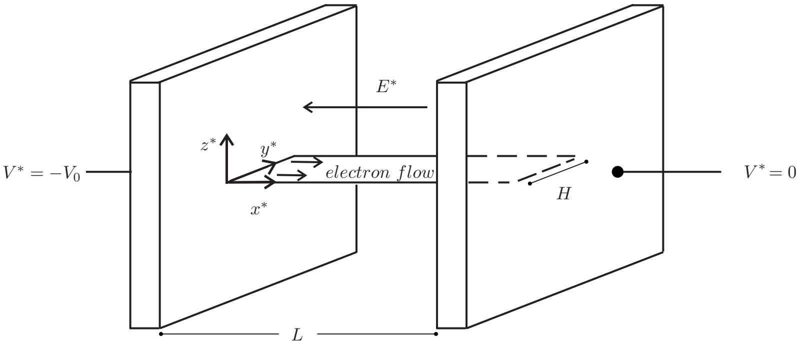

FIG. 1. Schematic of semiconductor material.

their study is the goal of this work. Analysis of instabilities in semiconductors based on a two-dimensional model, which may be very useful for the study of potential terahertz sources, has not been done. The two-dimensional geometry can be implemented experimentally due to the simplicity of the configuration and the boundary conditions can be prescribed by specifying the kind of contacts at the source and at the drain. In this paper we analyze the electrostatic and hydrodynamic equations in a high FET and characterize the instabilities present in them.

II. MATHEMATICAL MODEL

The problem is defined by a doped two-dimensional semiconductor as shown in Fig. 1. The semiconductor has a length L and width H and represents the (x^*,y^*) plane at $z^*=0$. The two contacts at $x^*=0$ and $x^*=L$ are larger than the semiconductor width.

The driving force through the device is given by an electric field due to a voltage difference between the two contacts at $x^*=0$ and $x^*=L$. The electric field surrounds the device due to the size of the contacts. x^*, y^*, and z^* are the Cartesian coordinates and t^* is the time. The electric field can have components in the three Cartesian coordinates. The imposed electric field in the x^*-direction implies that the (y^*,z^*) plane describes an equipotential area. We are interested in capturing instabilities in the (x^*,y^*) plane, neglecting variations in the direction normal to that plane. Electron-lattice interactions are neglected, which implies that the heat generation sources due to electron transport in the semiconductor are not taken into account. Under these assumptions, the two-dimensional hydrodynamic equations for electron flow in the semiconductor are[25]

$$\frac{\partial^2 V^*}{\partial x^{*2}} + \frac{\partial^2 V^*}{\partial y^{*2}} + \frac{\partial^2 V^*}{\partial z^{*2}} - \frac{e}{\epsilon_s}(n^* - N_D) = 0, \tag{1a}$$

$$\frac{\partial n^*}{\partial t^*} + \frac{\partial (u^* n^*)}{\partial x^*} + \frac{\partial (v^* n^*)}{\partial y^*} = 0, \tag{1b}$$

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} - \frac{e}{m_e}\frac{\partial V^*}{\partial x^*} + \frac{u^*}{\tau} = 0, \tag{1c}$$

$$\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} - \frac{e}{m_e}\frac{\partial V^*}{\partial y^*} + \frac{v^*}{\tau} = 0, \tag{1d}$$

where $V^*(x^*,y^*,z^*,t^*)$ is the voltage, $n^*(x^*,y^*,t^*)$ is the charge density, and $u^*(x^*,y^*,t^*)$ and $v^*(x^*,y^*,t^*)$ are the drift electron velocities in the x^* and y^* directions, respectively. The system of Eq. (1) includes Gauss' law Eq. (1a), the continuity equation Eq. (1b), and the momentum conservation equations in the x^* and y^* directions, Eqs. (1c) and (1d), respectively. The system parameters are the doping concentration N_D, the permittivity ϵ_s, the charge of an electron e, its effective mass m_e, and the momentum relaxation time τ. The boundary conditions establish a voltage gradient along the x^*-direction, a fixed charge density at $x^*=0$, a constant charge in the semiconductor as a whole, and charge reflexion along the edges $y^*=0$ and $y^*=H$. This is made possible by imposing an Ohmic contact between the semiconductor and the metal at $x^*=0$ and an inductive boundary condition at $x^*=L$. The Ohmic contact does not allow fluctuations in the majority carrier density, i.e., electrons, due to the infinitely high surface recombination velocity, whereas an inductive boundary condition does. These forms of contacts allow prescription of the voltage at both ends, the charge density at the source, and charge neutrality of the semiconductor layer. This last condition is enforced at all times and implies the absence of space charge injection effects.[3,24] Then, the boundary and charge neutrality conditions are

$$V^*(0,y^*,z^*,t^*) = -V_0, \quad V^*(L,y^*,z^*,t^*) = 0, \quad n^*(0,y^*,t^*) = n_0, \tag{2a}$$

$$v^*(x^*,0,t^*) = 0, \quad v^*(x^*,H,t^*) = 0, \tag{2b}$$

$$\frac{\partial V^*}{\partial y^*}(x^*,0,0,t^*) = 0, \quad \frac{\partial V^*}{\partial y^*}(x^*,H,0,t^*) = 0,$$

$$\int_0^L \int_0^H n^*(x^*,y^*,t^*)dx^* dy^* = N_D L H. \tag{2c}$$

For convenience, the governing equations can be nondimensionalized. Defining the aspect ratio as $R=H/L$ and writing $V=V^*/V_0$, $n=n^*/N_D$, $x=x^*/L$, $y=y^*/H$, $z=z^*/H$, $u=u^*\sqrt{m_e/eV_0}$, $v=v^*\sqrt{m_e/eV_0}$, and $t=t^*\sqrt{eV_0/m_e L^2}$, the nondimensional version of Eq. (1) is

