Temperature influence on hydrodynamic instabilities in a one-dimensional electron flow in semiconductors

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Hydrodynamic instabilities in one-dimensional electron flow in semiconductor and their dependency with the electron and lattice temperatures are studied here. The driving force for the electrons is imposed by a voltage difference, and the hydrodynamic and electrostatic equations are linearized with respect to the steady-flow solution. A two-temperature hydrodynamic model predicts a stable electron flow through the semiconductor. A one-temperature hydrodynamic model is obtained by neglecting the electron energy losses due to heat conduction and scattering. This model shows that the electron flow can become unstable and establishes a criterion for that. Applied voltage and temperature can play the role of tunable parameters in the stability of the electron flow.
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I. INTRODUCTION

The instabilities of electron flow in semiconductors may be useful as a basis for new technological applications. A particular example of interest is a compact and portable source of electromagnetic radiation. This radiation can be generated through instabilities in the electron density to accelerate or decelerate the electrons. That fact has motivated the use of theoretical and experimental techniques to study instabilities in the electron flow in semiconductors. Among the theoretical approaches, a hydrodynamic model provides a description of the electron flow in semiconductors through dependent variables such as voltage, electron density, electron velocity, electron and lattice temperatures. Due to the nonlinearities in this model, a linearized analysis is useful for the study of instabilities.

Electrons in semiconductors scatter as a consequence of collisions between themselves and with the lattice and impurities. Typically, the linear stability analysis of electron flow in semiconductors is done by neglecting thermal energy changes in electrons and the lattice, for which hydrodynamic models provide an useful description. This perspective has been useful to explain physical phenomena involved in electron transport in semiconductors, and several studies have been made in this direction. In 1993, Dyakonov and Shur[1] proposed that the current flow in a two-dimensional electron gas channel in traditional high electron mobility transistors can become unstable and fluidlike flow of electrons can generate plasma waves, much like the formation of waves on the surface of shallow water. Based on a gradual channel approximation for the relationship between the gate potential and the local channel charge, they showed that plasma waves could be generated using certain bias conditions. Subsequently, a close analogy of this hydrodynamic model with shallow-water equations was obtained.[2,3] Crowne[4,5] analyzed the changes that happen in the shallow water instability in high electron mobility transistors under modifications in the boundary conditions and channel nonuniformity. A plasma-wave response was also found for the shallow water instability at high frequencies in high electron mobility transistors.[6] Current instabilities and plasma waves can be generated in an ungated two-dimensional electron layer.[7,8] A hydrodynamic model describing a two-dimensional electron plasma in a field effect transistor showed a nonlinear dynamic response and the dependency on the boundary conditions in the nonlinear effects.[9] Electron drift across the high field region in high electron mobility transistors is related to transit-time effects in plasma instability.[10] Plasma oscillations in both striped and gated two-dimensional electron layers in high electron mobility transistors have been recently analyzed.[11,12] The presence of instabilities in multilayered semiconductor structures have been studied numerically and theoretically.[13] Recently, drift wave instabilities in semiconductor electron-hole plasma have been reported.[14]

In 2007, Calderón-Muñoz *et al.*[15] determined analytically the spatial and time dependent instabilities in one-dimensional electron flow in ungated semiconductors by using a hydrodynamic model. Only the instabilities in the direction of electron flow were allowed by assuming plane-wave type of propagation of electrons between the contacts. Any instability perpendicular to the direction of electron flow was neglected. Using a complete solution of the one-dimensional hydrodynamic transport equations, it was shown that the spectrum of temporal eigenmodes is defined by an out of phase Lambert *W* function and can become unstable depending on operating conditions. Recently, Calderón-Muñoz *et al.*[16] analyzed the spatial and temporal instabilities in a confined two-dimensional electron flow. The results showed that the eigenvalue spectrum includes the modes of the one-dimensional flow and also a set of modes depending on physical parameters and operating conditions such as aspect ratio and applied voltage. A mathematical model involving the thermal energy changes in the electron flow and lattice, and its stability analysis would be very useful to

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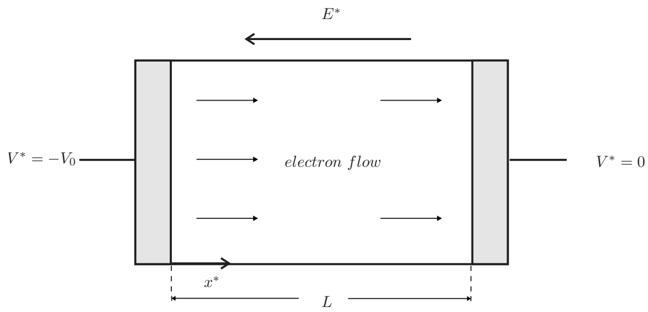

FIG. 1. Schematic of one-dimensional semiconductor material.

understand the behavior of the electron flow in semiconductors. In this paper we perform the linear stability analysis on two models: two-temperature and one-temperature. The two-temperature model includes both electron and lattice temperatures and the one-temperature model assumes that they are the same. This is designed to determine the influence of heat conduction and energy loss due to scattering of the electrons on the stability of one-dimensional electron flow in semiconductors.

II. TWO-TEMPERATURE MODEL

There are several semiconductor geometries and operating conditions that can be modeled and studied based on an one-dimensional electron transport. In the one-dimensional problem, the voltage drop, temperature changes, and transport of electrons are just allowed along the x-direction. The schematic of the one-dimensional configuration is shown in Fig. 1.

A. Governing equations and steady-state solution

The one-dimensional model equations[17–21] are

$$\frac{\partial^2 V^*}{\partial x^{*2}} = -\frac{e}{\epsilon_s}(N_D - n^*), \tag{1a}$$

$$\frac{\partial n^*}{\partial t^*} + \frac{\partial (u^* n^*)}{\partial x^*} = 0, \tag{1b}$$

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} = \frac{e}{m_e}\frac{\partial V^*}{\partial x^*} - \frac{1}{n^* m_e}\frac{\partial (n^* k_B T_e^*)}{\partial x^*} - \frac{u^*}{\tau_p}, \tag{1c}$$

$$\frac{\partial T_e^*}{\partial t^*} + u^*\frac{\partial T_e^*}{\partial x^*} = -\frac{2}{3}T_e^*\frac{\partial u^*}{\partial x^*} + \frac{2}{3n^* k_B}\frac{\partial}{\partial x^*}\left(k_e\frac{\partial T_e^*}{\partial x^*}\right) - \frac{T_e^* - T_L^*}{\tau_E} + \frac{2m_e(u^{*2})}{3k_B\tau_p}\left(1 - \frac{\tau_p}{2\tau_E}\right), \tag{1d}$$

$$C_L\frac{\partial T_L^*}{\partial t^*} = \frac{\partial}{\partial x^*}\left(k_L\frac{\partial T_L^*}{\partial x^*}\right) + \frac{3n^* k_B}{2}\left(\frac{T_e^* - T_L^*}{\tau_E}\right) + \frac{n^* m_e}{2\tau_E}(u^{*2}), \tag{1e}$$

where $V^*(x^*,t^*)$ is the voltage, $n^*(x^*,t^*)$ is the electron concentration, $u^*(x^*,t^*)$ is the x-component electron drift velocity, $T_e^*(x^*,t^*)$ is the electron temperature, and $T_L^*(x^*,t^*)$ is the lattice temperature.

The physical parameters are the electron charge e, the permittivity of the semiconductor ϵ_s, the doping concentration N_D, the effective electron mass m_e, the Boltzmann constant k_B, the electron thermal conductivity k_e, the lattice thermal conductivity k_L, the momentum relaxation time τ_p, the energy relaxation time τ_E, and the heat capacity for lattice C_L. In this analysis we consider k_e, k_L, τ_E, and C_L as constant values. The momentum relaxation time can be expressed as[22]

$$\tau_p = \frac{m_e \mu_{no}}{e}\frac{T_L^*}{T_e^*}, \tag{2}$$

where μ_{no} is the low field mobility.

