Stark-effect scattering in rough quantum wells

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A scattering mechanism stemming from the Stark-shift of energy levels by electric fields due to interface roughness in semiconductor quantum wells is identified. This scattering mechanism feeds off interface roughness and electric fields and modifies the well known “sixth-power” law of electron mobility degradation. This work first treats Stark-effect scattering in rough quantum wells as a perturbation for small electric fields and then directly absorbs it into the Hamiltonian for large fields. The major result is the existence of a window of quantum well widths for which the combined roughness scattering is minimum. Carrier scattering and mobility degradation in wide quantum wells are thus expected to be equally severe as in narrow wells due to Stark-effect scattering in electric fields. © 2011 American Institute of Physics. [doi:10.1063/1.3607485]

High-mobility 2-dimensional electron gases (2DEGs) have proven invaluable for fundamental discoveries in condensed matter physics such as the quantum Hall effect, quantized conductance, and ballistic transport among many others, and thus continuous improvement in the mobilities and mean free paths of carriers are highly desirable. For high-speed and low-power field-effect transistors (FETs), a high degree of vertical scaling is essential to support lateral scaling, requiring one to move towards highly confined 2DEGs in ultrathin quantum wells such as in silicon-on-insulator (SOI) and III-V quantum well (QW) FETs (Refs. 3 and 4) to avoid short-channel effects. Thus, interface roughness scattering assumes increasing importance in high-performance transistors. In their seminal work in 1987 Sakaki et al. identified the importance of interface roughness scattering on electron transport in 2DEGs confined in narrow quantum wells. They showed that in the presence of quantum well width ($L_w$) variations in the 2-dimensional (2D) plane, the electron mobility ($\mu$) limited by interface roughness (IR) scattering degrades in thin wells as the sixth power of the well-width ($\mu_{IR} \sim L_w^6$).

Sakaki et al. assumed a QW with no electric field. In typical QW FETs, the electric field indeed goes to zero when the carriers are depleted (when the device is in the “off” state) and increases to high values in the “on” state of the device. For high-performance devices, a high 2DEG density is essential for boosting the drive current—which results in high electric fields in the QW. In this work, we show that the electric field in the QW leads to an enhanced quantum-confined “Stark-effect” scattering that feeds off interface roughness and degrades electron mobility in rough quantum wells. We first evaluate the effect of Stark-effect scattering in a QW in cases where the potential fluctuation due to the electric field is small enough to be treated as a perturbation. Then, we discuss situations where the field is so large that a perturbative treatment does not do justice, and a modified treatment that treats IR + Stark-effect scattering on equal footing captures the role of this mobility degradation mechanism. We note that this form of scattering is incorporated in recent numerical approaches (see Ref. 6), therefore the purpose of this work is to offer an analytical framework for clear visualization of the physics and for ease of design.

The central problem is illustrated in Fig. 1. Following Ref. 5, the QW is visualized to be of width $L_w(r) = L_w + \Delta(r)$, where $r = (x, y)$ is the in-plane coordinate and $\Delta(r)$ is the fluctuation function with a correlation $\langle \Delta(r) \Delta(r + r') \rangle = \Delta^2 \exp[-(r'/L)^2]$ and mean $\langle \Delta(r) \rangle_r = 0$. The QW width fluctuation is parametrized by the height $\Delta$ and the in-plane correlation length $L$ as shown in Fig. 1. Assuming an infinite quantum well, the ground-state ($n = 1$) energy at zero vertical field $(F_w = 0)$ is $E_1(F_w = 0) = \pi^2 \hbar^2 / 2m^* L_w^2$, where $\hbar = h/2\pi$ is the reduced Planck’s constant and $m^*$ is the electron effective mass. Variations in the QW width by $\Delta(r)$ changes the ground state energy by

$$\delta E_1(r, F_w = 0) = \frac{\partial E_1(0)}{\partial L_w} \Delta(r) = - \frac{\hbar^2 \pi^2}{m^* L_w^2} \Delta(r),$$

which was the premise of Ref. 5, leading to a $\sim L_w^6$ mobility variation. In the presence of an electric field in the well, the ground state energy shifts. This quantum-confined Stark-