$$\frac{\partial^2 V}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 V}{\partial y^2} + \frac{1}{R^2}\frac{\partial^2 V}{\partial z^2} - \alpha(n-1) = 0, \tag{3a}$$

$$\frac{\partial n}{\partial t}+\frac{\partial(un)}{\partial x}+\frac{1}{R}\frac{\partial(vn)}{\partial y}=0, \tag{3b}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{R}v\frac{\partial u}{\partial y}-\frac{\partial V}{\partial x}+\frac{\sqrt{\alpha}}{\gamma}u=0, \tag{3c}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+\frac{1}{R}v\frac{\partial v}{\partial y}-\frac{1}{R}\frac{\partial V}{\partial y}+\frac{\sqrt{\alpha}}{\gamma}v=0. \tag{3d}$$

The dimensionless parametric groups are $\alpha = eN_D L^2/V_0\epsilon_s$, $\gamma = \sqrt{\tau^2 e^2 N_D/\epsilon_s m_e} = \omega_p \tau$, and $\omega_p = \sqrt{e^2 N_D/\epsilon_s m_e}$, which is the fundamental plasma frequency of free electrons in the semiconductor. The only tunable parameter is α and it is inversely proportional to the applied bias V_0; the phenomena related to collective excitations of the electron gas are captured by γ. The nondimensional boundary and charge neutrality conditions are

$$V(0,y,z,t)=-1, \quad V(1,y,z,t)=0, \quad n(0,y,t)=1, \tag{4a}$$

$$v(x,0,t)=0, \quad v(x,1,t)=0, \tag{4b}$$

$$\frac{\partial V}{\partial y}(x,0,0,t)=0, \quad \frac{\partial V}{\partial y}(x,1,0,t)=0, \tag{4c}$$

$$\int_0^1\int_0^1 n(x,y,t)\,dx\,dy=1, \tag{4d}$$

where we use $n_0 = N_D$.

The charge neutrality condition Eq. (4d) can be rewritten by integrating Eq. (3a) in the (x^*, y^*) plane to give

$$\int_0^1\left(\frac{\partial V}{\partial x}(1,y,0,t)-\frac{\partial V}{\partial x}(0,y,0,t)\right)dy+\int_0^1\left(\frac{\partial V}{\partial y}(x,1,0,t)-\frac{\partial V}{\partial y}(x,0,0,t)\right)dx=0. \tag{5}$$

Substituting Eqs. (4c) into Eq. (5), we get

$$\int_0^1\left(\frac{\partial V}{\partial x}(1,y,0,t)-\frac{\partial V}{\partial x}(0,y,0,t)\right)dy=0. \tag{6}$$

III. TWO-DIMENSIONAL ELECTRON FLOW

The steady-state solution of Eq. (3) in the semiconductor satisfying the boundary conditions (4) is

$$\bar{V}(x,y,0)=x-1, \quad \bar{n}(x,y)=1, \quad \bar{u}(x,y)=\frac{\gamma}{\sqrt{\alpha}}, \quad \bar{v}(x,y)=0, \tag{7}$$

where $\overline{(\,)}$ indicates time independence. The steady-state solution captures the drift flow and the electron velocity is independent of position. Also, the electron density and electric field are independent of position, since the potential varies linearly with x.[26,27] In dimensional form the electron concentration is $\bar{n}^*(x,y) = N_D$, which is the doping density of the semiconductor and the electron velocity $\bar{u}^*(x,y) = (e\tau/m_e)(V_0/L)$ is proportional to the electric field. Any instability that may exist is due to the growth of fluctuations from this steady state.

Applying small perturbations to the time-independent solution, we have $V = \bar{V}(x,y,0) + V'(x,y,z,t)$, $n = \bar{n}(x,y) + n'(x,y,z,t)$, $u = \bar{u}(x,y) + u'(x,y,t)$, and $v = \bar{v}(x,y) + v'(x,y,t)$. Substituting in Eq. (3) and linearizing, we get

$$\frac{\partial^2 V'}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 V'}{\partial y^2}-\alpha n'=0, \tag{8a}$$

$$\frac{\partial n'}{\partial t}+\frac{\gamma}{\sqrt{\alpha}}\frac{\partial n'}{\partial x}+\frac{\partial u'}{\partial x}+\frac{1}{R}\frac{\partial v'}{\partial y}=0, \tag{8b}$$

$$\frac{\partial u'}{\partial t}+\frac{\gamma}{\sqrt{\alpha}}\frac{\partial u'}{\partial x}-\frac{\partial V'}{\partial x}+\frac{\sqrt{\alpha}}{\gamma}u'=0, \tag{8c}$$

$$\frac{\partial v'}{\partial t}+\frac{\gamma}{\sqrt{\alpha}}\frac{\partial v'}{\partial x}-\frac{1}{R}\frac{\partial V'}{\partial y}+\frac{\sqrt{\alpha}}{\gamma}v'=0. \tag{8d}$$

In a two-dimensional semiconductor slab, the perturbation in the electron density in the z^*-direction can be considered very small. This implies that any variation in the electric field in the semiconductor in the z^*-direction is small in magnitude and can be neglected if compared to the steady-state electric field and its perturbations in the (x^*, y^*) plane. The electric field outside the semiconductor generated by the charge in the semiconductor can be neglected since the strong electric field in the x^*-direction is predominant. Due to this, Eq. (8) only includes the perturbations for the voltage, electron density, and longitudinal and transversal velocities in the (x^*, y^*) plane.