The system of Eqs. (1) includes Gauss' law Eq. (1a), the continuity equation Eq. (1b), the momentum conservation equation in the x^* direction Eq. (1c), the energy conservation equation for electrons Eq. (1d), and the energy conservation equation for the lattice Eq. (1e) that relates the energy loss of the electrons due to scattering with the heat conduction in the lattice.

The boundary conditions include an imposed voltage at both the source, $V^*(0,t^*)=-V_0$, and the drain, $V^*(L,t^*)=0$. A constant electron density at the source, $n^*(0,t^*)=N_D$, the charge neutrality condition through the semiconductor, $(\partial V^*/\partial x^*)(0,t^*)=(\partial V^*/\partial x^*)(L,t^*)$, a constant electron temperature at the source, $T_e^*(0,t^*)=T_1$, and an adiabatic condition for the electrons at the drain, $(\partial T_e^*/\partial x^*)(L,t^*)=0$. An imposed lattice temperature at both the source, $T_L^*(0,t^*)=T_2$, and the drain $T_L^*(L,t^*)=T_3$.

For convenience, nondimensional versions of the governing Eqs. (1) are obtained. By writing $V=V^*/V_0$, $n=n^*/N_D$, $x=x^*/L$, $u=u^*\sqrt{m_e/eV_0}$, $t=t^*\sqrt{eV_0/m_eL^2}$, $T_e=(T_e^*-T_{\mathrm{room}})k_B/eV_0$, and $T_L=(T_L^*-T_{\mathrm{room}})k_B/eV_0$. The nondimensional version of Eqs. (1) is

$$\frac{\partial^2 V}{\partial x^2} = \alpha(n-1), \tag{3a}$$

$$\frac{\partial n}{\partial t} + \frac{\partial (un)}{\partial x} = 0, \tag{3b}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial V}{\partial x} - \frac{\beta}{n}\frac{\partial n}{\partial x} - \frac{\partial T_e}{\partial x} - \frac{T_e}{n}\frac{\partial n}{\partial x} - \left(\frac{T_e+\beta}{T_L+\beta}\right)\frac{\sqrt{\alpha}}{\gamma_{pn}}u, \tag{3c}$$

$$\frac{\partial T_e}{\partial t} + u\frac{\partial T_e}{\partial x} = -\frac{2}{3}\beta\frac{\partial u}{\partial x} - \frac{2}{3}T_e\frac{\partial u}{\partial x} + \frac{2}{3}\frac{\sigma}{n}\frac{\partial^2 T_e}{\partial x^2} - \frac{\sqrt{\alpha}}{\gamma_E}(T_e - T_L) + \frac{2}{3}\left(\frac{T_e+\beta}{T_L+\beta}\right)\frac{\sqrt{\alpha}}{\gamma_{pn}}u^2 - \frac{1}{3}\frac{\sqrt{\alpha}}{\gamma_E}u^2, \tag{3d}$$

$$\frac{\partial T_L}{\partial t} = \theta_1\frac{\partial^2 T_L}{\partial x^2} + \theta_2 n\left(T_e - T_L - \frac{1}{3}u^2\right). \tag{3e}$$

The nondimensional parametric groups are $\alpha=eN_DL^2/V_0\epsilon_s$, $\beta=k_BT_{\mathrm{room}}/eV_0$, $\sigma=(k_e/N_Dk_BL^2)\sqrt{m_eL^2/eV_0}$, $\gamma_{pn}=\sqrt{m_e\mu_{no}^2N_D/\epsilon_s}$, $\gamma_E=\sqrt{\tau_E^2e^2N_D/m_e\epsilon_s}$, $\theta_1=(k_L/L^2C_L)\sqrt{m_eL^2/eV_0}$, and $\theta_2=(3N_Dk_B/2\tau_EC_L)\sqrt{m_eL^2/eV_0}$.

TABLE I. Physical properties for GaAs (Refs. 18, 19, 28, and 29).

Constant	Value
k_e	3 W/(m K)
k_L	42.61 W/(m K)
N_D	5×10^{23} m^{-3}
m_e	6.01×10^{-32} kg
C_L	8.73×10^{5} J/(m^3 K)
τ_E	5×10^{12} s
μ_{no}	0.45 m^2/(V s)

TABLE III. Dimensionless parameters for GaAs.

Parameter	Value
α	7.072
γ_{pn}	7.331
γ_E	217.057
β	0.026
σ	2.662
θ_1	2.989×10^{-4}
θ_2	1.453×10^{-7}

The nondimensional version of the boundary conditions is

$$V(0,t) = -1, \quad V(1,t) = 0, \quad n(0,t) = 1, \quad \frac{\partial V}{\partial x}(0,t) = \frac{\partial V}{\partial x}(1,t), \tag{4a}$$

$$T_e(0,t) = \frac{T_1 - T_{\text{room}}}{k_B/eV_0}, \tag{4b}$$

$$\frac{\partial T_e}{\partial x}(1,t) = 0, \tag{4c}$$

$$T_L(0,t) = \frac{T_2 - T_{\text{room}}}{k_B/eV_0}, \tag{4d}$$

$$T_L(1,t) = \frac{T_3 - T_{\text{room}}}{k_B/eV_0}. \tag{4e}$$

Based on experimental results,[23,24] the voltage varies linearly with x. Thus we start with $\bar{V}(x)=x-1$, which on substituting in Eq. (3a) gives $\bar{n}(x)=1$. Thus, Eq. (3b) implies that $\bar{u}(x)$ is constant. Based on these, the steady-state version of Eqs. (3) is reduced to

$$-1 + \frac{\partial \bar{T}_e}{\partial x} + \bar{u}\left(\frac{\bar{T}_e+\beta}{\bar{T}_L+\beta}\right)\frac{\sqrt{\alpha}}{\gamma_{pn}} = 0, \tag{5a}$$

$$\bar{u}\frac{\partial \bar{T}_e}{\partial x} - \frac{2}{3}\sigma\frac{\partial^2 \bar{T}_e}{\partial x^2} + \frac{\sqrt{\alpha}}{\gamma_E}(\bar{T}_e - \bar{T}_L) - \frac{2}{3}\left(\frac{\bar{T}_e+\beta}{\bar{T}_L+\beta}\right)\frac{\sqrt{\alpha}}{\gamma_{pn}}\bar{u}^2 + \frac{1}{3}\frac{\sqrt{\alpha}}{\gamma_E}\bar{u}^2 = 0, \tag{5b}$$

$$\theta_1\frac{\partial^2 \bar{T}_L}{\partial x^2} - \theta_2\left(\bar{T}_e - \bar{T}_L - \frac{1}{3}\bar{u}^2\right) = 0. \tag{5c}$$

TABLE II. System parameters.

Constant	Value
L	100 nm
V_0	1.0 V
T_{room}	300 K

We consider the physical properties for GaAs (Refs. 18 and 19) shown in Table I, the system parameters shown in Table II, and the nondimensional parameters shown in Table III. We assume that the system is operating at a saturation velocity $\bar{u}^*(x)=5.44\times10^5$ m/s, i.e., $\bar{u}(x)=1/3$, and the temperature boundary conditions are $T_1=2250$ K, $T_2=301$ K, and $T_3=346$ K. This was designed to maintain the lattice temperature above T_{room}. We solve Eqs. (5) numerically for the electron and lattice temperatures by using MATHEMATICA. The solutions for $\bar{V}$, $\bar{n}$, $\bar{u}$, $\bar{T}_e$, and $\bar{T}_L$ are shown in Fig. 2.

The electron temperature, T_e, is always growing in the direction of the electron flow as shown in Fig. 2. However, the lattice temperature, T_L, reaches its maximum value about the middle of the right half (drain side) of the semiconductor as shown in Fig. 2. This is due to the boundary conditions. The lattice temperature is fixed at both source and drain, allowing heat flux through the contacts and the cooling at the drain.