![FIG. 1. Schematic figure illustrating interface roughness. (a) and (b) are square QWs without and with an electric field respectively and (c) is the case of a triangular QW at a high field. Dashed lines indicate wider wells and corresponding eigenvalue fluctuations. Interface roughness parameters $\Delta$ and $L$ are illustrated.](image-url)
Moving to radial coordinates and using the property of the delta function, the integral converts to one over scattering angles

$$\frac{1}{\tau_m(k)} = \frac{m^* \hbar^2 \Delta^2 L^2}{2 \hbar^2} \int_0^{2\pi} d\theta \frac{e^{-q^2L^2\sin^2\theta}}{\epsilon_{SD}^2(2k \sin \frac{\theta}{2})(1 - \cos \theta)}.$$ (4)

For typically degenerate 2DEG carriers, transport occurs at the Fermi level, and averaging the momentum relaxation rate over the carrier distribution amounts to evaluating it at the Fermi wavevector determined by the 2DEG carrier density $|k| = k_F = \sqrt{2\pi n_e}$. The net electron mobility is then obtained as $\mu_{BE} = e\sigma(k_F)/m^*$. Using the same example as in Ref. 5, we choose 2DEGs in GaAs QWs (with $m^* = 0.067m_0$, $\Delta = 2.83A$, $\epsilon_e = 12.9$) to illustrate the effect of Stark-effect scattering. The results are shown in Fig. 2.

FIG. 2. (a) Mobility as a function of well width $L_w$ for various electric fields $F_w$. Dashed line indicates a decreasing peak mobility with increasing applied electric field. (b) Mobility with correlation length $L$ for various electric fields. (c) Mobility vs. electric field for various QW thicknesses.
more prudent to absorb the field $F$ directly into the unperturbed Hamiltonian. To do so, we assume the unperturbed Hamiltonian of the form $H_0 = -\hbar^2 \nabla^2 + e F z$, which yields eigenvalues $E_n = (\hbar^2/2m^*)^{1/3} (3\pi e F/2)^{2/3} (n + 3/4)^{2/3}$ for the $n$th eigenstate, with corresponding eigenfunctions as Airy functions $\psi_n(z) = A_n[(2m^*/\hbar^2)(eFz - E_n)]^{1/3}$. As shown in Refs. 1 and 9, the Airy-eigenfunction can be closely approximated by the Fang-Howard variational function $\psi_1(z) = \sqrt{b^2/2} e^{-b/2} F z$ for the ground state, where $b = (33m^* e^2 n_2/8\hbar^2 e) ^{1/3}$ is the variational parameter of inverse length unit. We note that whereas the Airy function assumes a combination of polar optical phonon (POP) and acoustic phonon (AP) and IR scattering using Matthiessen’s rule. Very high polarization-induced fields exist in AlN/GaN polar heterostructures. In such structures, the effect of increased IR scattering at room temperature reduces an intrinsic phonon-limited mobility of $\sim 2200$ cm$^2$/V·s for $F_z = 1.8$ MV/cm ($n_2 \sim 10^{12}$ cm$^{-2}$) to $\sim 1600$ cm$^2$/V·s for $F_z = 5.5$ MV/cm ($n_2 \sim 3 \times 10^{13}$ cm$^{-2}$). These numbers are in good agreement with experiments and indicate the strong mobility degradation by Stark-effect scattering. In a high electron mobility transistor (HEMT)-type device, the implication is that the electron mobility will initially increase as the gate pinches off the channel due to the reduction of IR/Stark scattering but saturates below a certain density due to intrinsic phonon scattering limitations.

In summary, this letter identifies and quantitatively evaluates the effect of Stark-effect scattering in the presence of an electric field on electron mobility in rough quantum wells. When the field is small, a perturbative treatment shows that mobility reduces as $\mu_{IR} \sim L_n^5$ for thin wells ($L_n < L_n^5$) but switches over to $\mu_{IR} \sim L_n^0$ above this critical width. The implication is that Stark-effect scattering enforces a window of QW widths for high mobility. On the other hand, for QWs where the field is too high to be treated perturbatively (such as in highly polar AlN/GaN QWs), the IR limited mobility degrades as the square of the peak electric field ($\mu_{IR} \sim 1/F_z^2$) resulting in low mobilities for high carrier densities. Since Stark-effect scattering feeds off interface roughness, it can be reduced by making smoother interfaces. For a given interface roughness, it can be reduced by careful band-diagram engineering such that the field in the QW or at the heterojunction is minimized.

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7. If the electric field is not constant, the energy shift $\Delta E = (\int \psi_n^2(z) e F_z(z) dz)^2 / (E_1 - E_n)$ is calculated by generalized 2nd order perturbation theory using the varying electric field along the well and the stark shift depends on the field distribution.

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