The boundary and charge neutrality conditions for the perturbations in the semiconductor are

$$V'(0,y,t)=0, \quad V'(1,y,t)=0, \quad n'(0,y,t)=0, \tag{9a}$$

$$v'(x,0,t)=0, \quad v'(x,1,t)=0, \tag{9b}$$

$$\frac{\partial V'}{\partial y}(x,0,t)=0, \quad \frac{\partial V'}{\partial y}(x,1,t)=0, \tag{9c}$$

$$\int_0^1\left(\frac{\partial V'}{\partial x}(1,y,t)-\frac{\partial V'}{\partial x}(0,y,t)\right)dy=0. \tag{9d}$$

The system in Eq. (8) and the boundary conditions in Eq. (9) describe the evolution of the perturbations in voltage, electron density, and longitudinal and transversal velocities from the steady state of the two-dimensional electron flow in the semiconductor.

In order to find the temporal and spatial modes that characterize the perturbations, we use normal modes of the form

$$\{V' n' u' v'\}^T=\{\tilde{V}(y)\tilde{n}(y)\tilde{u}(y)\tilde{v}(y)\}^T\exp(k^x x+\omega t), \tag{10}$$

where the wave number vector $\mathbf{k} = k^x\mathbf{i} + k^y\mathbf{j}$, the frequency is ω, and the amplitudes denoted by ~ are all complex. We will write $k^x = k_r^x + ik_i^x$, $k^y = k_r^y + ik_i^y$, and $\omega = \omega_r + i\omega_i$. Therefore, Eq. (8) becomes

$$\frac{d^2\tilde{V}}{dy^2}+(k^x)^2R^2\tilde{V}-\alpha R^2\tilde{n}=0, \tag{11a}$$

$$\frac{d\tilde{v}}{dy}+R\left(\omega+k^x\frac{\gamma}{\sqrt{\alpha}}\right)\tilde{n}+k^xR\tilde{u}=0, \tag{11b}$$

$$k^x\tilde{V}-\left(\omega+k^x\frac{\gamma}{\sqrt{\alpha}}+\frac{\sqrt{\alpha}}{\gamma}\right)\tilde{u}=0, \tag{11c}$$

$$\frac{d\tilde{V}}{dy}-R\left(\omega+k^x\frac{\gamma}{\sqrt{\alpha}}+\frac{\sqrt{\alpha}}{\gamma}\right)\tilde{v}=0. \tag{11d}$$

If $(\omega+k^x\gamma/\sqrt{\alpha}+\sqrt{\alpha}/\gamma)=0$, then Eqs. (11a) and (11c) give $\tilde{V}=\tilde{n}=0$, which is not of interest, so we will assume that $(\omega+k^x\gamma/\sqrt{\alpha}+\sqrt{\alpha}/\gamma)\neq 0$. By differentiating Eq. (11d), the system of equations can be reduced to

$$\frac{d\tilde{V}^2}{dy^2}-(k^y)^2\tilde{V}=0, \tag{12}$$

with

$$(k^y)^2=-(k^x)^2R^2. \tag{13}$$

By using Eq. (11d) with the boundary conditions in Eqs. (9b) and (9c), we get

$$\frac{d\tilde{V}}{dy}(0)=0, \quad \frac{d\tilde{V}}{dy}(1)=0. \tag{14}$$

The general solution for Eq. (12) is

$$\tilde{V}=C_1\exp(k^yy)+C_2\exp(-k^yy). \tag{15}$$

By applying the boundary conditions (14), this takes the form

$$\tilde{V}=D\cosh(k^{y,m}y), \tag{16}$$

with $k^{y,m}=\pm i2\pi m$. Therefore, the general solution can be written as

$$\tilde{V}=\sum_{m=0}^{\infty}D_m\cos(ik^{y,m}y). \tag{17}$$

It is easy to show that the proposed solution in Eq. (10) can satisfy the boundary conditions in Eqs. (9a) and (9d) independently of the value of k^y. Rewriting

$$\{\tilde{V}\tilde{n}\tilde{u}\tilde{v}\}^T=\{\hat{V}\hat{n}\hat{u}\hat{v}\}^T\exp(k^yy),$$

where the amplitudes denoted by ˆ are also all complex, Eq. (8) becomes

$$\begin{bmatrix} -(k^x)^2-(k^y)^2/R^2 & \alpha & 0 & 0 \\ 0 & \omega+k^x\gamma/\sqrt{\alpha} & k^x & k^y/R \\ -k^x & 0 & \omega+k^x\gamma/\sqrt{\alpha}+\sqrt{\alpha}/\gamma & 0 \\ -k^y/R & 0 & 0 & \omega+k^x\gamma/\sqrt{\alpha}+\sqrt{\alpha}/\gamma \end{bmatrix}\begin{Bmatrix}\hat{V}\\ \hat{n}\\ \hat{u}\\ \hat{v}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\end{Bmatrix}.$$

For a nontrivial solution, the determinant should vanish, so that the characteristic equation is

$$-[(k^x)^2R^2+(k^y)^2](\omega\sqrt{\alpha}\gamma+\gamma^2k^x+\alpha)\times\left[\left(\omega+\frac{\gamma}{\sqrt{\alpha}}k^x\right)\left(\omega+\frac{\gamma}{\sqrt{\alpha}}k^x+\frac{\sqrt{\alpha}}{\gamma}\right)+\alpha\right]=0. \tag{18}$$

The roots of Eq. (18) are

$$k_1^x=i\frac{k^y}{R}, \quad k_2^x=-i\frac{k^y}{R}, \quad k_3^x=a+ib, \quad k_4^x=a-ib, \tag{19}$$

where $a=-(\alpha/2\gamma^2)(2\gamma\omega/\sqrt{\alpha}+1)$ and $b=(\alpha/2\gamma^2)\sqrt{4\gamma-1}$. Since $k^{y,m}=\pm i2\pi m$, we have two cases.