B. Linear stability analysis

We are interested in determining if the steady-state solution, known in the hydrodynamic stability literature as base flow, is stable or unstable. To do that, we introduce small perturbations to the steady-state solution of the form

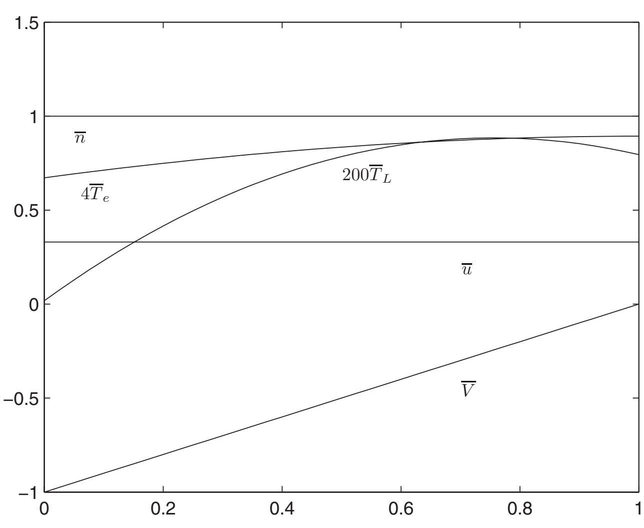

FIG. 2. Two-temperature model; nondimensional steady-state solution.

$$V=\bar{V}(x)+V'(x,t),\quad n=\bar{n}(x)+n'(x,t),\quad u=\bar{u}(x)+u'(x,t),$$

$$T_e=\bar{T}_e(x)+T'_e(x,t),\quad T_L=\bar{T}_L(x)+T'_L(x,t), \tag{6}$$

where $'$ denotes the perturbed variable.

Substituting Eqs. (6) into the system of Eqs. (3) and linearizing, we get the following perturbed system of equations

$$\frac{\partial^2 V'}{\partial x^2}-\alpha n'=0, \tag{7a}$$

$$\frac{\partial n'}{\partial t}+\bar{u}\frac{\partial n'}{\partial x}+\frac{\partial u'}{\partial x}=0, \tag{7b}$$

$$\begin{aligned}&(\beta+\bar{T}_L)\frac{\partial u'}{\partial t}+\left(\frac{\bar{u}\sqrt{\alpha}}{\gamma_{pn}}\right)T'_e+\left(\frac{\sqrt{\alpha}\beta}{\gamma_{pn}}+\frac{\sqrt{\alpha}\bar{T}_e}{\gamma_{pn}}\right)u'\\&+(\beta^2+\beta\bar{T}_e+\beta\bar{T}_L+\bar{T}_e\bar{T}_L)\frac{\partial n'}{\partial x}+(\beta+\bar{T}_L)\frac{\partial T'_e}{\partial x}\\&+\left(-1+\frac{d\bar{T}_e}{dx}\right)T'_L+\left(-\beta+\frac{\bar{u}\sqrt{\alpha}\beta}{\gamma_{pn}}+\frac{\bar{u}\sqrt{\alpha}\bar{T}_e}{\gamma_{pn}}\right.\\&\left.-\bar{T}_L+\beta\frac{d\bar{T}_e}{dx}+\bar{T}_L\frac{d\bar{T}_e}{dx}\right)n'+(\bar{u}\beta+\bar{u}\bar{T}_L)\frac{\partial u'}{\partial x}\\&-(\beta+\bar{T}_L)\frac{\partial V'}{\partial x}=0,\end{aligned} \tag{7c}$$

$$\begin{aligned}&(\beta+\bar{T}_L)\frac{\partial T'_e}{\partial t}+\left(\frac{\sqrt{\alpha}\beta}{\gamma_E}-\frac{2\bar{u}^2\sqrt{\alpha}}{3\gamma_{pn}}+\frac{\sqrt{\alpha}\bar{T}_L}{\gamma_E}\right)T'_e+(\bar{u}\beta\\&+\bar{u}\bar{T}_L)\frac{\partial T'_e}{\partial x}+\left(\frac{2\bar{u}\sqrt{\alpha}\beta}{3\gamma_E}-\frac{4\bar{u}\sqrt{\alpha}\beta}{3\gamma_{pn}}-\frac{4\bar{u}\sqrt{\alpha}\bar{T}_e}{3\gamma_{pn}}\right.\\&\left.+\frac{2\bar{u}\sqrt{\alpha}\bar{T}_L}{3\gamma_E}+\beta\frac{d\bar{T}_e}{dx}+\bar{T}_L\frac{d\bar{T}_e}{dx}\right)u'+\left(\frac{\bar{u}^2\sqrt{\alpha}\beta}{3\gamma_E}\right.\\&\left.-\frac{2\bar{u}^2\sqrt{\alpha}\beta}{3\gamma_{pn}}+\frac{\sqrt{\alpha}\beta\bar{T}_e}{\gamma_E}-\frac{2\bar{u}^2\sqrt{\alpha}\bar{T}_e}{3\gamma_{pn}}+\frac{\bar{u}^2\sqrt{\alpha}\bar{T}_L}{3\gamma_E}\right)n'\\&-\left(\frac{\sqrt{\alpha}\beta\bar{T}_L}{\gamma_E}+\frac{\sqrt{\alpha}\bar{T}_e\bar{T}_L}{\gamma_E}-\frac{\sqrt{\alpha}\bar{T}_L^2}{\gamma_E}+\bar{u}\beta\frac{d\bar{T}_e}{dx}\right.\\&\left.+\bar{u}\bar{T}_L\frac{d\bar{T}_e}{dx}\right)n'+\left(\frac{2}{3}\beta^2+\frac{2}{3}\beta\bar{T}_e+\frac{2}{3}\beta\bar{T}_L\right.\\&\left.+\frac{2}{3}\bar{T}_e\bar{T}_L\right)\frac{\partial u'}{\partial x}-\frac{2}{3}\left(\beta\sigma+\frac{2}{3}\sigma\bar{T}_L\right)\frac{\partial^2 T'_e}{\partial x^2}+\left(\frac{\bar{u}^2\sqrt{\alpha}}{3\gamma_E}\right.\\&\left.-\frac{\sqrt{\alpha}\beta}{\gamma_E}+\frac{\sqrt{\alpha}\bar{T}_e}{\gamma_E}-\frac{2\sqrt{\alpha}\bar{T}_L}{\gamma_E}+\bar{u}\frac{d\bar{T}_e}{dx}-\frac{2}{3}\sigma\frac{d^2\bar{T}_e}{dx^2}\right)T'_L=0,\end{aligned} \tag{7d}$$

$$\begin{aligned}&\frac{\partial T'_L}{\partial t}-\theta_2 T'_e+\theta_2 T'_L+\left(-\frac{\bar{u}^2\theta_2}{3}-\theta_2\bar{T}_e+\theta_2\bar{T}_L\right)n'-\frac{2}{3}\bar{u}\theta_2 u'\\&-\theta_1\frac{\partial^2 T'_L}{\partial x^2}=0.\end{aligned} \tag{7e}$$