A. $m\neq 0$

Now there are oscillations in both x- and y-directions. The modes in the x-direction are described in Eq. (19) and the modes in the y-direction are $k^{y,m}=\pm i2\pi m$. We can write the amplitudes of the perturbations as

$$V'=[Ae^{ik^{y,m}x/R}+Be^{-ik^{y,m}x/R}+Ce^{(a+ib)x}+De^{(a-ib)x}]e^{k^yy+\omega t},$$

$$\frac{\partial V'}{\partial x}=\left[Ai\frac{ik^{y,m}}{R}e^{ik^{y,m}x/R}-Bi\frac{k^{y,m}}{R}e^{-ik^{y,m}x/R}+C(a+ib)e^{(a+ib)x}+D(a-ib)e^{(a-ib)x}\right]e^{k^yy+\omega t},$$

$$n'=\frac{1}{\alpha}\left[Ce^{(a+ib)x}\left((a+ib)^2+\frac{(k^{y,m})^2}{R^2}\right)+De^{(a-ib)x}\left((a-ib)^2+\frac{(k^{y,m})^2}{R^2}\right)\right]e^{k^yy+\omega t}.$$

The boundary conditions can be written as

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ e^{ik^{y,m}/R} & e^{-ik^{y,m}/R} & e^{a+ib} & e^{a-ib} \\ 0 & 0 & (a+ib)^2+\dfrac{(k^{y,m})^2}{R^2} & (a-ib)^2+\dfrac{(k^{y,m})^2}{R^2} \\ i\dfrac{k^{y,m}}{R}(1-e^{ik^{y,m}/R}) & -i\dfrac{k^{y,m}}{R}(1-e^{-ik^{y,m}/R}) & (a+ib)(1-e^{a+ib}) & (a-ib)(1-e^{a-ib}) \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$

For a nontrivial solution the determinant must vanish, which gives

$$\left((a+ib)^2+\frac{(k^{y,m})^2}{R^2}\right)\Bigg[(a-ib)(1-e^{a-ib})(e^{-ik^{y,m}/R} - e^{ik^{y,m}/R})+i\frac{k^{y,m}}{R}(e^{a-ib}-e^{-ik^{y,m}/R})(1-e^{ik^{y,m}/R}) + i\frac{k^{y,m}}{R}(e^{a-ib}-e^{ik^{y,m}/R})(1-e^{-ik^{y,m}/R})\Bigg]-\left((a-ib)^2 + \frac{(k^{y,m})^2}{R^2}\right)\Bigg[(a+ib)(1-e^{a+ib})(e^{-ik^{y,m}/R}-e^{ik^{y,m}/R}) + i\frac{k^{y,m}}{R}(e^{a+ib}-e^{-ik^{y,m}/R})(1-e^{ik^{y,m}/R})+i\frac{k^{y,m}}{R}(e^{a+ib} - e^{ik^{y,m}/R})(1-e^{-ik^{y,m}/R})\Bigg]=0. \tag{20}$$

Equation (20) is satisfied by

$$a+ib=\pm i\frac{k^{y,m}}{R}, \quad a-ib=\pm i\frac{k^{y,m}}{R}.$$

If $b=\pm 2\pi p$, with p being a natural number, Eq. (20) is also satisfied by $a=0$. Due to the complexity of Eq. (20) other solutions may be possible, in which case the spectra may include additional temporal modes. b may be real, zero, or imaginary if $4\gamma^2$ is greater than, equal to, or less than unity.

1. For $4\gamma^2>1$

The solution for ω is

$$\omega_m=\frac{\sqrt{\alpha}}{2\gamma}\left[\pm\frac{4\gamma^2\pi m}{\alpha R}-1\pm i\sqrt{4\gamma^2-1}\right], \tag{21}$$

which indicates temporal modes that are either growing or decaying with an oscillatory component. Also, if $b=\pm 2\pi p$, the temporal mode is $\omega=-\sqrt{\alpha}/2\gamma$.

TABLE I. Temporal modes.

Condition	ω
$4\gamma^2>1$	$\omega=-(\gamma/\sqrt{\alpha})\{(\alpha/2\gamma^2)+b\cot b+W[-(be^{-b\cot b}/\sin b)]\}$
$4\gamma^2=1$	$\omega=-(\gamma/\sqrt{\alpha})[(\alpha/2\gamma^2)+1+W(-e^{-1})]$
$4\gamma^2<1$	$\omega=-(\gamma/\sqrt{\alpha})\{(\alpha/2\gamma^2)+b'\coth b'+W[-(b'e^{-b'\coth b'}/\sinh b')]\}$

2. For $4\gamma^2\leq 1$

Now,

$$\omega_m=\frac{\sqrt{\alpha}}{2\gamma}\left[\pm\frac{4\gamma^2\pi m}{\alpha R}-1\pm\sqrt{1-4\gamma^2}\right]. \tag{22}$$

In addition to this, if $b=\pm i2\pi p$, the temporal mode is $\omega=-\sqrt{\alpha}/2\gamma$. The temporal modes ω are real and therefore the evolution in time is either growing or decaying without oscillations. Furthermore, a and b are real, which implies no spatial oscillations. From the nonlinear nature of the problem, the growth must have an upper limit, though the linear analysis does not provide this value.