Since the system (7) has variable coefficients in spatial derivatives, we use normal modes of the form

$$\begin{Bmatrix}V'\\ n'\\ u'\\ T'_e\\ T'_L\end{Bmatrix}=\begin{Bmatrix}\hat{V}(x)\\ \hat{n}(x)\\ \hat{u}(x)\\ \hat{T}_e(x)\\ \hat{T}_L(x)\end{Bmatrix}e^{\omega t},$$

where the frequency ω and the amplitudes denoted by ^ are all complex. Thus, we get a system of differential equations with variable coefficients. This is

$$A_1(x)\frac{d^2\hat{V}}{dx^2}+A_2(x)\hat{n}=0, \tag{8a}$$

$$B_1(x)\hat{n}+B_2(x)\frac{d\hat{n}}{dx}+B_3(x)\frac{d\hat{u}}{dx}=0, \tag{8b}$$

$$\begin{aligned}&C_1(x)\hat{T}_e+C_2(x)\hat{u}+C_3(x)\frac{d\hat{n}}{dx}+C_4(x)\frac{d\hat{T}_e}{dx}+C_5(x)\hat{T}_L\\&+C_6(x)\hat{n}+C_7(x)\frac{d\hat{u}}{dx}+C_8(x)\frac{d\hat{V}}{dx}=0,\end{aligned} \tag{8c}$$

$$\begin{aligned}&D_1(x)\hat{T}_e+D_2(x)\frac{d\hat{T}_e}{dx}+D_3(x)\hat{u}+D_4(x)\hat{n}+D_5(x)\frac{d\hat{u}}{dx}\\&+D_6(x)\frac{d^2\hat{T}_e}{dx^2}+D_7(x)\hat{T}_L=0,\end{aligned} \tag{8d}$$

$$E_1(x)\hat{T}_e+E_2(x)\hat{T}_L+E_3(x)\hat{n}+E_4(x)\hat{u}+E_5(x)\frac{d^2\hat{T}_L}{dx^2}=0, \tag{8e}$$

where the coefficients are defined in the Appendix. The boundary conditions are

$$\hat{V}(0)=0,\quad \hat{V}(1)=0,\quad \hat{n}(0)=0,\quad \frac{d\hat{V}}{dx}(0)=\frac{d\hat{V}}{dx}(1), \tag{9a}$$

$$\hat{T}_e(0)=0,\quad \frac{d\hat{T}_e}{dx}(1)=0,\quad \hat{T}_L(0)=0,\quad \hat{T}_L(1)=0. \tag{9b}$$

C. Temporal eigenmodes

The system of Eq. (8) can be rewritten as a system of first-order differential equations

$$\frac{d\mathbf{Y}}{dx} = \mathbf{J}^{-1}\mathbf{P}\mathbf{Y}, \tag{10}$$

where $\mathbf{Y}(x)=\{\hat{V},\hat{E},\hat{n},\hat{u},\hat{T}_e,\hat{q}_e,\hat{T}_L,\hat{q}_L\}^T$,

$$\mathbf{J}(\omega,x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & B_2(x) & B_3(x) & 0 & 0 & 0 & 0 \\ C_8(x) & 0 & C_3(x) & C_7(x) & C_4(x) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_5(x) & D_2(x) & D_6(x) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_5(x) \end{bmatrix},$$

$$\mathbf{P}(\omega,x) = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_2(x)/A_1(x) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -B_1(x) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_6(x) & -C_2(x) & -C_1(x) & 0 & -C_5(x) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -D_4(x) & -D_3(x) & -D_1(x) & 0 & D_7(x) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -E_3(x) & -E_4(x) & -E_1(x) & 0 & -E_2(x) & 0 \end{bmatrix},$$

with $\hat{q}_e=d\hat{T}_e/dx$ and $\hat{q}_L=d\hat{T}_L/dx$. $\mathbf{J}^{-1}\mathbf{P}$ is a square matrix. This is a boundary-value problem where the boundary conditions in Eq. (9) for Eq. (10) are $\mathbf{Y}(0)=\{0,\hat{E}(0),0,\hat{u}(0),0,\hat{q}_e(0),0,\hat{q}_L(0)\}^T$ and $\mathbf{Y}(1)=\{0,\hat{E}(0),\hat{n}(1),\hat{u}(1),\hat{T}_e(1),0,0,\hat{q}_L(1)\}^T$. The relation can be defined to be of the form $\mathbf{Y}(1)=\mathbf{A}(\omega)\mathbf{Y}(0)$, where $\mathbf{A}$ is a transfer matrix. To get the columns of the transfer matrix $\mathbf{A}$ (Ref. 25) for a given value of ω, the differential equation (10) can be integrated from $x=0$ to $x=1$ using as $\mathbf{Y}(0)$ the orthonormal vectors $\{1,0,0,0,0,0,0,0\}^T$, $\{0,1,0,0,0,0,0,0\}^T$, $\{0,0,1,0,0,0,0,0\}^T$, $\{0,0,0,1,0,0,0,0\}^T$, $\{0,0,0,0,1,0,0,0\}^T$, $\{0,0,0,0,0,1,0,0\}^T$, $\{0,0,0,0,0,0,1,0\}^T$, $\{0,0,0,0,0,0,0,1\}^T$ in turn. A fourth-order Runge–Kutta method with 100 integration steps was used.

Once we have the transfer matrix $\mathbf{A}$, a nontrivial solution is required to be satisfied for the system of four algebraic equations in terms of $\hat{E}(0)$, $\hat{u}(0)$, $\hat{q}_e(0)$, and $\hat{q}_L(0)$ that contains the boundary conditions at $x=1$. Thus

$$\begin{bmatrix} a_{12} & a_{14} & a_{16} & a_{18} \\ a_{22}-1 & a_{24} & a_{26} & a_{28} \\ a_{62} & a_{64} & a_{66} & a_{68} \\ a_{72} & a_{74} & a_{76} & a_{78} \end{bmatrix} \begin{Bmatrix} \hat{E}(0) \\ \hat{u}(0) \\ \hat{q}_e(0) \\ \hat{q}_L(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{11}$$

where a_{ij} are the coefficients (i,j) of the matrix $\mathbf{A}$.

By applying Muller's method to find the roots of the determinant of the matrix that contains the boundary conditions, we get the eigenmodes in time ω around the origin. This is an extension of the secant method, where a second-degree polynomial is used to interpolate three points, instead of a linear polynomial;[26,27] and its order of convergence is almost quadratic. The purpose of using this method is to find the closest unstable eigenmode in time to the origin. The first five eigenfrequencies are shown in Table IV. Unstable complex eigenfrequencies with positive real part were not found using the procedure described before. Also, oscillatory eigenmodes with imaginary part were not found, even though oscillatory eigenmodes were used to start the iteration process. It is important to emphasize that the method requires that starting values be neighbors to the value of interest.

TABLE IV. Temporal eigenmodes of two-temperature model.

ω_1	−0.002 950 23
ω_2	−0.011 800 9
ω_3	−0.026 552 1
ω_4	−0.047 203 9
ω_5	−0.073 756 5

According to the eigenmodes in Table IV, the system is stable and without an oscillatory component. The main difference between the mathematical model described here and the others in earlier works[1,2,7,8,15] is the thermal coupling. In this analysis we are considering conduction effects and scattering in the electron transport in the semiconductor. These phenomena, which involve energy dissipation, can determine the stability or instability of the electron flow.

III. ONE-TEMPERATURE APPROXIMATION WITH COLLISION

The model takes into account several effects that determine the electron flow in the semiconductor. With the purpose of exploring which effect contributes to the stability of system, in this section we study a simplified model in which we assume that $T_e = T_L$. The assumption is valid if $\gamma_E/\sqrt{\alpha} \ll 1$. This condition means that the energy relaxation time, τ_E, is much lower than the transit time of the electrons, τ_{tr}, and implies that electrons get sufficient time to cool and approach T_L. We also neglect the heat conduction of the electrons, i.e., $k_e = 0$.