B. $m=0$

This represents oscillations only along the x-direction. A complete treatment of this has been described in Ref. 24. The temporal modes are described by Lambert W functions [The Lambert W function is defined as the solution of the equation $W(z)e^{W(z)}=z$]. There are three operating conditions that can be determined by the value of $4\gamma^2$. This also defines the nature of the spatial modes in the x-direction: no oscillations (purely real), constant amplitude oscillations (purely imaginary), and oscillations with spatial growth or decay (complex). By requiring a nontrivial solution over a wavelike solution for the perturbed system, the temporal modes are

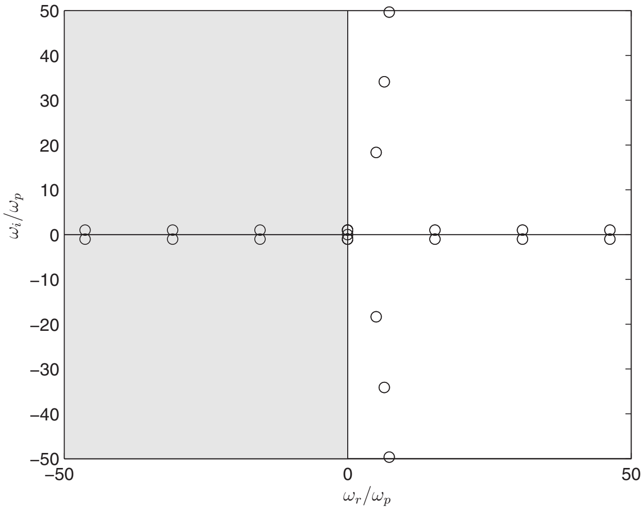

FIG. 2. Spectrum of eigenvalues for GaAs semiconductor where effective mass of electron is 6.6% of its actual mass, $\varepsilon_s=113.28\times 10^{-12}$ C^2/m^2 N, $L=100$ nm, $n_0=5\times 10^{17}$ cm^{-3}, $N_D=5\times 10^{17}$ cm^{-3}, $\tau=0.4\times 10^{-12}$ s, and $V_0=1$ V, which gives $\alpha=7.072$, $\gamma=17.365$, and $\omega_p=43.41\times 10^{12}$ s^{-1}. The shaded area represents the stable region and the unshaded the unstable.

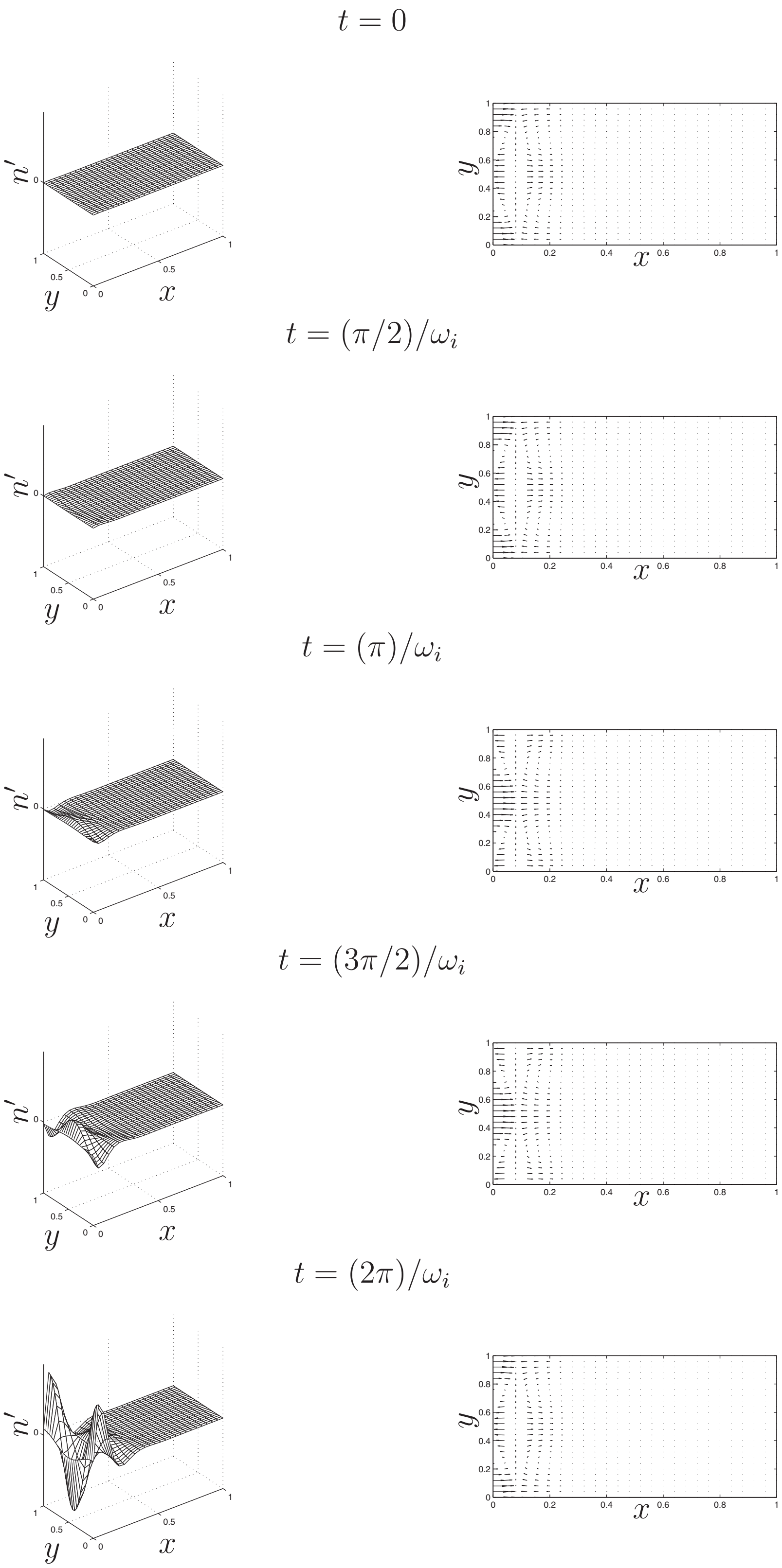

FIG. 3. Electron density eigenfunction (left) and electric field (right) for $\alpha=10$, $\gamma=1$, $R=0.5$, and $\omega_i \sim 0.87\omega_p$ (arbitrary scale) at the first unstable eigenmode $m=1$.