A. Governing equations and steady-state solution

$$\frac{\partial^2 V}{\partial x^2} = \alpha(n-1), \tag{12a}$$

$$\frac{\partial n}{\partial t} + \frac{\partial(un)}{\partial x} = 0, \tag{12b}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial V}{\partial x} - \frac{\beta}{n}\frac{\partial n}{\partial x} - \frac{\partial T_e}{\partial x} - \frac{T_e}{n}\frac{\partial n}{\partial x} - \frac{\sqrt{\alpha}}{\gamma_{pn}}u, \tag{12c}$$

$$\frac{\partial T_e}{\partial t} + u\frac{\partial T_e}{\partial x} = -\frac{2}{3}\beta\frac{\partial u}{\partial x} - \frac{2}{3}T_e\frac{\partial u}{\partial x} - \frac{\sqrt{\alpha}}{3}\left(\frac{1}{\gamma_E} - \frac{2}{\gamma_{pn}}\right)u^2. \tag{12d}$$

The new energy conservation equation Eq. (12d) considers only the terms for energy advection, work done by the electron pressure and a part of the collision term.[20] $T_e = T_L$ implies that Eq. (3e) is uncoupled and does not provide information about the dependent variables. The boundary conditions for Eqs. (12) are Eqs. (4a) and (4b) with $T_1 = T_{\text{room}}$. This last condition imposes that the electron temperature at the source must be constant and equal to the room temperature T_{room}. The steady-state solution for Eqs. (12) is

$$\bar{V}(x) = x - 1, \quad \bar{n}(x) = 1, \quad \bar{u}(x) = \frac{1}{\frac{5}{3}\frac{\sqrt{\alpha}}{\gamma_{pn}} - \frac{1}{3}\frac{\sqrt{\alpha}}{\gamma_E}}, \quad \bar{T}_e(x) = \left(\frac{\frac{2}{3}\frac{\sqrt{\alpha}}{\gamma_{pn}} - \frac{1}{3}\frac{\sqrt{\alpha}}{\gamma_E}}{\frac{5}{3}\frac{\sqrt{\alpha}}{\gamma_{pn}} - \frac{1}{3}\frac{\sqrt{\alpha}}{\gamma_E}}\right)x.$$

The only dependent variable that varies along the semiconductor is $\bar{T}_e$.

B. Linear stability analysis

To do the linear stability analysis of the steady-state solution we introduce small perturbations for V, n, u, and T_e of the form shown in Eq. (6). Substituting the perturbations into the system (12) and linearizing, we get

$$\frac{\partial^2 V'}{\partial x^2} - \alpha n' = 0, \tag{13a}$$

$$\frac{\partial n'}{\partial t} + \bar{u}\frac{\partial n'}{\partial x} + \frac{\partial u'}{\partial x} = 0, \tag{13b}$$

$$\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} - \frac{\partial V'}{\partial x} + \beta\frac{\partial n'}{\partial x} + \frac{\partial T_e'}{\partial x} + \bar{T}_e\frac{\partial n'}{\partial x} + \frac{\sqrt{\alpha}}{\gamma_{pn}}u' = 0, \tag{13c}$$

$$\frac{\partial T_e'}{\partial t} + \bar{u}\frac{\partial T_e'}{\partial x} + \frac{2}{3}\beta\frac{\partial u'}{\partial x} + \frac{2}{3}\bar{T}_e\frac{\partial u'}{\partial x} + \frac{2}{3}\sqrt{\alpha}\left(\frac{1}{\gamma_E} - \frac{2}{\gamma_{pn}}\right)\bar{u}u' = 0. \tag{13d}$$

The boundary conditions take the form

$$V'(0,t) = 0, \quad V'(1,t) = 0, \quad n'(0,t) = 0, \quad \frac{\partial V'}{\partial x}(0,t) = \frac{\partial V'}{\partial x}(1,t), \quad T_e'(0,t) = 0. \tag{14}$$

Since the system (13) has variable coefficients in spatial derivatives we use normal modes of the form

$$\begin{Bmatrix} V' \\ n' \\ u' \\ T_e' \end{Bmatrix} = \begin{Bmatrix} \hat{V}(x) \\ \hat{n}(x) \\ \hat{u}(x) \\ \hat{T}_e(x) \end{Bmatrix} e^{\omega t}.$$

The system in Eq. (13) can be rewritten as

$$\frac{d\mathbf{Y}}{dx} = \mathbf{J}_s^{-1}\mathbf{P}_s\mathbf{Y}, \tag{15}$$

where $\mathbf{Y}(x) = \{\hat{V}, \hat{E}, \hat{n}, \hat{u}, \hat{T}_e\}^T$,

$$\mathbf{J}_s(\omega, x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \bar{u} & 1 & 0 \\ 0 & 0 & \beta + \bar{T}_e & \bar{u} & 1 \\ 0 & 0 & 0 & \frac{2}{3}(\beta + \bar{T}_e) & \bar{u} \end{bmatrix},$$

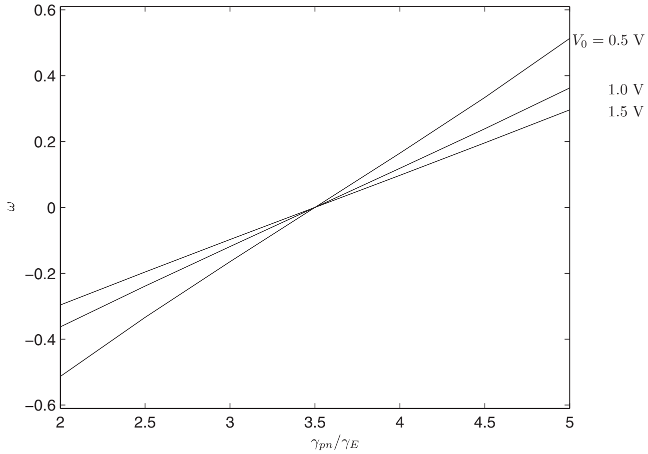

FIG. 3. One-temperature approximation with collision; closest eigenfrequency to the origin at $T_{\mathrm{room}}=300$ K.

$$\mathbf{P}_s(\omega,x)=\begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & 0 & 0 \\ 0 & 0 & -\omega & 0 & 0 \\ 0 & -1 & 0 & -\left(\omega+\dfrac{\sqrt{\alpha}}{\gamma_{pn}}\right) & 0 \\ 0 & 0 & 0 & -\dfrac{2}{3}\sqrt{\alpha}\left(\dfrac{1}{\gamma_E}-\dfrac{2}{\gamma_{pn}}\right)\bar{u} & -\omega \end{bmatrix},$$

and $\mathbf{J}_s^{-1}\mathbf{P}_s$ is a square matrix. This is also a boundary-value problem where the boundary conditions for Eq. (15) are $\mathbf{Y}(0)=\{0,\hat{E}(0),0,\hat{u}(0),0\}^T$ and $\mathbf{Y}(1)=\{0,\hat{E}(0),\hat{n}(1),\hat{u}(1),\hat{T}_e(1)\}^T$. The same numerical procedure described before was used to satisfy the boundary conditions and get the eigenfrequencies ω. We take the nondimensional parameters $\alpha=7.072$ and $\beta=0.026$ from Table III and explore the necessary condition between γ_E and γ_{pn} to make the system in Eqs. (12) unstable.

The steady-state electron velocity must be greater than zero. That implies the condition $\gamma_{pn}/\gamma_E<5$. The closest eigenfrequencies to the origin when γ_{pn}/γ_E varies are shown in Fig. 3 for different values of applied voltage V_0. The relation is linear and shows that the system is more unstable when γ_E decreases with respect to γ_{pn}. There is a critical value $\gamma_{pn}/\gamma_E=3.5$ that defines a transition of the eigenfrequency from stable to unstable. Choosing $\gamma_E=\gamma_{pn}/4$, the system is unstable and some eigenfrequencies are shown in Table V. If we impose this critical value, the collision term [last term in Eq. (12d)] is rewritten as $-\sqrt{\alpha}u^2/2\gamma_{pn}$. Physically, this means that the electrons are losing energy due to collisions. Also, the critical value implies $\bar{u}=2\gamma_{pn}/\sqrt{\alpha}$ and $\bar{T}_e=-x$. The magnitude of the collision term also depends on α. When the applied voltage V_0 increases, α decreases and the energy loss due to collisions also decreases. In conclusion, the condition

$$3.5<\frac{\gamma_{pn}}{\gamma_E}<5 \tag{16}$$

guarantees that the system is unstable. In terms of electron velocity and temperature Eq. (16) implies that

$$\bar{u}>2\frac{\gamma_{pn}}{\sqrt{\alpha}}, \quad \frac{d\bar{T}_e}{dx}<-1.$$

TABLE V. Temporal eigenmodes of one-temperature approximation with collision $\gamma_E=\gamma_{pn}/4$.

ω_1	0.118 627
ω_2	417.697
ω_3	940.637
ω_4	1607.64
ω_5	1887.8

IV. ONE-TEMPERATURE APPROXIMATION WITHOUT COLLISION

To understand if the effects of heat conduction of electrons and energy loss due to scattering of electrons are responsible for the stability of the system, we study the system when these effects are neglected. We also assume that $T_e=T_L$. The last four terms in the electron energy equation, Eq. (3d), where the first is the heat conduction term and the last three are due to collision,[20] are neglected.

A. Governing equations and steady-state solution

Under the assumptions described before, the system of Eq. (3) is rewritten

$$\frac{\partial^2 V}{\partial x^2}=\alpha(n-1), \tag{17a}$$

$$\frac{\partial n}{\partial t}+\frac{\partial(un)}{\partial x}=0, \tag{17b}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\frac{\partial V}{\partial x}-\frac{\beta}{n}\frac{\partial n}{\partial x}-\frac{\partial T_e}{\partial x}+\frac{T_e}{n}\frac{\partial n}{\partial x}-\frac{\sqrt{\alpha}}{\gamma_{pn}}u, \tag{17c}$$

$$\frac{\partial T_e}{\partial t}+u\frac{\partial T_e}{\partial x}=-\frac{2}{3}\beta\frac{\partial u}{\partial x}-\frac{2}{3}T_e\frac{\partial u}{\partial x}. \tag{17d}$$

The new energy conservation equation, Eq. (17d), considers just the terms for energy advection and work done by the electron pressure. As before and due to the assumptions, Eq. (3e) is uncoupled and does not provide information about the dependent variables. The boundary conditions for Eqs. (17) are in Eqs. (4a) and (4b) with $T_1=T_{\mathrm{room}}$. The steady-state solution for Eqs. (17) is

$$\bar{V}(x)=x-1, \quad \bar{n}(x)=1, \quad \bar{u}(x)=\frac{\gamma_{pn}}{\sqrt{\alpha}}, \quad \bar{T}_e(x)=0.$$

B. Linear stability analysis

As before, to do the linear stability analysis of the steady-state solution we introduce small perturbations for V, n, u, and T_e of the form shown in Eq. (6). Substituting the perturbations into the system (17) and linearizing, we get

$$\frac{\partial^2 V'}{\partial x^2} - \alpha n' = 0, \tag{18a}$$

$$\frac{\partial n'}{\partial t} + \bar{u}\frac{\partial n'}{\partial x} + \frac{\partial u'}{\partial x} = 0, \tag{18b}$$

$$\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} - \frac{\partial V'}{\partial x} + \beta\frac{\partial n'}{\partial x} + \frac{\partial T_e'}{\partial x} + \frac{\sqrt{\alpha}}{\gamma_{pn}}u' = 0, \tag{18c}$$

$$\frac{\partial T_e'}{\partial t} + \bar{u}\frac{\partial T_e'}{\partial x} + \frac{2}{3}\beta\frac{\partial u'}{\partial x} = 0. \tag{18d}$$

The boundary conditions take the form

$$V'(0,t) = 0, \quad V'(1,t) = 0, \quad n'(0,t) = 0, \quad \frac{\partial V'}{\partial x}(0,t) = \frac{\partial V'}{\partial x}(1,t), \quad T_e'(0,t) = 0. \tag{19}$$

The system (18) has constant coefficients in spatial derivatives. This allows us to do the linear stability analysis of the steady-state solution using the normal modes

$$\begin{Bmatrix} V' \\ n' \\ u' \\ T_e \end{Bmatrix} = \begin{Bmatrix} \tilde{V} \\ \tilde{n} \\ \tilde{u} \\ \tilde{T}_e \end{Bmatrix} e^{kx+\omega t},$$

where the frequency ω, the wave vector k, and the amplitudes denoted by $\tilde{}$ are all complex and constant. Thus, we can rewrite the system of Eqs. (18) as the following matrix:

$$\begin{bmatrix} k^2 & -\alpha & 0 & 0 \\ 0 & \omega + k\bar{u} & k & 0 \\ -k & k\beta & \omega + k\bar{u} + \dfrac{\sqrt{\alpha}}{\gamma_{pn}} & k \\ 0 & 0 & \dfrac{2}{3}k\beta & \omega + k\bar{u} \end{bmatrix} \begin{Bmatrix} \tilde{V} \\ \tilde{n} \\ \tilde{u} \\ \tilde{T}_e \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}. \tag{20}$$

By imposing a nontrivial solution, we get the characteristic equation

$$k^5\left(\bar{u}^3 - \frac{5\bar{u}\beta}{3}\right) + k^4\left(\frac{\bar{u}^2\sqrt{\alpha}}{\gamma_{pn}} + 3\bar{u}^2\omega - \frac{5\beta\omega}{3}\right) + k^3\left(\bar{u}\alpha + \frac{2\bar{u}\sqrt{\alpha}\omega}{\gamma_{pn}} + 3\bar{u}\omega^2\right) + k^2\left(\alpha\omega + \frac{\sqrt{\alpha}\omega^2}{\gamma_{pn}} + \omega^3\right) = 0 \tag{21}$$

whose solutions are

$$k_1 = 0, \tag{22a}$$

$$k_2 = 0, \tag{22b}$$

$$k_3 = -\frac{\omega}{\bar{u}}, \tag{22c}$$

$$k_4 = \frac{-3\bar{u}(\sqrt{\alpha} + 2\gamma_{pn}\omega) - \sqrt{9\bar{u}^2\alpha(1 - 4\gamma_{pn}^2) + 60\beta\gamma_{pn}(\alpha\gamma_{pn} + \sqrt{\alpha}\omega + \gamma_{pn}\omega^2)}}{2(3\bar{u}^2 - 5\beta)\gamma_{pn}}, \tag{22d}$$

$$k_5 = \frac{-3\bar{u}(\sqrt{\alpha} + 2\gamma_{pn}\omega) + \sqrt{9\bar{u}^2\alpha(1 - 4\gamma_{pn}^2) + 60\beta\gamma_{pn}(\alpha\gamma_{pn} + \sqrt{\alpha}\omega + \gamma_{pn}\omega^2)}}{2(3\bar{u}^2 - 5\beta)\gamma_{pn}}. \tag{22e}$$

Therefore, the solutions for V', n', u', and T_e' can be written as

$$V'(x,t) = (Z_1 + Z_2 x + Z_3 e^{k_3 x} + Z_4 e^{k_4 x} + Z_5 e^{k_5 x})e^{\omega t}, \tag{23a}$$

$$n'(x,t) = \frac{1}{\alpha}(Z_3 k_3^2 e^{k_3 x} + Z_4 k_4^2 e^{k_4 x} + Z_5 k_5 e^{k_5 x})e^{\omega t}, \tag{23b}$$

$$u'(x,t) = \frac{-1}{\alpha}[Z_3 k_3(k_3\bar{u} + \omega)e^{k_3 x} + Z_4 k_4(k_4\bar{u} + \omega)e^{k_4 x} + Z_5 k_5(k_5\bar{u} + \omega)e^{k_5 x}]e^{\omega t}, \tag{23c}$$

$$T_e'(x,t) = -\frac{e^{\omega t}}{3\alpha\gamma_{pn}\omega}[3Z_2\bar{u}\alpha\gamma_{pn} + Z_3 e^{k_3 x}k_3 \times (k_3^2\phi_1 + 3\phi_2 + k_3\phi_3)] - \frac{e^{\omega t}}{3\alpha\gamma_{pn}\omega}[Z_4 e^{k_4 x}k_4(k_4^2\phi_1 + 3\phi_2 + k_4\phi_3) + Z_5 e^{k_5 x}k_5(k_5^2\phi_1 + 3\phi_2 + k_5\phi_3)], \tag{23d}$$

where $[Z_1, Z_2, Z_3, Z_4, Z_5]^T$ is the vector of unknown amplitudes and $\phi_1 = \bar{u}(3\bar{u}^2 - 5\beta)\gamma_{pn}$, $\phi_2 = \bar{u}(\alpha\gamma_{pn} + \sqrt{\alpha}\omega + \gamma_{pn}\omega^2)$, and $\phi_3 = -2\beta\gamma_{pn}\omega + 3\bar{u}^2(\sqrt{\alpha} + 2\gamma_{pn}\omega)$.