obtained. The solutions of Eq. (20) are in Table I, where $b' = (\alpha\sqrt{1-4\gamma^2})/2\gamma^2$.

IV. DISCUSSION

A. Spectrum of eigenmodes

The spectrum presents both stable and unstable regions. It is tunable mainly through the applied voltage, but the aspect ratio R can also determine instability when we have spatial oscillations along the y-direction, i.e., if $k^{y,m} \neq 0$. Since $k^{y,m}$ is purely imaginary, we get oscillations along the y-direction with a constant amplitude in space. The number of oscillatory temporal modes over a range of temporal mode amplitudes is independent of the value of the oscillatory component for $k^{y,m}=0$. Otherwise, for $k^{y,m} \neq 0$ it presents two

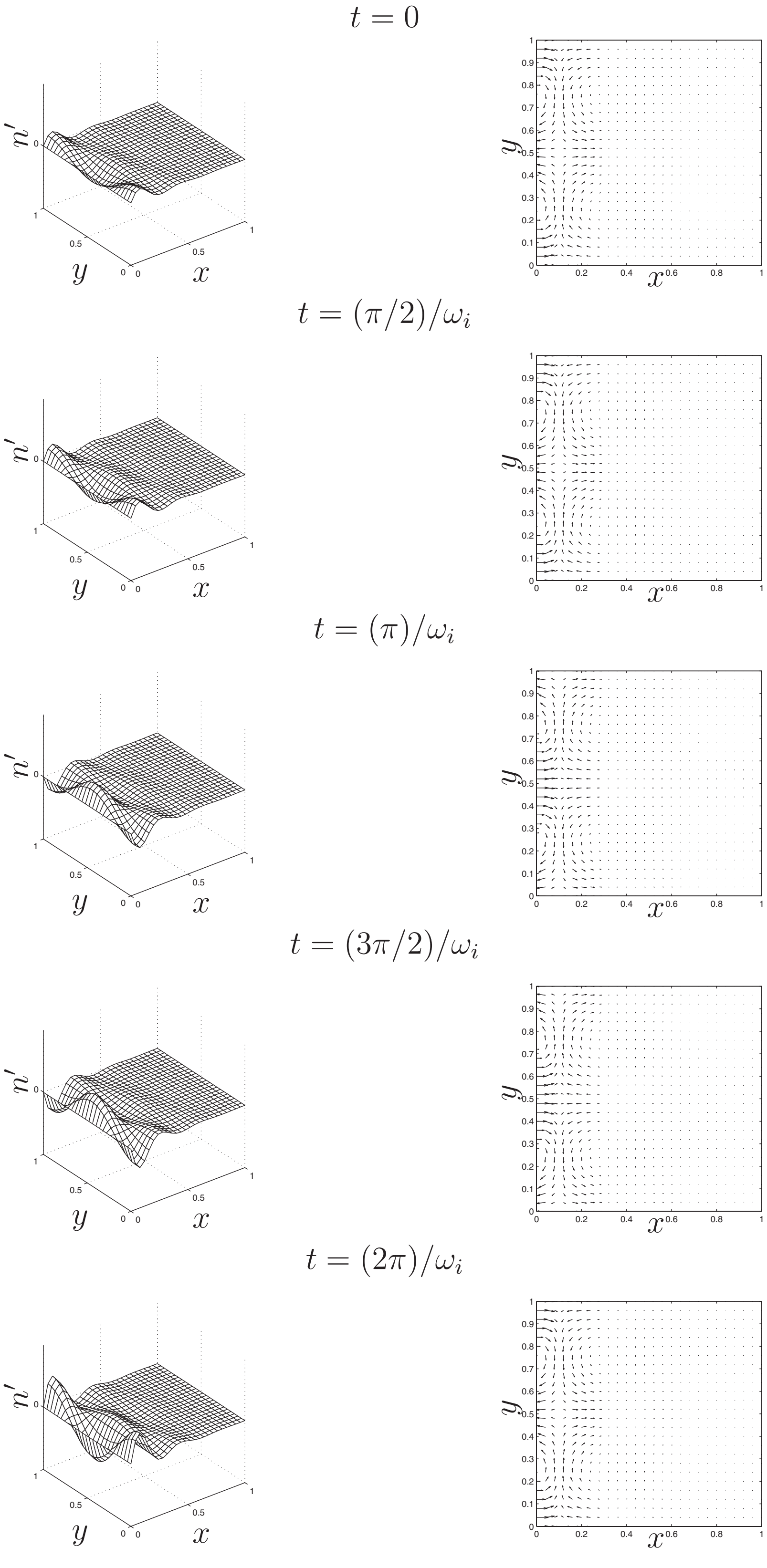

FIG. 4. Electron density eigenfunction (left) and electric field (right) for $\alpha=10$, $\gamma=1$, $R=1$, and $\omega_i \sim 0.87\omega_p$ (arbitrary scale) at the first unstable eigenmode $m=1$.

delta functions at $\pm(\sqrt{\alpha}/\omega_p)\sqrt{\omega_p - 1/4\tau^2}$. Taking $4\gamma^2 > 1$, the spectrum for GaAs is shown in Fig. 2.

B. Aspect ratio dependency

We are interested in imaginary components of the temporal modes ω, which describe the oscillatory behavior in time. Given an aspect ratio R, there is a critical positive mode above which the system is unstable. From Eq. (21), the critical positive mode is

$$m^c = \frac{\alpha R}{4\gamma^2 \pi}. \tag{23}$$

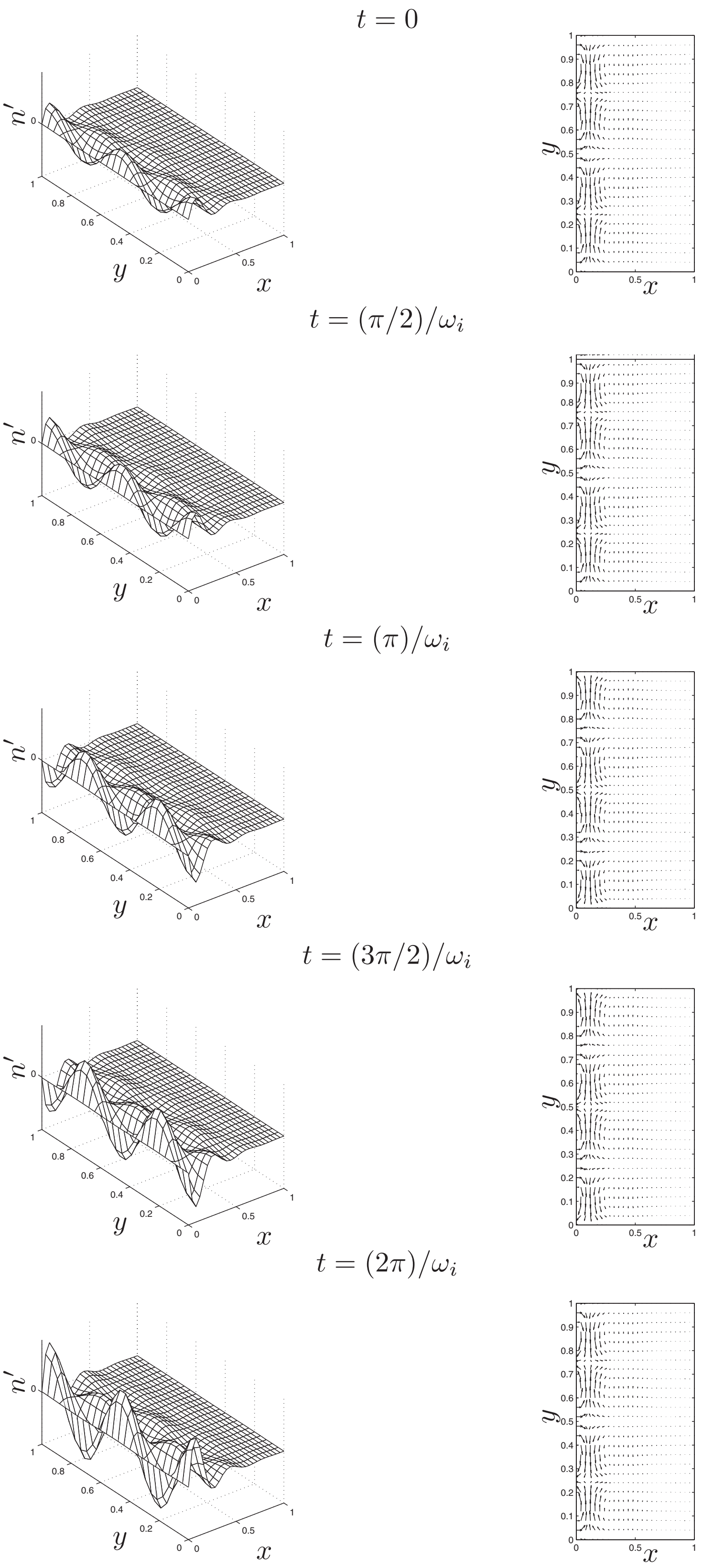

FIG. 5. Electron density eigenfunction (left) and electric field (right) for $\alpha=10$, $\gamma=1$, $R=2$, and $\omega_i \sim 0.87\omega_p$ (arbitrary scale) at the first unstable eigenmode $m=2$.

Moreover, this critical mode can be written in terms of the plasma frequency as

$$m^c = \frac{\alpha R}{4\pi(\omega_p \tau)^2}. \qquad (24)$$

It can also be written as

$$m^c = \frac{1}{\bar{u}^*}\left(\frac{H}{4\pi\tau}\right). \qquad (25)$$

As an example, choosing $\alpha=10$, $\gamma=1$, and $R=1$, we get $m^c \sim 0.8$, therefore $m \geq 1$ guarantees an unstable regime. As is shown in Fig. 3, which describes the evolution of electron density and electric field through the first period of oscilla-