By applying the boundary conditions in Eqs. (19) to Eq. (23), we get

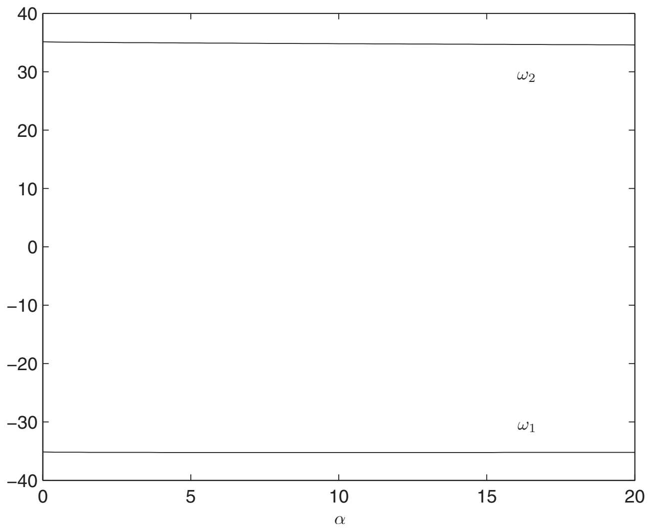

FIG. 4. One-temperature approximation without collision; ω_1 and ω_2 eigenmodes as a function of α with γ_{pn}=7.331 and β=0.026.

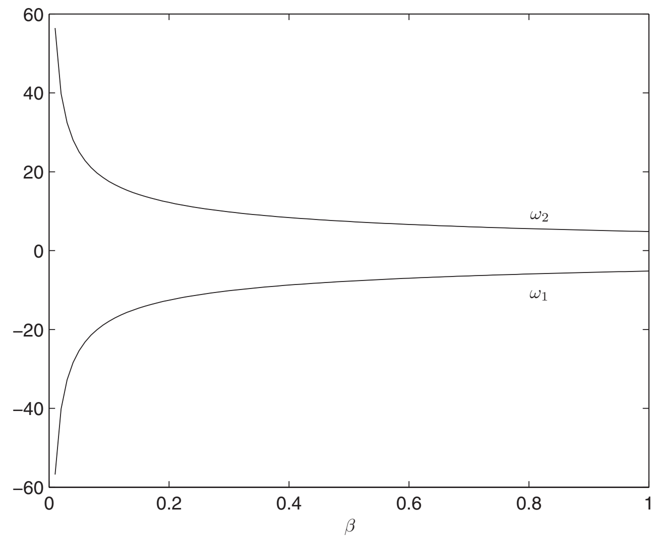

FIG. 5. One-temperature approximation without collision; ω_1 and ω_2 eigenmodes as a function of β with α=7.072 and γ_{pn}=7.331.

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & e^{k_3} & e^{k_4} & e^{k_5} \\ 0 & 0 & \dfrac{k_3^2}{\alpha} & \dfrac{k_4^2}{\alpha} & \dfrac{k_5^2}{\alpha} \\ 0 & 0 & k_3(1-e^{k_3}) & k_4(1-e^{k_4}) & k_5(1-e^{k_5}) \\ 0 & 3\bar{u}\alpha\gamma_{pn} & k_3\eta_1 & k_4\eta_2 & k_5\eta_3 \end{bmatrix} \times \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \tag{24}$$

where $\eta_1 = k_3^2\phi_1 + 3\phi_2 + k_3\phi_3$, $\eta_2 = k_4^2\phi_1 + 3\phi_2 + k_4\phi_3$, and $\eta_3 = k_5^2\phi_1 + 3\phi_2 + k_5\phi_3$.

By imposing a nontrivial solution to Eq. (24), we get the characteristic equation

$$\begin{aligned} &3(-1+e^{k_3})k_4k_5[(-1+e^{k_5})k_4 + k_5 - e^{k_4}k_5]\bar{u}\alpha\gamma_{pn} + k_3\{3(\\ &-1+e^{k_3})(-1+e^{k_4})k_5^2\bar{u}\alpha\gamma_{pn} + k_4k_5^2[(-1+e^{k_4})\eta_1 \\ &+\eta_2 - e^{k_3}\eta_2]\} - k_3[k_4^2\{3(-1+e^{k_3})(-1+e^{k_5})\bar{u}\alpha\gamma_{pn} \\ &+k_5[(-1+e^{k_5})\eta_1 + \eta_3 - e^{k_3}\eta_3]\}] + k_3^2[-3(-1+e^{k_4}) \\ &\times(-1+e^{k_5})k_5\bar{u}\alpha\gamma_{pn}] + k_3^2[k_4\{3(-1+e^{k_4})(-1 \\ &+e^{k_5})\bar{u}\alpha\gamma_{pn} + k_5[(-1+e^{k_5})\eta_2 + \eta_3 - e^{k_4}\eta_3]\}] = 0. \end{aligned} \tag{25}$$

C. Temporal eigenmodes

Two of the solutions of Eq. (25) are

$$\omega_1 = \frac{-20\sqrt{\alpha}\beta - \sqrt{400\alpha\beta^2 - 80\beta\gamma_{pn}(3\gamma_{pn} + 20\alpha\beta\gamma_{pn} - 12\gamma_{pn}^3)}}{40\beta\gamma_{pn}}, \tag{26}$$

$$\omega_2 = \frac{-20\sqrt{\alpha}\beta + \sqrt{400\alpha\beta^2 - 80\beta\gamma_{pn}(3\gamma_{pn} + 20\alpha\beta\gamma_{pn} - 12\gamma_{pn}^3)}}{40\beta\gamma_{pn}}. \tag{27}$$

We are interested in finding unstable temporal eigenmodes, i.e., values of ω with positive real part. The real part of ω_1 is always negative and the real part of ω_2 can be positive if

$$\gamma_{pn} > \frac{1}{2}, \quad \alpha < \frac{12\gamma_{pn}^2 - 3}{20\beta}. \tag{28}$$

Taking γ_{pn}=7.331 and β=0.026 from Table III, we need α <1234.47 to have at least one unstable eigenmode. With α =7.072 from Table III we get ω_2=34.8533, which is unstable. By using the definitions of the nondimensional groups α, β and γ_{pn}, condition (28) can be rewritten as $V_0 > L\sqrt{20N_Dk_BT_{\text{room}}/(12m_e\mu_{no}^2N_D - 3\epsilon_s)}$.

The temporal eigenmodes ω_1 and ω_2 vary lightly with the value of α when β and γ take a constant value as shown in Fig. 4. ω_1 remains stable and ω_2 unstable. ω_1 turns out to be more stable and ω_2 more unstable when β decreases, with α and γ_{pn} constant, as shown in Fig. 5. Besides, ω_1 turns to be more stable and ω_2 more unstable when γ_{pn} goes to zero

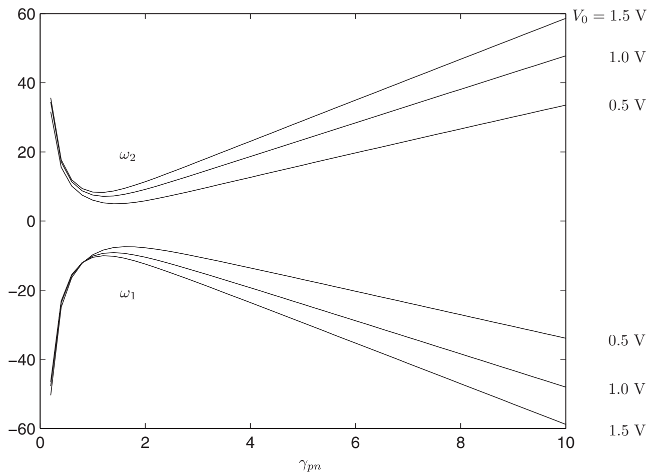

FIG. 6. One-temperature approximation without collision; ω_1 and ω_2 eigenmodes as a function of γ_{pn} at $T_{\text{room}}=300$ K.

or gets a high value as shown in Fig. 6. The applied voltage at the source, V_0, and the room temperature T_{room} are part of the tunable parameters of the system. They can be specified locally which makes them easy to impose and vary at the source and the drain. α and β cannot vary independently if just V_0 varies. Figure 6 shows the dependency of ω_1 and ω_2 with γ_{pn} for different applied voltages. ω_1 is more stable and ω_2 more unstable when the applied voltage is higher. The room temperature can vary by changing the value of β. Figure 7 shows the dependency of ω_1 and ω_2 with the absolute value of the applied voltage at the source, V_0, for different room temperatures. ω_1 is more stable and ω_2 is more unstable when T_{room} is lower and the applied voltage varies. The value of T_{room} is the condition for the electron temperature at the source as shown in Eq. (4b) with $T_1=T_{\text{room}}$.

V. CONCLUSIONS

The analysis shows that heat conduction and energy loss contribute to the stability of the electron flow in semiconductors. If thermal equilibrium between the lattice and the electrons is assumed, and the heat conduction and energy loss due to scattering of the electrons are neglected, it is possible to have an operating condition to make the system unstable.

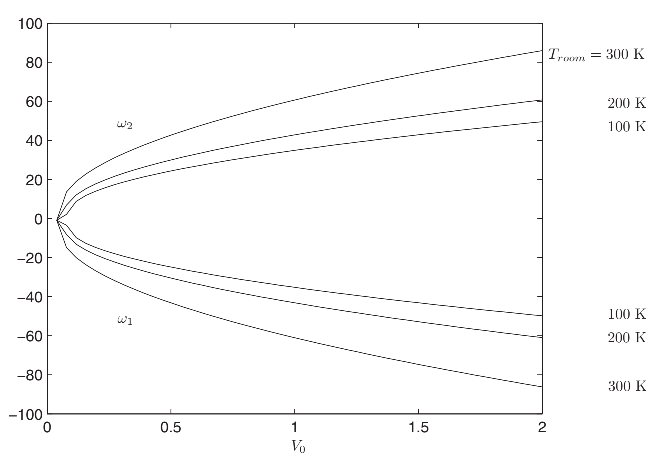

FIG. 7. One-temperature approximation without collision; ω_1 and ω_2 eigenmodes as a function of applied voltage V_0 with $\gamma_{pn}=7.331$.

The one-temperature model considers the electron temperature as a variable that can have perturbations. When α decreases, the one-temperature system becomes more unstable and the two-temperature system tends to be less stable. As a consequence, a higher applied voltage contributes to make the electron flow unstable.

ACKNOWLEDGMENTS

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APPENDIX: COEFFICIENTS OF SYSTEM OF PERTURBED DIFFERENTIAL EQUATIONS

$$A_1(x) = 1, \quad A_2(x) = -\alpha,$$

$$B_1(x) = \omega, \quad B_2(x) = \bar{u}, \quad B_3(x) = \bar{n} = 1,$$

$$C_1(x) = \frac{\bar{u}\sqrt{\alpha}}{\gamma_{pn}}, \quad C_2(x) = \frac{\sqrt{\alpha}\beta}{\gamma_{pn}} + \beta\omega + \frac{\sqrt{\alpha}\bar{T}_e}{\gamma_{pn}} + \omega\bar{T}_L,$$

$$C_3(x) = \beta^2 + \beta\bar{T}_e + \beta\bar{T}_L + \bar{T}_e\bar{T}_L,$$

$$C_4(x) = \beta + \bar{T}_L, \quad C_5(x) = -1 + \frac{d\bar{T}_e}{dx},$$

$$C_6(x) = -\beta + \frac{\bar{u}\sqrt{\alpha}\beta}{\gamma_{pn}} + \frac{\bar{u}\sqrt{\alpha}\bar{T}_e}{\gamma_{pn}} - \bar{T}_L + \beta\frac{d\bar{T}_e}{dx} + \bar{T}_L\frac{d\bar{T}_e}{dx},$$

$$C_7(x) = \bar{u}\beta + \bar{u}\bar{T}_L, \quad C_8(x) = -\beta - \bar{T}_L,$$

$$D_1(x) = \frac{\sqrt{\alpha}\beta}{\gamma_E} - \frac{2\bar{u}^2\sqrt{\alpha}}{3\gamma_{pn}} + \beta\omega + \frac{\sqrt{\alpha}\bar{T}_L}{\gamma_E} + \omega\bar{T}_L,$$

$$D_2(x) = \bar{u}\beta + \bar{u}\bar{T}_L,$$

$$D_3(x) = \frac{2\bar{u}\sqrt{\alpha}\beta}{3\gamma_E} - \frac{4\bar{u}\sqrt{\alpha}\beta}{3\gamma_{pn}} - \frac{4\bar{u}\sqrt{\alpha}\bar{T}_e}{3\gamma_{pn}} + \frac{2\bar{u}\sqrt{\alpha}\bar{T}_L}{3\gamma_E} + \beta\frac{d\bar{T}_e}{dx} + \bar{T}_L\frac{d\bar{T}_e}{dx},$$

$$D_4(x) = \frac{\bar{u}^2\sqrt{\alpha}\beta}{3\gamma_E} - \frac{2\bar{u}^2\sqrt{\alpha}\beta}{3\gamma_{pn}} + \frac{\sqrt{\alpha}\beta\bar{T}_e}{\gamma_E} - \frac{2\bar{u}^2\sqrt{\alpha}\bar{T}_e}{3\gamma_{pn}} + \frac{\bar{u}^2\sqrt{\alpha}\bar{T}_L}{3\gamma_E} - \frac{\sqrt{\alpha}\beta\bar{T}_L}{\gamma_E} + \frac{\sqrt{\alpha}\bar{T}_e\bar{T}_L}{\gamma_E} - \frac{\sqrt{\alpha}\bar{T}_L^2}{\gamma_E} + \bar{u}\beta\frac{d\bar{T}_e}{dx} + \bar{u}\bar{T}_L\frac{d\bar{T}_e}{dx},$$

$$D_5(x) = \frac{2}{3}\beta^2 + \frac{2}{3}\beta\bar{T}_e + \frac{2}{3}\beta\bar{T}_L + \frac{2}{3}\bar{T}_e\bar{T}_L,$$

$$D_6(x) = -\frac{2}{3}(\beta\sigma + \sigma\bar{T}_L),$$

$$D_7(x) = \frac{\bar{u}^2\sqrt{\alpha}}{3\gamma_E} - \frac{\sqrt{\alpha}\beta}{\gamma_E} + \frac{\sqrt{\alpha}\bar{T}_e}{\gamma_E} - \frac{2\sqrt{\alpha}\bar{T}_L}{\gamma_E} + \bar{u}\frac{d\bar{T}_e}{dx} - \frac{2}{3}\sigma\frac{d^2\bar{T}_e}{dx^2},$$

$$E_1(x) = -\theta_2, \quad E_2(x) = \theta_2 + \omega,$$

$$E_3(x) = -\frac{\bar{u}^2\theta_2}{3} - \theta_2\bar{T}_e + \theta_2\bar{T}_L,$$

$$E_4(x) = -\frac{2}{3}\bar{u}\theta_2, \quad E_5(x) = -\theta_1.$$

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