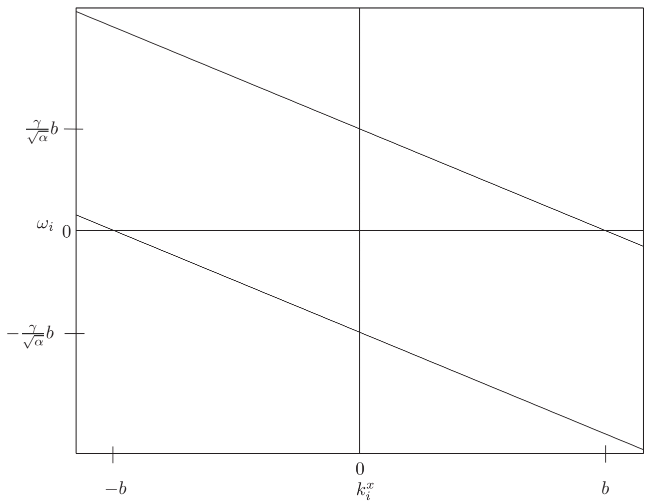

FIG. 6. Dispersion relation for $4\gamma^2>1$.

tions in time for a configuration with $R=0.5$, the first unstable mode is $m=1$. When $R=1$, the first unstable mode is $m=1$ and the evolution of electron density and electric field are shown in Fig. 4. Otherwise, when $R=2$ the first unstable mode is $m=2$ as is shown in Fig. 5. It illustrates how the first unstable mode is determined by R. The magnitude of the oscillatory components of the spectrum for $k^{y,m}\neq 0$ is $(\sqrt{\alpha}/\omega_p)\sqrt{\omega_p^2-1/4\tau^2}$. It depends only on α, γ, and τ and tends to the plasma frequency ω_p in the ballistic limit $\tau\to\infty$. This limit is reached when there is very low electron-impurity scattering.

C. Dispersion relation

By definition, the dispersion relation provides a relationship between the oscillatory component of the temporal modes, ω_i, and the oscillatory component of the spatial modes, k_i^x and k_i^y. Many situations of physical interest may have multiple and discrete roots of ω_i. In the proposed problem, the relation can be obtained from Eq. (18).

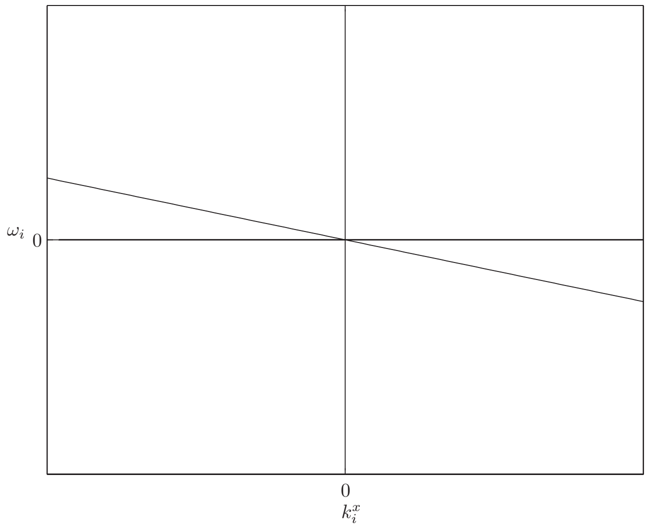

FIG. 7. Dispersion relation for $4\gamma^2\leq 1$.

For $4\gamma^2>1$, the dispersion relation takes the form

$$\frac{\alpha}{2\gamma^2}\left[-\frac{2\gamma\omega_i}{\sqrt{\alpha}}\pm\sqrt{4\gamma^2-1}\right]-k_i^x=0. \tag{26}$$

This linear relationship between ω_i and k_i^x is two parallel lines as shown in Fig. 6. The values of ω_i are determined by Eq. (21) and the corresponding values in Table I. From this, the phase and group velocities can be deduced to be

$$c_p=\frac{\gamma}{\sqrt{\alpha}}\frac{2\gamma\omega_i}{2\gamma\omega_i+\sqrt{\alpha(4\gamma^2-1)}},\quad c_g=\frac{\gamma}{\sqrt{\alpha}}, \tag{27}$$

respectively.

For $4\gamma^2\leq 1$, the expression for the dispersion relation is

$$\frac{\sqrt{\alpha}\omega_i}{\gamma}+k_i^x=0. \tag{28}$$

The relation is linear as in the previous case, but it is just one straight line crossing the origin, as shown in Fig. 7. This expression is valid only for $k^{y,m}=k^{y,0}=0$ since $\omega_i\neq 0$, otherwise for $k^{y,m}\neq 0$, $\omega_i=0$. From this, the phase and group velocities can be deduced to be

$$c_p=c_g=\frac{\gamma}{\sqrt{\alpha}}, \tag{29}$$

respectively. It can be noticed that the steady state electron velocity in Eq. (7) and the group velocity of the instability waves are equal.

V. CONCLUSIONS

The instabilities in the hydrodynamic model of a two-dimensional electron flow in ungated semiconductors are analyzed. Analytical expressions for the spatial and temporal plasma oscillation modes are derived. The spectrum of temporal modes shows a predominant unstable region, which depends strongly in the applied voltage through the semiconductor. As the applied voltage decreases, the spectrum is more stable. Also, the aspect ratio determines how unstable the temporal modes can be. As the aspect ratio decreases, the unstable modes become more unstable. The unstable region, which means temporal modes with positive real part, has oscillatory components able to describe terahertz frequencies under specific parameter values.

In summary, the required operating condition to support semiconductors in a two-dimensional configuration (such as in a HEMT structure) as a radiative source can be obtained under the right set of parameters such as applied voltage, aspect ratio and doping density, among others. The theoretical formalism presented extends the earlier works on the subject by revealing a spectrum of both stable and unstable modes for plasma-mode oscillations and presents a direct method for analyzing their dependence on the material and geometrical parameters of the device. This will prove to be valuable in the design process of compact electronic terahertz sources of the future.

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